Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

3-Logarithms/61-3.2.3-u-log-e-f-a+b-x-^p-c+d-x-^q-^r-^s

Nasser M. Abbasi

December 8, 2023 Compiled on December 8, 2023 at 6:07pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [108]. This is test number [61].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.07 (107)	0.93 (1)
Rubi	98.15 (106)	1.85 (2)
Maxima	62.96 (68)	37.04 (40)
Maple	41.67 (45)	58.33 (63)
Fricas	37.96 (41)	62.04 (67)
Giac	33.33 (36)	66.67 (72)
Mupad	32.41 (35)	67.59 (73)
Sympy	18.52 (20)	81.48 (88)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

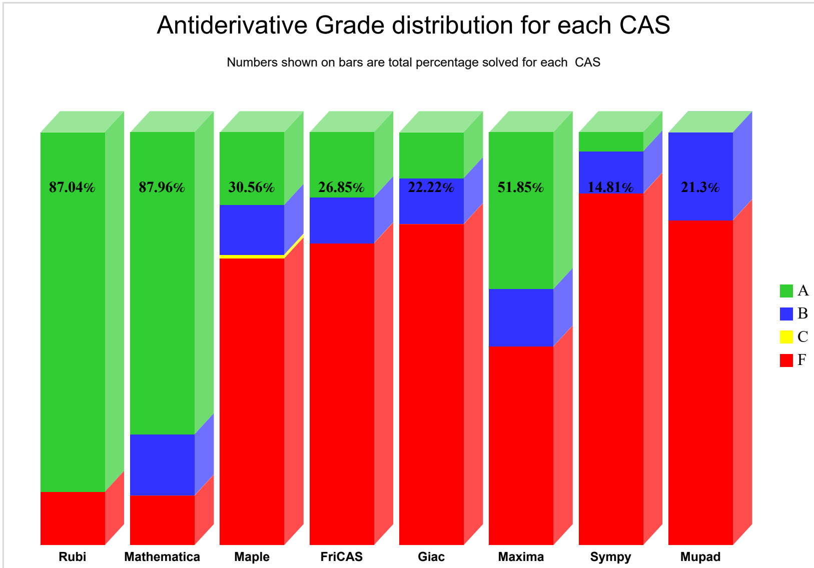
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

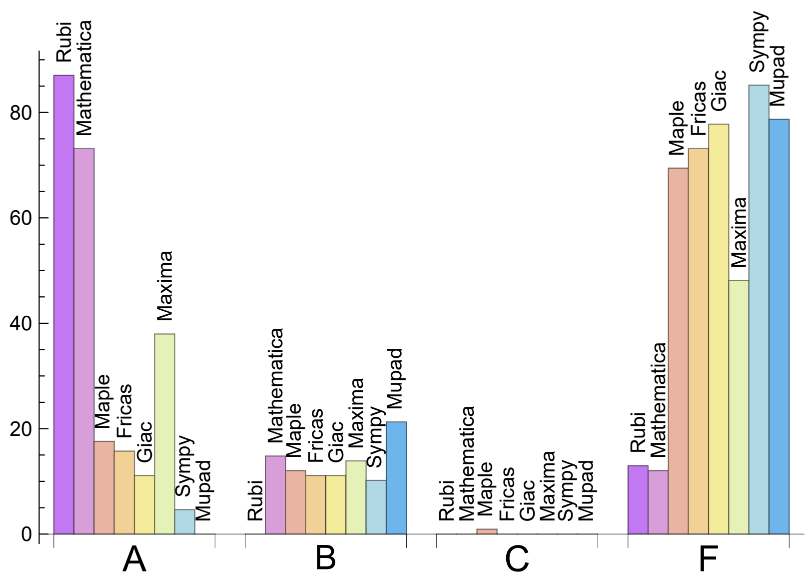
System	% A grade	% B grade	% C grade	% F grade
Rubi	87.037	0.000	0.000	12.963
Mathematica	73.148	14.815	0.000	12.037
Maxima	37.963	13.889	0.000	48.148
Maple	17.593	12.037	0.926	69.444
Fricas	15.741	11.111	0.000	73.148
Giac	11.111	11.111	0.000	77.778
Sympy	4.630	10.185	0.000	85.185
Mupad	0.000	21.296	0.000	78.704

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	1	100.00	0.00	0.00
Rubi	2	100.00	0.00	0.00
Maxima	40	75.00	0.00	25.00
Maple	63	100.00	0.00	0.00
Fricas	67	95.52	4.48	0.00
Giac	72	97.22	2.78	0.00
Mupad	73	0.00	100.00	0.00
Sympy	88	37.50	56.82	5.68

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.45
Mathematica	0.49
Rubi	0.79
Fricas	2.07
Mupad	2.28
Giac	2.45
Sympy	30.14
Maple	36.64

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	163.75	2.17	65.00	1.98
Fricas	186.63	1.66	59.00	1.29
Maple	240.00	1.88	130.00	1.15
Mupad	275.66	1.69	50.00	1.10
Giac	310.47	2.12	63.50	1.13
Rubi	326.62	0.97	168.50	1.00
Maxima	453.97	2.19	199.00	1.48
Mathematica	1211.81	2.10	164.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

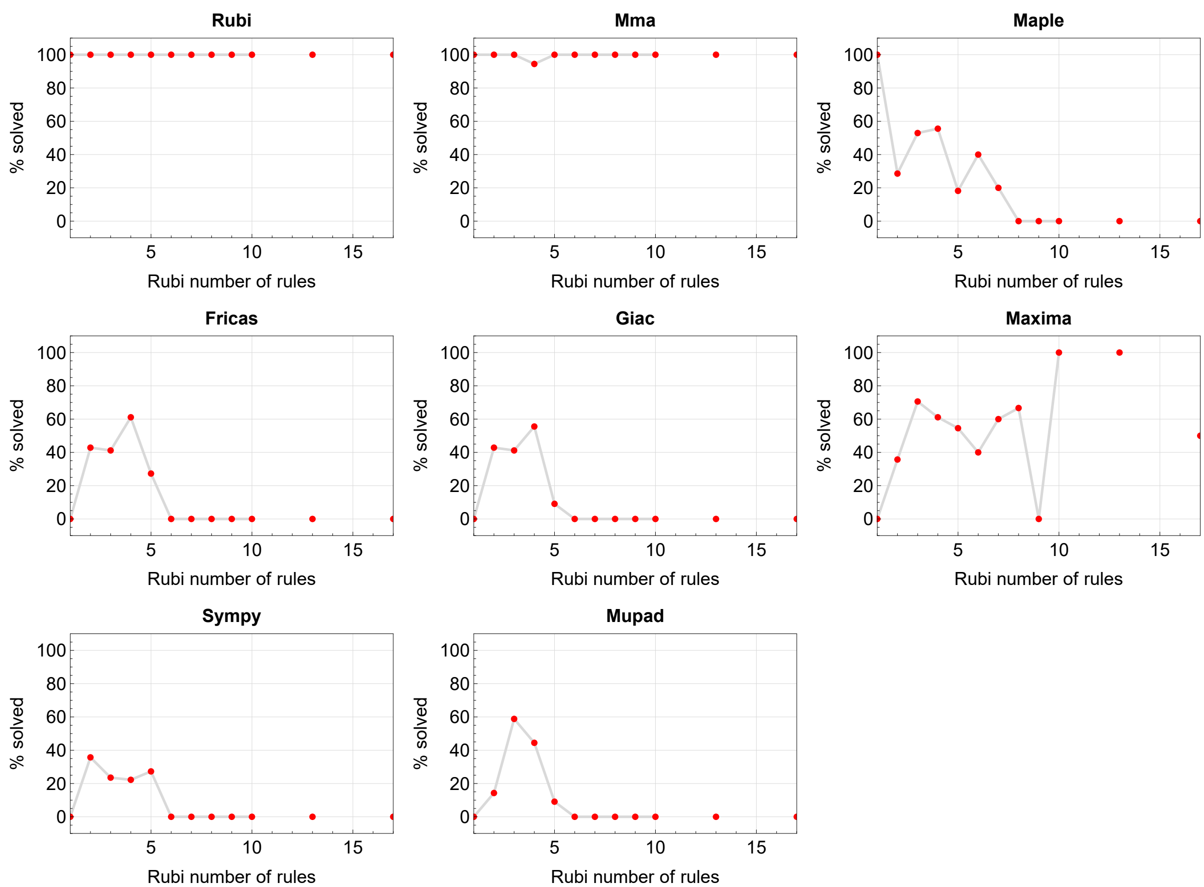


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

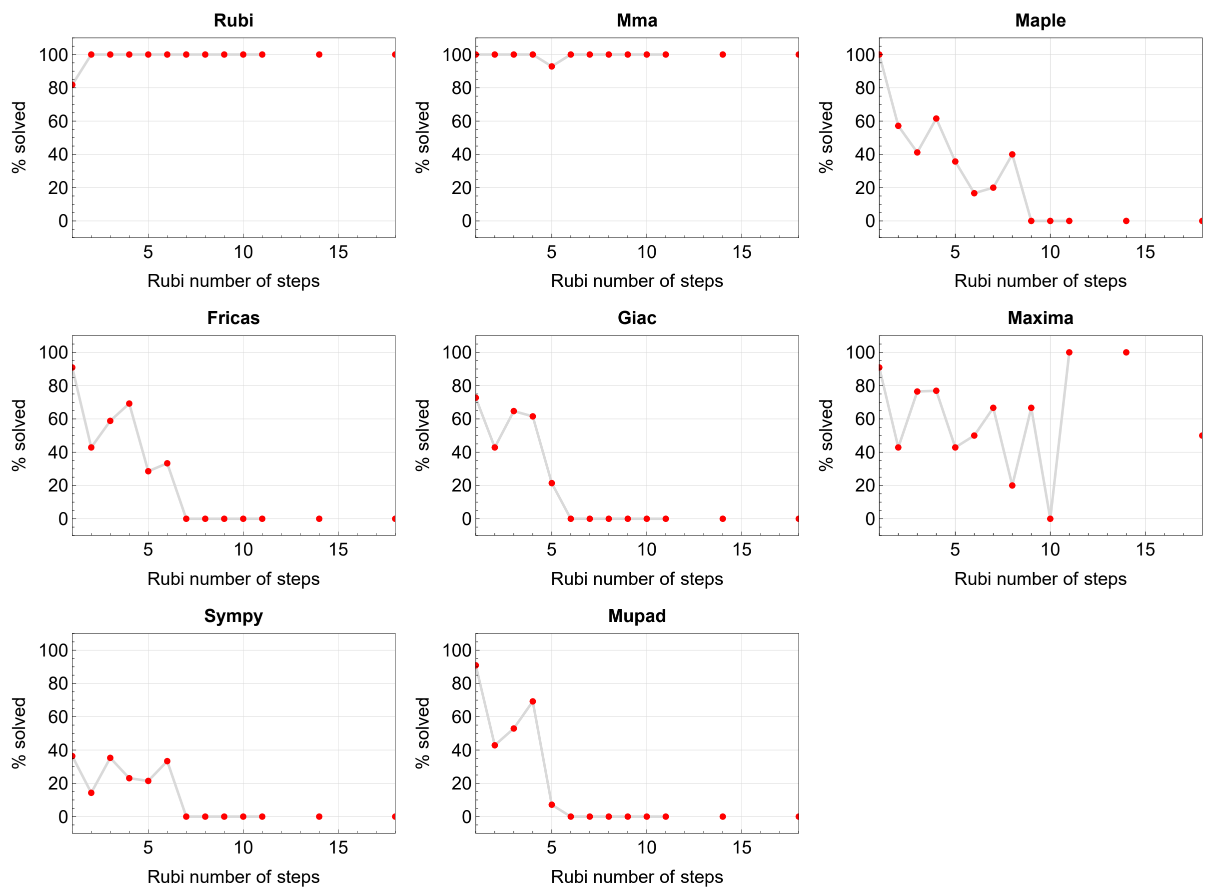


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

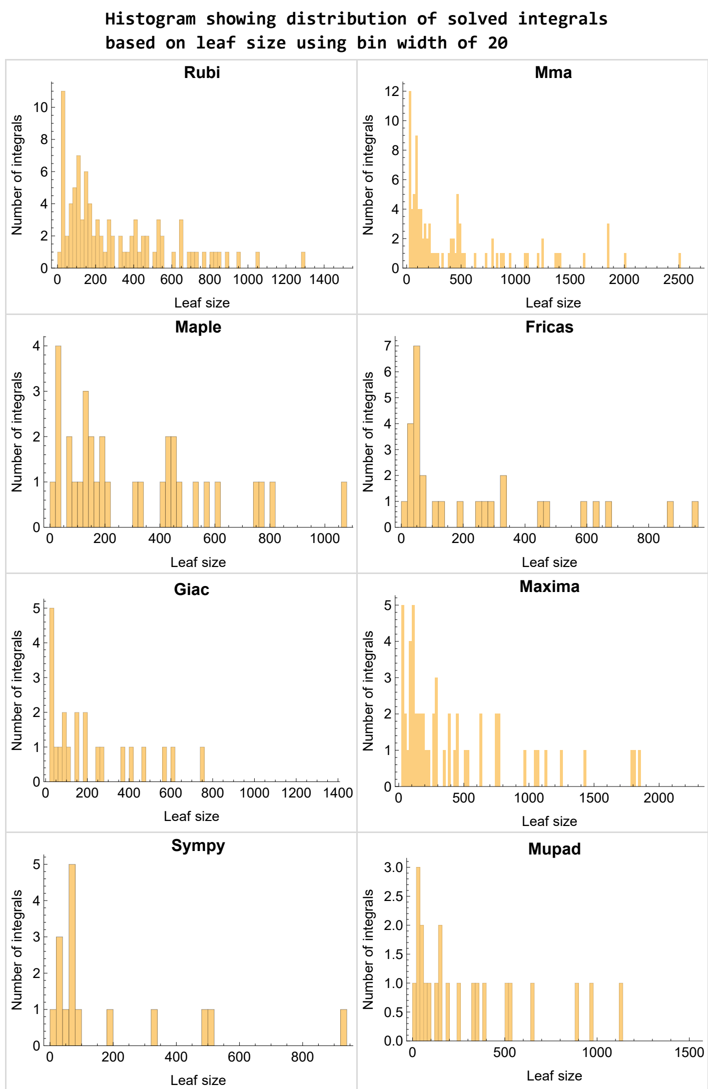


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

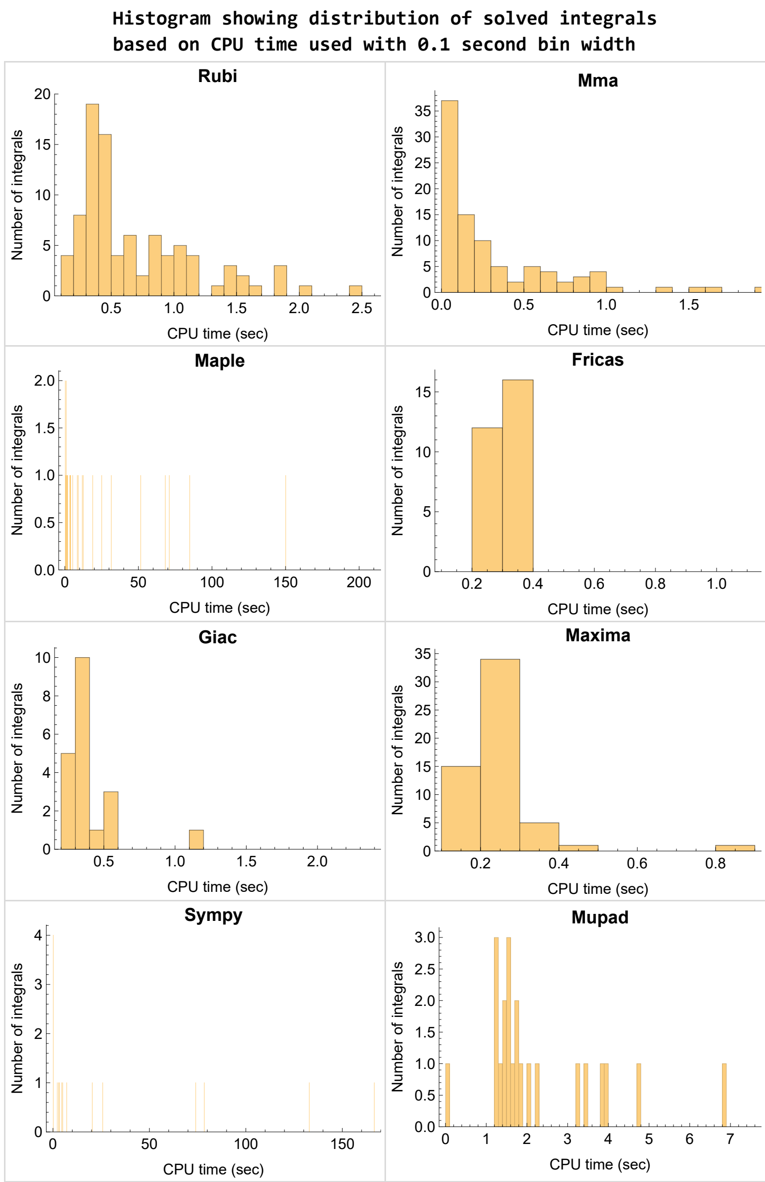


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

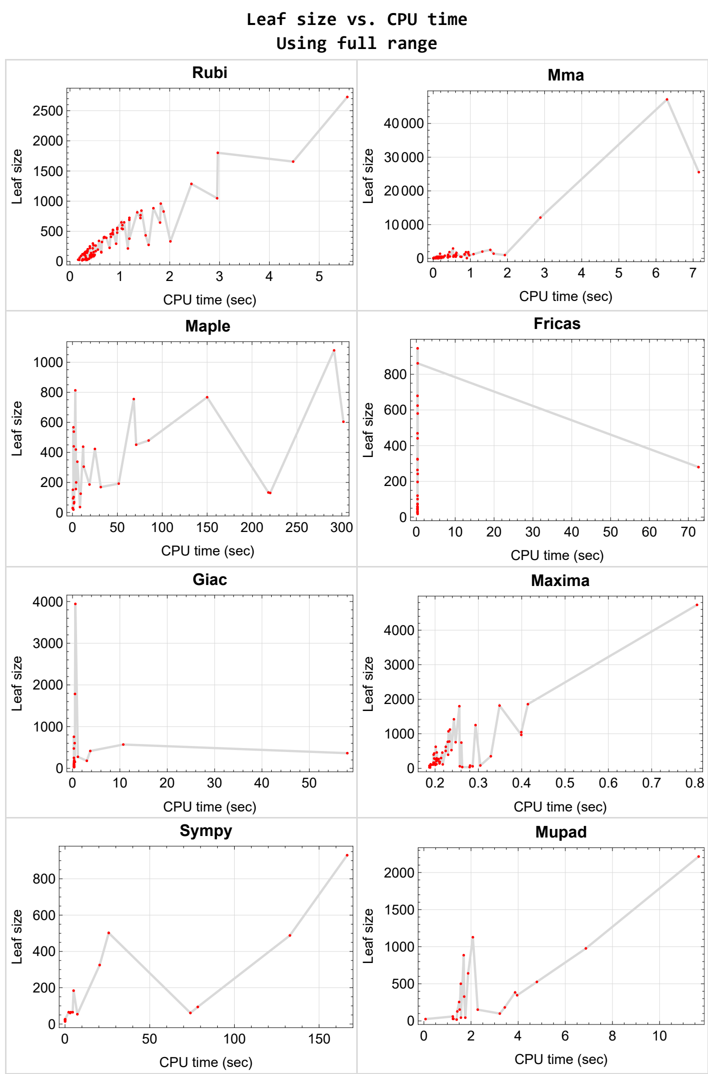


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{44, 45, 46, 50, 89\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

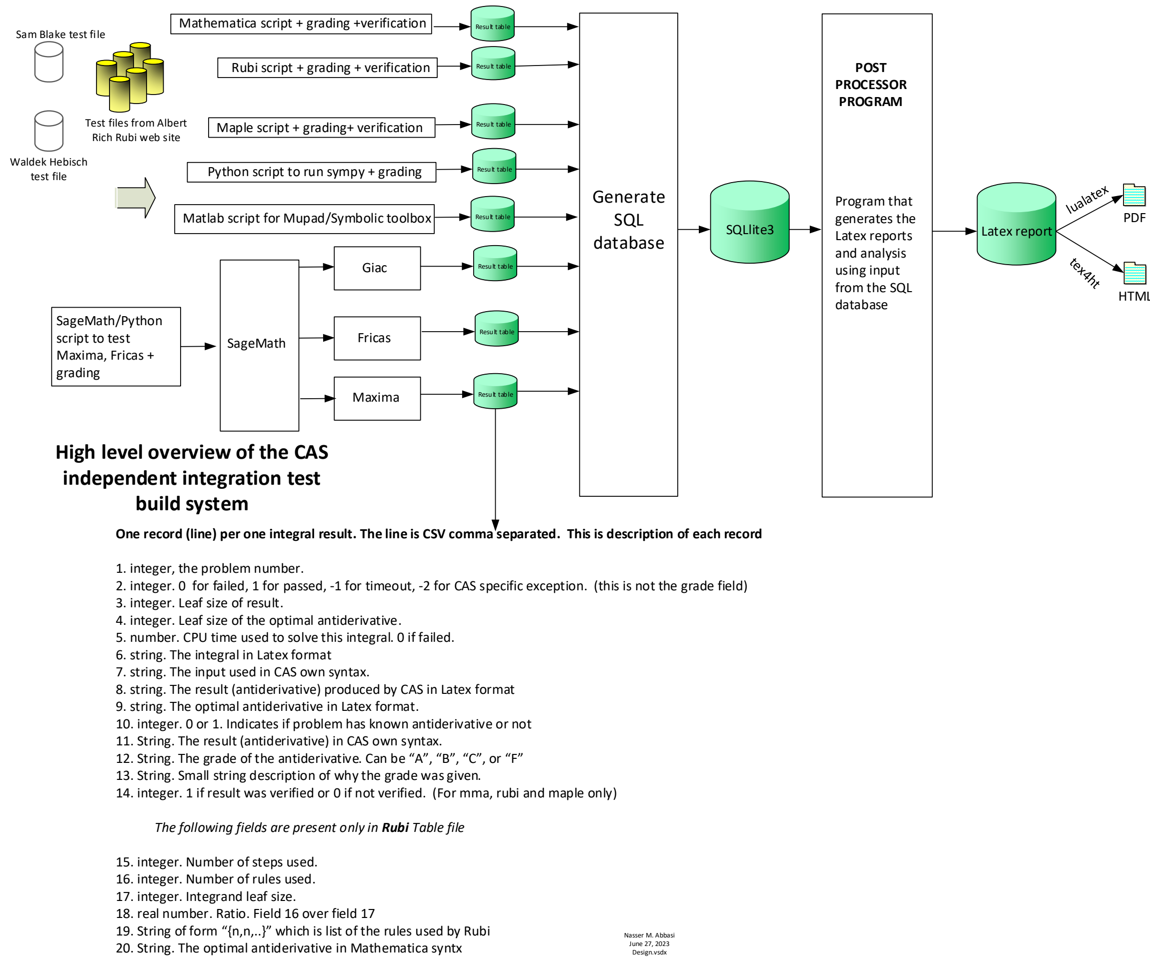
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade { }

C grade { }

F normal fail { 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 59, 64, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105 }

B grade { 16, 17, 24, 40, 41, 42, 51, 56, 57, 58, 68, 69, 92, 106, 107, 108 }

C grade { }

F normal fail { 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 11, 12, 29, 30, 53, 59, 64, 76, 78, 80, 88, 89, 91, 93, 96, 99, 102, 103, 106 }

B grade { 8, 9, 10, 13, 26, 27, 28, 31, 74, 75, 104, 105, 107 }

C grade { 50 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 56, 57, 58, 67, 68, 69, 77, 79, 81, 82, 83, 84, 85, 86, 87, 90, 92, 94, 95, 97, 98, 100, 101, 108 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 10, 12, 28, 29, 43, 46, 47, 48, 49, 50, 74, 75, 89, 93, 96, 99, 106 }

B grade { 7, 8, 9, 13, 14, 15, 25, 26, 27, 31, 44, 45 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 107, 108 }

F(-1) timedout fail { 32, 33, 34 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 47, 48, 53, 59, 74, 75, 79, 93, 94, 96, 97, 99, 100 }

B grade { 7, 15, 24, 25, 34, 42, 44, 45, 46, 49, 50, 77, 81, 88, 89 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 20, 39, 43, 51, 52, 56, 57, 58, 67, 68, 69, 76, 78, 80, 90, 91, 92, 95, 98, 101, 102, 103, 104, 105 }

F(-1) timedout fail { }

F(-2) exception fail { 64, 82, 83, 84, 85, 86, 87, 106, 107, 108 }

2.1.6 Giac

A grade { 10, 12, 13, 27, 28, 29, 31, 43, 47, 48, 96, 99 }

B grade { 7, 8, 9, 14, 15, 32, 33, 34, 46, 49, 50, 93 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 35, 37, 38, 39, 40, 41, 42, 44, 45, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

F(-1) timedout fail { 26, 36 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 7, 8, 9, 10, 12, 13, 14, 15, 25, 26, 27, 28, 29, 31, 32, 33, 34, 74, 75, 89, 93, 96, 99 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 47, 89, 93, 96, 99 }

B grade { 9, 10, 27, 28, 29, 43, 44, 45, 46, 48, 50 }

C grade { }

F normal fail { 1, 2, 3, 11, 17, 18, 19, 20, 21, 22, 23, 24, 35, 36, 37, 38, 59, 64, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 103, 104, 105, 106, 107 }

F(-1) timedout fail { 4, 5, 6, 7, 8, 13, 14, 15, 16, 25, 26, 30, 31, 32, 33, 34, 39, 40, 41, 42, 49, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 67, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 108 }

F(-2) exception fail { 12, 72, 73, 74, 75 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	404	404	336	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.662	0.275	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	217	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	135	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	185	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	295	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.627	0.272	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	531	531	470	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.918	0.608	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	184	185	0	395	624	0	570	886
N.S.	1	0.92	0.92	0.00	1.97	3.10	0.00	2.84	4.41
time (sec)	N/A	0.344	0.194	0.000	0.197	0.315	0.000	10.717	1.691

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	160	154	604	285	469	0	415	501
N.S.	1	0.93	0.90	3.51	1.66	2.73	0.00	2.41	2.91
time (sec)	N/A	0.306	0.135	301.793	0.198	0.297	0.000	3.743	1.568

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	136	127	451	194	325	488	275	255
N.S.	1	0.95	0.89	3.15	1.36	2.27	3.41	1.92	1.78
time (sec)	N/A	0.282	0.092	70.987	0.209	0.287	132.776	1.128	1.497

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	112	105	305	118	197	325	152	128
N.S.	1	0.97	0.91	2.63	1.02	1.70	2.80	1.31	1.10
time (sec)	N/A	0.251	0.139	12.557	0.190	0.317	20.453	0.503	1.419

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	110	93	125	164	0	0	0	0
N.S.	1	1.03	0.87	1.17	1.53	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.070	9.167	0.198	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	91	89	169	99	120	0	113	99
N.S.	1	0.96	0.94	1.78	1.04	1.26	0.00	1.19	1.04
time (sec)	N/A	0.223	0.041	31.532	0.201	0.302	0.000	0.293	3.222

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	119	116	767	165	323	0	250	182
N.S.	1	0.88	0.86	5.68	1.22	2.39	0.00	1.85	1.35
time (sec)	N/A	0.280	0.146	150.010	0.196	0.317	0.000	0.292	3.434

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	144	141	0	289	580	0	475	346
N.S.	1	0.88	0.86	0.00	1.76	3.54	0.00	2.90	2.11
time (sec)	N/A	0.306	0.221	0.000	0.204	0.344	0.000	0.281	3.958

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	169	164	0	459	861	0	756	526
N.S.	1	0.88	0.85	0.00	2.38	4.46	0.00	3.92	2.73
time (sec)	N/A	0.342	0.210	0.000	0.204	0.348	0.000	0.290	4.795

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	920	843	2508	0	1421	0	0	0	0
N.S.	1	0.92	2.73	0.00	1.54	0.00	0.00	0.00	0.00
time (sec)	N/A	1.364	1.538	0.000	0.244	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	805	723	1853	0	1071	0	0	0	0
N.S.	1	0.90	2.30	0.00	1.33	0.00	0.00	0.00	0.00
time (sec)	N/A	1.190	0.945	0.000	0.231	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	686	601	1211	0	769	0	0	0	0
N.S.	1	0.88	1.77	0.00	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	1.008	0.606	0.000	0.229	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	540	477	781	0	504	0	0	0	0
N.S.	1	0.88	1.45	0.00	0.93	0.00	0.00	0.00	0.00
time (sec)	N/A	0.877	0.398	0.000	0.225	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	431	432	460	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.460	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	465	407	411	0	392	0	0	0	0
N.S.	1	0.88	0.88	0.00	0.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.812	0.396	0.000	0.230	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	632	545	872	0	755	0	0	0	0
N.S.	1	0.86	1.38	0.00	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.997	0.624	0.000	0.247	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	764	690	1407	0	1252	0	0	0	0
N.S.	1	0.90	1.84	0.00	1.64	0.00	0.00	0.00	0.00
time (sec)	N/A	1.180	0.944	0.000	0.293	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	884	814	2003	0	1816	0	0	0	0
N.S.	1	0.92	2.27	0.00	2.05	0.00	0.00	0.00	0.00
time (sec)	N/A	1.322	1.322	0.000	0.349	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	334	298	275	0	624	945	0	0	1128
N.S.	1	0.89	0.82	0.00	1.87	2.83	0.00	0.00	3.38
time (sec)	N/A	0.440	0.201	0.000	0.202	0.326	0.000	0.000	2.079

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	250	231	1079	431	679	0	0	641
N.S.	1	0.91	0.84	3.91	1.56	2.46	0.00	0.00	2.32
time (sec)	N/A	0.398	0.177	291.420	0.198	0.296	0.000	0.000	1.878

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	202	209	755	269	441	930	363	328
N.S.	1	0.93	0.96	3.46	1.23	2.02	4.27	1.67	1.50
time (sec)	N/A	0.351	0.142	68.290	0.200	0.316	166.570	58.017	1.711

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	154	120	438	143	242	502	180	153
N.S.	1	0.96	0.75	2.74	0.89	1.51	3.14	1.12	0.96
time (sec)	N/A	0.291	0.132	11.889	0.199	0.345	25.799	3.018	1.520

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	75	72	184	69	60
N.S.	1	1.00	0.93	1.00	1.23	1.18	3.02	1.13	0.98
time (sec)	N/A	0.192	0.041	1.665	0.187	0.311	5.056	0.314	1.228

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	154	163	186	186	0	0	0	0
N.S.	1	1.04	1.10	1.26	1.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	0.062	18.984	0.211	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	120	93	479	123	280	0	191	152
N.S.	1	0.94	0.73	3.74	0.96	2.19	0.00	1.49	1.19
time (sec)	N/A	0.234	0.088	84.840	0.200	72.606	0.000	0.345	2.285

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	168	206	0	232	0	0	603	384
N.S.	1	0.83	1.02	0.00	1.15	0.00	0.00	2.99	1.90
time (sec)	N/A	0.342	0.225	0.000	0.205	0.000	0.000	0.448	3.871

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	220	254	0	456	0	0	1783	977
N.S.	1	0.85	0.98	0.00	1.75	0.00	0.00	6.86	3.76
time (sec)	N/A	0.389	0.395	0.000	0.217	0.000	0.000	0.519	6.873

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	318	268	480	0	776	0	0	3943	2215
N.S.	1	0.84	1.51	0.00	2.44	0.00	0.00	12.40	6.97
time (sec)	N/A	0.439	0.733	0.000	0.233	0.000	0.000	0.599	11.656

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2240	1802	1386	0	1799	0	0	0	0
N.S.	1	0.80	0.62	0.00	0.80	0.00	0.00	0.00	0.00
time (sec)	N/A	2.913	1.627	0.000	0.256	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1645	1286	899	0	1123	0	0	0	0
N.S.	1	0.78	0.55	0.00	0.68	0.00	0.00	0.00	0.00
time (sec)	N/A	2.377	0.971	0.000	0.234	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1063	828	480	0	623	0	0	0	0
N.S.	1	0.78	0.45	0.00	0.59	0.00	0.00	0.00	0.00
time (sec)	N/A	1.809	0.540	0.000	0.225	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	225	389	0	298	0	0	0	0
N.S.	1	0.84	1.45	0.00	1.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.767	0.124	0.000	0.214	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1471	2728	1370	0	0	0	0	0	0
N.S.	1	1.85	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.271	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	832	644	2930	0	745	0	0	0	0
N.S.	1	0.77	3.52	0.00	0.90	0.00	0.00	0.00	0.00
time (sec)	N/A	1.814	0.533	0.000	0.261	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1304	1048	12086	0	1857	0	0	0	0
N.S.	1	0.80	9.27	0.00	1.42	0.00	0.00	0.00	0.00
time (sec)	N/A	2.995	2.886	0.000	0.414	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1957	1656	47127	0	4732	0	0	0	0
N.S.	1	0.85	24.08	0.00	2.42	0.00	0.00	0.00	0.00
time (sec)	N/A	4.464	6.296	0.000	0.804	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	B	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	56	94	38	0
N.S.	1	1.00	1.00	0.00	0.00	1.33	2.24	0.90	0.00
time (sec)	N/A	0.367	0.025	0.000	0.000	0.319	78.374	0.298	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	B	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	25	37	0	526	101	65	0	0
N.S.	1	0.68	1.00	0.00	14.22	2.73	1.76	0.00	0.00
time (sec)	N/A	0.331	0.009	0.000	0.238	0.309	4.506	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	B	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	25	37	0	268	74	65	0	0
N.S.	1	0.68	1.00	0.00	7.24	2.00	1.76	0.00	0.00
time (sec)	N/A	0.326	0.010	0.000	0.208	0.295	3.364	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	B	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	35	37	0	105	47	61	86	0
N.S.	1	0.95	1.00	0.00	2.84	1.27	1.65	2.32	0.00
time (sec)	N/A	0.300	0.007	0.000	0.195	0.299	3.014	0.317	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	36	30	54	31	0
N.S.	1	1.00	1.00	0.00	1.06	0.88	1.59	0.91	0.00
time (sec)	N/A	0.340	0.144	0.000	0.263	0.304	7.281	0.328	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	34	29	61	34	0
N.S.	1	1.00	1.00	0.00	1.00	0.85	1.79	1.00	0.00
time (sec)	N/A	0.341	0.010	0.000	0.280	0.276	74.019	0.349	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	80	59	0	85	0
N.S.	1	1.00	1.00	0.00	2.16	1.59	0.00	2.30	0.00
time (sec)	N/A	0.337	0.009	0.000	0.281	0.280	0.000	0.310	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	30	151	83	24	65	58	0
N.S.	1	0.93	1.00	5.03	2.77	0.80	2.17	1.93	0.00
time (sec)	N/A	0.277	0.008	0.487	0.305	0.291	2.155	0.306	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	386	958	0	0	0	0	0	0
N.S.	1	0.94	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.190	1.929	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	302	436	0	0	0	0	0	0
N.S.	1	0.99	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.878	0.758	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	166	192	204	0	0	0	0
N.S.	1	1.00	0.97	1.12	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	0.087	51.527	0.203	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	48	50	57	0	50	50
N.S.	1	1.00	1.04	1.00	1.04	1.19	0.00	1.04	1.04
time (sec)	N/A	0.228	0.512	0.394	3.443	0.310	0.000	0.326	1.606

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	48	222	91	0	50	50
N.S.	1	1.00	1.04	1.00	4.62	1.90	0.00	1.04	1.04
time (sec)	N/A	0.232	1.505	0.276	2.930	0.300	0.000	0.352	1.753

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	332	1241	0	0	0	0	0	0
N.S.	1	1.01	3.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.018	1.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	274	839	0	0	0	0	0	0
N.S.	1	1.05	3.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.539	0.551	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	214	451	0	0	0	0	0	0
N.S.	1	1.10	2.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.155	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	93	78	103	126	0	0	0	0
N.S.	1	1.15	0.96	1.27	1.56	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.047	1.368	0.196	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	41	39	41	41	0	41	41
N.S.	1	1.00	1.05	1.00	1.05	1.05	0.00	1.05	1.05
time (sec)	N/A	0.622	0.308	1.010	3.373	0.307	0.000	0.372	1.479

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	41	39	231	41	0	41	41
N.S.	1	1.00	1.05	1.00	5.92	1.05	0.00	1.05	1.05
time (sec)	N/A	0.570	1.612	1.044	3.123	0.311	0.000	0.358	1.510

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	272	30	71	30	31
N.S.	1	1.00	1.07	1.00	9.71	1.07	2.54	1.07	1.11
time (sec)	N/A	0.187	0.084	1.818	0.274	0.296	4.145	0.585	1.360

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	154	30	66	30	31
N.S.	1	1.00	1.07	1.00	5.50	1.07	2.36	1.07	1.11
time (sec)	N/A	0.184	0.070	0.554	0.268	0.301	6.484	0.415	1.224

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	95	66	94	0	0	0	0	0
N.S.	1	1.16	0.80	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	0.030	0.694	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	32	30	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	1.14	1.07	1.11
time (sec)	N/A	0.187	0.079	0.444	0.247	0.289	27.260	0.351	1.166

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	83	30	126	30	31
N.S.	1	1.00	1.07	1.00	2.96	1.07	4.50	1.07	1.11
time (sec)	N/A	0.185	0.231	33.559	0.245	0.291	41.297	0.351	1.211

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	620	647	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	496	519	25557	0	0	0	0	0	0
N.S.	1	1.05	51.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.850	7.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	391	1842	0	0	0	0	0	0
N.S.	1	1.05	4.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.723	0.881	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	47	45	47	54	0	47	47
N.S.	1	1.00	1.04	1.00	1.04	1.20	0.00	1.04	1.04
time (sec)	N/A	0.778	1.243	0.250	1.719	0.300	0.000	0.413	1.353

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	47	45	181	54	0	47	47
N.S.	1	1.00	1.04	1.00	4.02	1.20	0.00	1.04	1.04
time (sec)	N/A	0.361	0.798	0.053	0.741	0.304	0.000	0.318	1.302

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	50	154	62	0	50	50
N.S.	1	1.00	1.04	1.04	3.21	1.29	0.00	1.04	1.04
time (sec)	N/A	0.751	0.512	0.936	0.269	0.284	0.000	0.499	1.937

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	50	155	61	0	50	50
N.S.	1	1.00	1.04	1.04	3.23	1.27	0.00	1.04	1.04
time (sec)	N/A	0.736	0.392	0.992	0.267	0.301	0.000	0.545	1.774

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	A	A	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	44	133	59	50	0	0	44
N.S.	1	0.00	0.98	2.96	1.31	1.11	0.00	0.00	0.98
time (sec)	N/A	0.000	0.905	218.217	0.257	0.298	0.000	0.000	1.758

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	A	A	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	45	130	58	49	0	0	44
N.S.	1	0.00	1.00	2.89	1.29	1.09	0.00	0.00	0.98
time (sec)	N/A	0.000	0.056	220.254	0.287	0.285	0.000	0.000	1.572

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	536	461	538	0	0	0	0	0
N.S.	1	0.96	0.82	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.092	0.257	1.516	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	550	523	467	0	1047	0	0	0	0
N.S.	1	0.95	0.85	0.00	1.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.902	0.176	0.000	0.399	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	402	413	440	0	0	0	0	0
N.S.	1	1.00	1.02	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.706	0.091	1.444	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	340	421	0	349	0	0	0	0
N.S.	1	1.17	1.45	0.00	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	0.082	0.000	0.329	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	477	487	567	0	0	0	0	0
N.S.	1	0.92	0.94	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.955	0.148	1.083	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	555	479	0	969	0	0	0	0
N.S.	1	0.93	0.80	0.00	1.63	0.00	0.00	0.00	0.00
time (sec)	N/A	0.938	0.230	0.000	0.399	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1046	959	1240	0	0	0	0	0	0
N.S.	1	0.92	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.784	0.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	831	766	1105	0	0	0	0	0	0
N.S.	1	0.92	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.456	0.848	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	685	648	539	0	0	0	0	0	0
N.S.	1	0.95	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.123	0.450	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	450	515	0	0	0	0	0	0
N.S.	1	1.12	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.870	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	800	719	625	0	0	0	0	0	0
N.S.	1	0.90	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.419	0.569	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	995	883	721	0	0	0	0	0	0
N.S.	1	0.89	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.729	0.572	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	69	95	0	0	0	0
N.S.	1	1.00	1.83	1.50	2.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.014	1.898	0.191	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	17	20	19	141	18	14	0	18
N.S.	1	0.85	1.00	0.95	7.05	0.90	0.70	0.00	0.90
time (sec)	N/A	0.246	0.115	1.076	0.210	0.333	0.058	0.000	1.391

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	84	76	0	0	0	0	0	0
N.S.	1	1.02	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	149	110	200	0	0	0	0	0
N.S.	1	0.99	0.73	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	0.031	4.106	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	157	785	0	0	0	0	0	0
N.S.	1	0.98	4.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	28	26	25	29	20	153	25
N.S.	1	1.00	1.12	1.04	1.00	1.16	0.80	6.12	1.00
time (sec)	N/A	0.183	0.006	0.397	0.189	0.308	0.048	0.312	0.071

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	64	63	0	113	0	0	0	0
N.S.	1	0.96	0.94	0.00	1.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.011	0.000	0.218	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	108	91	0	0	0	0	0	0
N.S.	1	1.11	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	0.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	42	26	29	28
N.S.	1	1.00	1.00	1.04	1.00	1.50	0.93	1.04	1.00
time (sec)	N/A	0.158	0.003	0.227	0.188	0.288	0.057	0.315	1.254

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	69	106	0	118	0	0	0	0
N.S.	1	1.03	1.58	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.013	0.000	0.200	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	41	26	29	28
N.S.	1	1.00	1.00	1.04	1.00	1.46	0.93	1.04	1.00
time (sec)	N/A	0.162	0.004	0.399	0.187	0.303	0.063	0.313	1.233

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	69	96	0	118	0	0	0	0
N.S.	1	1.03	1.43	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.022	0.000	0.203	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	103	98	0	0	0	0	0	0
N.S.	1	1.05	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	0	0	0	0
N.S.	1	1.00	0.86	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.010	8.383	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	156	0	0	0	0	0
N.S.	1	1.00	0.80	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.021	3.803	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	139	135	419	0	0	0	0	0
N.S.	1	0.99	0.96	2.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.032	3.767	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	96	423	0	0	0	0	0
N.S.	1	1.00	0.88	3.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.022	25.005	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	195	1636	338	0	264	0	0	0
N.S.	1	0.96	8.02	1.66	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.593	0.625	5.451	0.000	0.285	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	392	1080	813	0	0	0	0	0
N.S.	1	1.22	3.35	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.663	0.400	3.178	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	433	454	1855	0	0	0	0	0	0
N.S.	1	1.05	4.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.817	0.435	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [95] had the largest ratio of [.6923080000000000034]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	32	0.062
2	A	2	2	1.00	32	0.062
3	A	2	2	1.00	30	0.067
4	A	2	2	1.00	32	0.062
5	A	2	2	1.00	32	0.062
6	A	2	2	1.00	32	0.062
7	A	4	4	0.92	29	0.138
8	A	4	4	0.93	29	0.138
9	A	4	4	0.95	29	0.138
10	A	4	4	0.97	27	0.148
11	A	7	6	1.03	29	0.207
12	A	4	4	0.96	29	0.138
13	A	4	4	0.88	29	0.138
14	A	4	4	0.88	29	0.138
15	A	4	4	0.88	29	0.138
16	A	7	7	0.92	31	0.226
17	A	7	7	0.90	31	0.226
18	A	7	7	0.88	31	0.226
19	A	7	7	0.88	29	0.241
20	A	8	7	1.00	31	0.226
21	A	7	7	0.88	31	0.226
22	A	7	7	0.86	31	0.226

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	7	0.90	31	0.226
24	A	7	7	0.92	31	0.226
25	A	3	3	0.89	29	0.103
26	A	3	3	0.91	29	0.103
27	A	3	3	0.93	29	0.103
28	A	3	3	0.96	27	0.111
29	A	3	3	1.00	21	0.143
30	A	5	4	1.04	29	0.138
31	A	3	3	0.94	29	0.103
32	A	3	3	0.83	29	0.103
33	A	3	3	0.85	29	0.103
34	A	3	3	0.84	29	0.103
35	A	11	10	0.80	31	0.323
36	A	11	10	0.78	31	0.323
37	A	11	10	0.78	29	0.345
38	A	11	10	0.84	23	0.435
39	A	18	17	1.85	31	0.548
40	A	11	10	0.77	31	0.323
41	A	14	13	0.80	31	0.419
42	A	18	17	0.85	31	0.548
43	A	3	2	1.00	40	0.050
44	A	6	5	0.68	40	0.125
45	A	6	5	0.68	40	0.125
46	A	5	4	0.95	38	0.105
47	A	3	2	1.00	40	0.050
48	A	3	2	1.00	40	0.050
49	A	3	2	1.00	40	0.050
50	A	5	4	0.93	34	0.118
51	A	7	6	0.94	48	0.125
52	A	6	5	0.99	46	0.109
53	A	6	5	1.00	32	0.156
54	N/A	1	0	1.00	48	0.000
55	N/A	1	0	1.00	48	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	9	8	1.01	39	0.205
57	A	8	7	1.05	39	0.179
58	A	7	6	1.10	37	0.162
59	A	3	3	1.15	25	0.120
60	N/A	1	0	1.00	39	0.000
61	N/A	1	0	1.00	39	0.000
62	N/A	1	0	1.00	28	0.000
63	N/A	1	0	1.00	28	0.000
64	A	7	7	1.16	26	0.269
65	N/A	1	0	1.00	28	0.000
66	N/A	1	0	1.00	28	0.000
67	A	5	4	1.04	45	0.089
68	A	5	4	1.05	45	0.089
69	A	5	4	1.05	43	0.093
70	N/A	2	0	1.00	45	0.000
71	N/A	2	0	1.00	45	0.000
72	N/A	2	0	1.00	48	0.000
73	N/A	2	0	1.00	48	0.000
74	F	0	0	N/A	0.000	N/A
75	F	0	0	N/A	0.000	N/A
76	A	7	6	0.96	32	0.188
77	A	5	5	0.95	32	0.156
78	A	4	4	1.00	30	0.133
79	A	4	3	1.17	29	0.103
80	A	8	7	0.92	32	0.219
81	A	5	5	0.93	32	0.156
82	A	5	5	0.92	34	0.147
83	A	5	5	0.92	34	0.147
84	A	8	7	0.95	32	0.219
85	A	4	3	1.12	31	0.097
86	A	10	9	0.90	34	0.265
87	A	7	7	0.89	34	0.206

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	8	7	1.00	19	0.368
89	A	4	3	0.85	24	0.125
90	A	3	3	1.02	34	0.088
91	A	3	3	0.99	55	0.055
92	A	3	3	0.98	58	0.052
93	A	5	5	1.00	11	0.455
94	A	7	6	0.96	13	0.462
95	A	10	9	1.11	13	0.692
96	A	2	2	1.00	13	0.154
97	A	9	8	1.03	15	0.533
98	A	6	5	1.00	15	0.333
99	A	2	2	1.00	13	0.154
100	A	9	8	1.03	15	0.533
101	A	6	5	1.05	15	0.333
102	A	1	1	1.00	38	0.026
103	A	2	2	1.00	50	0.040
104	A	5	4	0.99	42	0.095
105	A	2	2	1.00	62	0.032
106	A	5	4	0.96	49	0.082
107	A	4	3	1.22	42	0.071
108	A	5	4	1.05	65	0.062

CHAPTER 3

Listing of integrals

3.1 $\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

3.1.1 Optimal result

Integrand size = 32, antiderivative size = 404

$$
\begin{aligned}
&\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{B(bc-ad)g^3n}{2acx} + Af^3x - \frac{1}{2}B\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)g^3n\log(x) + \frac{b^2Bg^3n\log(a+bx)}{2a^2} \\
&\quad - 3Bf^2gn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} + \frac{3(bc-ad)fg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)} \\
&\quad + 3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \\
&\quad - \frac{B(bc-ad)f^3n\log(c+dx)}{bd} - \frac{Bd^2g^3n\log(c+dx)}{2c^2} \\
&\quad + 3Bf^2gn\log(x)\log\left(1+\frac{dx}{c}\right) + \frac{3B(bc-ad)fg^2n\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} \\
&\quad - 3Bf^2gn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right) + 3Bf^2gn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)
\end{aligned}
$$

output
```
-1/2*B*(-a*d+b*c)*g^3*n/a/c/x+A*f^3*x-1/2*B*(b^2/a^2-d^2/c^2)*g^3*n*ln(x)+
1/2*b^2*B*g^3*n*ln(b*x+a)/a^2-3*B*f^2*g*n*ln(x)*ln(1+b*x/a)+B*f^3*(b*x+a)*
ln(e*((b*x+a)/(d*x+c))^n)/b-1/2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/x^2+3*
(-a*d+b*c)*f*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b
*x+a)/(d*x+c))+3*f^2*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*
f^3*n*ln(d*x+c)/b/d-1/2*B*d^2*g^3*n*ln(d*x+c)/c^2+3*B*f^2*g*n*ln(x)*ln(1+d
*x/c)+3*B*(-a*d+b*c)*f*g^2*n*ln(a-c*(b*x+a)/(d*x+c))/a/c-3*B*f^2*g*n*polyl
og(2,-b*x/a)+3*B*f^2*g*n*polylog(2,-d*x/c)
```

3.1.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \left(f+\frac{g}{x}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}\\ &-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}\\ &+3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^3n\log(c+dx)}{bd}\\ &+\frac{3Bfg^2n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac}\\ &+\frac{Bg^3n((-b^2c^2x+a^2d^2x)\log(x)+b^2c^2x\log(a+bx)+a(-bc^2+acd-ad^2x\log(c+dx)))}{2a^2c^2x}\\ &-3Bf^2gn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\text{PolyLog}\left(2,-\frac{bx}{a}\right)\right.\\ &\left.-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\right)\end{aligned}$$

input
```
Integrate[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
A*f^3*x + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) - (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*f*g^2*n*((b*c - a*d)*
Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) + (B*g^3*n*((-(b^2*c^
2*x) + a^2*d^2*x)*Log[x] + b^2*c^2*x*Log[a + b*x] + a*(-(b*c^2) + a*c*d -
a*d^2*x*Log[c + d*x])))/(2*a^2*c^2*x) - 3*B*f^2*g*n*(Log[x]*(Log[1 + (b*x)
/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLog[2, -((d*x)/c)])
```

3.1.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+\frac{g}{x}\right)^3 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{3008}$$

$$\int \left(f^3 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{3f^2g\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{x}+\frac{3fg^2\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{x^2}+\frac{g^3\left(B\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}+3f^2g\log(x)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{3fg^2(a+bx)(bc-ad)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}-\frac{g^3\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2x^2}+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bf^3n(bc-ad)\log(c+dx)}{bd}+\frac{3Bfg^2n(bc-ad)\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac}-\frac{Bg^3n(bc-ad)}{2acx}-3Bf^2gn\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)-3Bf^2gn\log(x)\log\left(\frac{bx}{a}+1\right)+Af^3x-\frac{Bd^2g^3n\log(c+dx)}{2c^2}+3Bf^2gn\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)+3Bf^2gn\log(x)\log\left(\frac{dx}{c}+1\right)$$

input
```
Int[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*g^3*n)/(a*c*x) + A*f^3*x - (B*(b^2/a^2 - d^2/c^2)*g^3*
n*Log[x])/2 + (b^2*B*g^3*n*Log[a + b*x])/(2*a^2) - 3*B*f^2*g*n*Log[x]*Log[
1 + (b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) + (3*(b*c - a*d)*f*g^2*(a +
 b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*
x))/(c + d*x))) + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
(B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) - (B*d^2*g^3*n*Log[c + d*x])/(2*c
^2) + 3*B*f^2*g*n*Log[x]*Log[1 + (d*x)/c] + (3*B*(b*c - a*d)*f*g^2*n*Log[a
 - (c*(a + b*x))/(c + d*x)])/(a*c) - 3*B*f^2*g*n*PolyLog[2, -((b*x)/a)] +
3*B*f^2*g*n*PolyLog[2, -((d*x)/c)]
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

3.1.4 Maple [F]

$$\int \left(f+\frac{g}{x}\right)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

input
```
int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.1.5 Fricas [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

input
```
integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
integral((A*f^3*x^3 + 3*A*f^2*g*x^2 + 3*A*f*g^2*x + A*g^3 + (B*f^3*x^3 + 3
*B*f^2*g*x^2 + 3*B*f*g^2*x + B*g^3)*log(e*((b*x + a)/(d*x + c))^n))/x^3, x
)
```

3.1.6 Sympy [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)(fx+g)^3}{x^3}\,dx$$

input
```
integrate((f+g/x)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)**3/x**3
, x)
```

3.1.7 Maxima [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

input
```
integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - 3*B*f*g^2*n*(b*log(b*x + a
)/a - d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + 1/2*B*g^3*n*(b^2*log(
b*x + a)/a^2 - d^2*log(d*x + c)/c^2 - (b*c - a*d)/(a*c*x) - (b^2*c^2 - a^2
*d^2)*log(x)/(a^2*c^2)) + B*f^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
 A*f^3*x - 3*B*f^2*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log
(e))/x, x) + 3*A*f^2*g*log(x) - 3*B*f*g^2*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n)/x - 3*A*f*g^2/x - 1/2*B*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/
x^2 - 1/2*A*g^3/x^2
```

3.1.8 Giac [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

input
```
integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^3, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \left(f+\frac{g}{x}\right)^3 \left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)\,dx$$

input
```
int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.2 $\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

3.2.1 Optimal result

Integrand size = 32, antiderivative size = 263

$$\begin{aligned}
&\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= Af^2x-2Bfgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad+\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)} \\
&\quad+2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^2n\log(c+dx)}{bd} \\
&\quad+2Bfgn\log(x)\log\left(1+\frac{dx}{c}\right)+\frac{B(bc-ad)g^2n\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} \\
&\quad-2Bfgn\,\mathrm{PolyLog}\left(2,-\frac{bx}{a}\right)+2Bfgn\,\mathrm{PolyLog}\left(2,-\frac{dx}{c}\right)
\end{aligned}$$

output
```
A*f^2*x-2*B*f*g*n*ln(x)*ln(1+b*x/a)+B*f^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n
)/b+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*
(b*x+a)/(d*x+c))+2*f*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*
f^2*n*ln(d*x+c)/b/d+2*B*f*g*n*ln(x)*ln(1+d*x/c)+B*(-a*d+b*c)*g^2*n*ln(a-c*
(b*x+a)/(d*x+c))/a/c-2*B*f*g*n*polylog(2,-b*x/a)+2*B*f*g*n*polylog(2,-d*x/
c)
```

3.2.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.83

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}$$
$$+2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^2n\log(c+dx)}{bd}$$
$$+\frac{Bg^2n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac}$$
$$-2Bfgn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\text{PolyLog}\left(2,-\frac{bx}{a}\right)\right.$$
$$\left.-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\right)$$

input
```
Integrate[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
A*f^2*x + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + (B*g^2*n*((b*
c - a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - 2*B*f*g*n*
(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - P
olyLog[2, -((d*x)/c)])
```

3.2.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+\frac{g}{x}\right)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{ 3008}$$

$$\int \left(f^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{2fg\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{x}+\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{g^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bf^2n(bc-ad)\log(c+dx)}{bd}+\frac{Bg^2n(bc-ad)\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac}-2Bfgn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)-2Bfgn\log(x)\log\left(\frac{bx}{a}+1\right)+Af^2x+2Bfgn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)+2Bfgn\log(x)\log\left(\frac{dx}{c}+1\right)$$

input
```
Int[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
A*f^2*x - 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n])/b + ((b*c - a*d)*g^2*(a + b*x)*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c + d*x))) + 2*f*g*Log[
x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d
*x])/(b*d) + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] + (B*(b*c - a*d)*g^2*n*Log[
a - (c*(a + b*x))/(c + d*x)])/(a*c) - 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2
*B*f*g*n*PolyLog[2, -((d*x)/c)]
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

3.2.4 Maple [F]

$$\int \left(f+\frac{g}{x}\right)^{2} \left(A+B\ln\left(e{\left(\frac{bx+a}{dx+c}\right)}^{n}\right)\right)dx$$

input
```
int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.2.5 Fricas [F]

$$\int \left(f+\frac{g}{x}\right)^{2} \left(A+B\log\left(e{\left(\frac{a+bx}{c+dx}\right)}^{n}\right)\right)dx = \int \left(B\log\left(e{\left(\frac{bx+a}{dx+c}\right)}^{n}\right)+A\right)\left(f+\frac{g}{x}\right)^{2} dx$$

input
```
integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
integral((A*f^2*x^2 + 2*A*f*g*x + A*g^2 + (B*f^2*x^2 + 2*B*f*g*x + B*g^2)*
log(e*((b*x + a)/(d*x + c))^n))/x^2, x)
```

3.2.6 Sympy [F]

$$\int \left(f+\frac{g}{x}\right)^{2} \left(A+B\log\left(e{\left(\frac{a+bx}{c+dx}\right)}^{n}\right)\right)dx$$
$$= \int \frac{\left(A+B\log\left(e{\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}^{n}\right)\right)(fx+g)^{2}}{x^{2}}\,dx$$

input
```
integrate((f+g/x)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)**2/x**2
, x)
```

3.2.7 Maxima [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^2\,dx$$

input
```
integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
B*f^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - B*g^2*n*(b*log(b*x + a)/a
- d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + B*f^2*x*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) + A*f^2*x - 2*B*f*g*integrate(-(log((b*x + a)^n) - l
og((d*x + c)^n) + log(e))/x, x) + 2*A*f*g*log(x) - B*g^2*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n)/x - A*g^2/x
```

3.2.8 Giac [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^2\,dx$$

input
```
integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^2, x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \left(f+\frac{g}{x}\right)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

input
```
int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.3 $\int \left(f+\frac{g}{x}\right) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

3.3.1 Optimal result

Integrand size = 30, antiderivative size = 143

$$\begin{aligned}
&\int \left(f+\frac{g}{x}\right) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx \\
&= Afx - Bgn \log(x) \log \left(1+\frac{bx}{a}\right) + \frac{Bf(a+bx) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad + g \log(x) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{B(bc-ad)fn \log(c+dx)}{bd} \\
&\quad + Bgn \log(x) \log \left(1+\frac{dx}{c}\right) - Bgn \operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) + Bgn \operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)
\end{aligned}$$

output
```
A*f*x-B*g*n*ln(x)*ln(1+b*x/a)+B*f*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b+g*ln
(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f*n*ln(d*x+c)/b/d+B*g*n*l
n(x)*ln(1+d*x/c)-B*g*n*polylog(2,-b*x/a)+B*g*n*polylog(2,-d*x/c)
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)fn\log(c+dx)}{bd}-Bgn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\text{PolyLog}\left(2,-\frac{bx}{a}\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\right)$$

input
```
Integrate[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d)
- B*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)
/a)] - PolyLog[2, -((d*x)/c)])
```

3.3.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+\frac{g}{x}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{3008}$$
$$\int \left(f\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{g\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bfn(bc-ad)\log(c+dx)}{bd}-Bgn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)-Bgn\log(x)\log\left(\frac{bx}{a}+1\right)+Afx+Bgn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)+Bgn\log(x)\log\left(\frac{dx}{c}+1\right)$$

input
```
Int[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c
 - a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*Po
lyLog[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

3.3.4 Maple [F]

$$\int\left(f+\frac{g}{x}\right)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

input
```
int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.3.5 Fricas [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)\,dx$$

input
```
integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
integral((A*f*x + A*g + (B*f*x + B*g)*log(e*((b*x + a)/(d*x + c))^n))/x, x
)
```

3.3.6 Sympy [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)(fx+g)}{x}\,dx$$

input
```
integrate((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)/x, x)
```

3.3.7 Maxima [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)\,dx$$

input
```
integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + A*f*x - B*g*integrate(-(log((b*x + a)^n) - log((d*x + c
)^n) + log(e))/x, x) + A*g*log(x)
```

3.3.8 Giac [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)\,dx$$

input
```
integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(f+\frac{g}{x}\right)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

input
```
int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.4 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$

3.4.1 Optimal result

Integrand size = 32, antiderivative size = 217

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx &= \frac{Ax}{f}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{B(bc-ad)n\log(c+dx)}{bdf}\\ &\quad+\frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^2}\\ &\quad-\frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2}\\ &\quad-\frac{Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^2}\\ &\quad+\frac{Bgn\,\mathrm{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^2}-\frac{Bgn\,\mathrm{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^2}\end{aligned}$$

output
```
A*x/f+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f-B*(-a*d+b*c)*n*ln(d*x+c)/b/d
/f+B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^2-g*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))*ln(f*x+g)/f^2-B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^2+B*g*n*pol
ylog(2,-b*(f*x+g)/(a*f-b*g))/f^2-B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.85

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$$
$$=\frac{Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)fn\log(c+dx)}{bd}-g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)+Bgn\left(\left(\log\left(\frac{f}{c}\right.\right.\right.}{f^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

output
```
(A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)
*f*n*Log[c + d*x])/(b*d) - g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g
+ f*x] + B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)] - Log[(f*(c + d*x))/(c*f -
 d*g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/(-(a*f) + b*g)] - PolyLog[
2, (d*(g + f*x))/(-(c*f) + d*g)]))/f^2
```

3.4.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{f+\frac{g}{x}}\,dx$$
$$\Big\downarrow \text{3008}$$
$$\int \left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{f}-\frac{g\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f(fx+g)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{Bn(bc-ad)\log(c+dx)}{bdf}+\frac{Bgn\,\text{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^2}+\frac{Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^2}+\frac{Ax}{f}-\frac{Bgn\,\text{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^2}-\frac{Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

output
```
(A*x)/f + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f) - (B*(b*c - a
*d)*n*Log[c + d*x])/(b*d*f) + (B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g
+ f*x])/f^2 - (g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^2
- (B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^2 + (B*g*n*PolyLog
[2, -((b*(g + f*x))/(a*f - b*g))])/f^2 - (B*g*n*PolyLog[2, -((d*(g + f*x))
/(c*f - d*g))])/f^2
```

3.4.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

3.4.4 Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{f+\frac{g}{x}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)
```

3.4.5 Fricas [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{f + \frac{g}{x}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="fricas")
```

output
```
integral((B*x*log(e*((b*x + a)/(d*x + c))^n) + A*x)/(f*x + g), x)
```

3.4.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x),x)
```

output
```
Timed out
```

3.4.7 Maxima [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{f + \frac{g}{x}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="maxima")
```

output
```
A*(x/f - g*log(f*x + g)/f^2) - B*integrate(-(x*log((b*x + a)^n) - x*log((d
*x + c)^n) + x*log(e))/(f*x + g), x)
```

3.4.8 Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{f+\frac{g}{x}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x), x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx=\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f+\frac{g}{x}}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x), x)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx$

3.5.1 Optimal result

Integrand size = 32, antiderivative size = 322

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx &= \frac{Ax}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} \\ &\quad-\frac{g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(af-bg)(g+fx)} \\ &\quad-\frac{B(bc-ad)n\log(c+dx)}{bdf^2}+\frac{2Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^3} \\ &\quad-\frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} \\ &\quad-\frac{2Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^3} \\ &\quad+\frac{B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^2(af-bg)(cf-dg)}+\frac{2Bgn\operatorname{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^3} \\ &\quad-\frac{2Bgn\operatorname{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^3}\end{aligned}$$

output
```
A*x/f^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^2-g^2*(b*x+a)*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/f^2/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^2
+2*B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^3-2*g*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))*ln(f*x+g)/f^3-2*B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^3+B*(-a
*d+b*c)*g^2*n*ln((f*x+g)/(d*x+c))/f^2/(a*f-b*g)/(c*f-d*g)+2*B*g*n*polylog(
2,-b*(f*x+g)/(a*f-b*g))/f^3-2*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^3
```

3.5.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = \frac{Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g+fx}-\frac{B(bc-ad)fn\log(c+dx)}{bd}-2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

output
```
(A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(g + f*x) - (B*(b*c - a*d)*f*n*Log[c + d*x]
)/(b*d) - 2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (B*g^2
*n*(b*(-(c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*
d)*f*Log[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + 2*B*g*n*((Log[(f*(a + b*x)
)/(a*f - b*g)] - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2,
 (b*(g + f*x))/(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)])
)/f^3
```

3.5.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^2}dx$$
$$\Big\downarrow \text{3008}$$
$$\int \left(\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)^2}-\frac{2g\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{f^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3}-\frac{g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)(af-bg)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}+\frac{Bg^2n(bc-ad)\log\left(\frac{fx+g}{c+dx}\right)}{f^2(af-bg)(cf-dg)}-\frac{Bn(bc-ad)\log(c+dx)}{bdf^2}+\frac{2Bgn\,\text{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^3}+\frac{2Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^3}+\frac{Ax}{f^2}-\frac{2Bgn\,\text{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^3}-\frac{2Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

output
```
(A*x)/f^2 + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f^2) - (g^2*(a
 + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(f^2*(a*f - b*g)*(g + f*x)
) - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f^2) + (2*B*g*n*Log[(f*(a + b*x))/
(a*f - b*g)]*Log[g + f*x])/f^3 - (2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[g + f*x])/f^3 - (2*B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x
])/f^3 + (B*(b*c - a*d)*g^2*n*Log[(g + f*x)/(c + d*x)])/(f^2*(a*f - b*g)*(
c*f - d*g)) + (2*B*g*n*PolyLog[2, -((b*(g + f*x))/(a*f - b*g))])/f^3 - (2*
B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f - d*g))])/f^3
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

3.5.4 Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(f+\frac{g}{x}\right)^{2}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

3.5.5 Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{\left(f+\frac{g}{x}\right)^{2}}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(f+\frac{g}{x}\right)^{2}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="fricas"
)
```

output
```
integral((B*x^2*log(e*((b*x + a)/(d*x + c))^n) + A*x^2)/(f^2*x^2 + 2*f*g*x
 + g^2), x)
```

3.5.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{\left(f+\frac{g}{x}\right)^{2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**2,x)
```

output
```
Timed out
```

3.5.7 Maxima [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\, dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{\left(f + \frac{g}{x}\right)^2}\, dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="maxima"
)
```

output
```
-A*(g^2/(f^4*x + f^3*g) - x/f^2 + 2*g*log(f*x + g)/f^3) - B*integrate(-(x^
2*log((b*x + a)^n) - x^2*log((d*x + c)^n) + x^2*log(e))/(f^2*x^2 + 2*f*g*x
 + g^2), x)
```

3.5.8 Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\, dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{\left(f + \frac{g}{x}\right)^2}\, dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^2, x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\, dx = \int \frac{A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\, dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2, x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$

3.6.1 Optimal result

Integrand size = 32, antiderivative size = 531

$$
\begin{aligned}
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx &= \frac{Ax}{f^3}+\frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)}-\frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} \\
&+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3}+\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} \\
&-\frac{3g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(af-bg)(g+fx)} \\
&-\frac{B(bc-ad)n\log(c+dx)}{bdf^3}+\frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2} \\
&+\frac{B(bc-ad)g^3(bcf+adf-2bdg)n\log(g+fx)}{2f^3(af-bg)^2(cf-dg)^2} \\
&+\frac{3Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^4} \\
&-\frac{3g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^4} \\
&-\frac{3Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^4} \\
&+\frac{3B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^3(af-bg)(cf-dg)}+\frac{3Bgn\operatorname{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^4} \\
&-\frac{3Bgn\operatorname{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^4}
\end{aligned}
$$

output
```
A*x/f^3+1/2*B*(-a*d+b*c)*g^3*n/f^3/(a*f-b*g)/(c*f-d*g)/(f*x+g)-1/2*b^2*B*g
^3*n*ln(b*x+a)/f^4/(a*f-b*g)^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^3+1
/2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/f^4/(f*x+g)^2-3*g^2*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/f^3/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/
d/f^3+1/2*B*d^2*g^3*n*ln(d*x+c)/f^4/(c*f-d*g)^2+1/2*B*(-a*d+b*c)*g^3*(a*d*
f+b*c*f-2*b*d*g)*n*ln(f*x+g)/f^3/(a*f-b*g)^2/(c*f-d*g)^2+3*B*g*n*ln(f*(b*x
+a)/(a*f-b*g))*ln(f*x+g)/f^4-3*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(f*x+g)
/f^4-3*B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^4+3*B*(-a*d+b*c)*g^2*n*ln
((f*x+g)/(d*x+c))/f^3/(a*f-b*g)/(c*f-d*g)+3*B*g*n*polylog(2,-b*(f*x+g)/(a*
f-b*g))/f^4-3*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^4
```

3.6.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$$

$$= \frac{2Afx+\frac{2Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(g+fx)^2}-\frac{6g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g+fx}-\frac{2B(bc-ad)fn\log(c+dx)}{bd}-6g(}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

output
```
(2*A*f*x + (2*B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (g^3*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(g + f*x)^2 - (6*g^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(g + f*x) - (2*B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d)
 - 6*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (6*B*g^2*n*(b
*(-(c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f*
Log[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + B*(b*c - a*d)*g^3*n*(-((b^2*Log
[a + b*x])/((b*c - a*d)*(a*f - b*g)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d)
+ (f*(((a*f - b*g)*(c*f - d*g))/(g + f*x) + (b*c*f + a*d*f - 2*b*d*g)*Log[
g + f*x]))/(a*f - b*g)^2)/(c*f - d*g)^2) + 6*B*g*n*((Log[(f*(a + b*x))/(a*
f - b*g)] - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(
g + f*x))/(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/(2*
f^4)
```

3.6.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}dx$$

$$\Big\downarrow \text{3008}$$

$$\int \left(-\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)^3}+\frac{3g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)^2}-\frac{3g\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)}+\frac{B\log\left(e\left(\frac{a}{c}\right.\right.}{f}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2}-\frac{3g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^4}-\frac{3g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)(af-bg)}-\frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3}+\frac{Bg^3n(bc-ad)}{2f^3(fx+g)(af-bg)(cf-dg)}+\frac{Bg^3n(bc-ad)\log(fx+g)(adf+bcf-2bdg)}{2f^3(af-bg)^2(cf-dg)^2}+\frac{3Bg^2n(bc-ad)\log\left(\frac{fx+g}{c+dx}\right)}{f^3(af-bg)(cf-dg)}-\frac{Bn(bc-ad)\log(c+dx)}{bdf^3}+\frac{3Bgn\operatorname{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^4}+\frac{3Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^4}+\frac{Ax}{f^3}+\frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2}-\frac{3Bgn\operatorname{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^4}-\frac{3Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

output
```
(A*x)/f^3 + (B*(b*c - a*d)*g^3*n)/(2*f^3*(a*f - b*g)*(c*f - d*g)*(g + f*x)
) - (b^2*B*g^3*n*Log[a + b*x])/(2*f^4*(a*f - b*g)^2) + (B*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(b*f^3) + (g^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(2*f^4*(g + f*x)^2) - (3*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(f^3*(a*f - b*g)*(g + f*x)) - (B*(b*c - a*d)*n*Log[c + d*x])/(
b*d*f^3) + (B*d^2*g^3*n*Log[c + d*x])/(2*f^4*(c*f - d*g)^2) + (B*(b*c - a*
d)*g^3*(b*c*f + a*d*f - 2*b*d*g)*n*Log[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f
 - d*g)^2) + (3*B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g + f*x])/f^4 - (
3*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^4 - (3*B*g*n*Lo
g[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^4 + (3*B*(b*c - a*d)*g^2*n*Lo
g[(g + f*x)/(c + d*x)])/(f^3*(a*f - b*g)*(c*f - d*g)) + (3*B*g*n*PolyLog[2
, -((b*(g + f*x))/(a*f - b*g))])/f^4 - (3*B*g*n*PolyLog[2, -((d*(g + f*x))
/(c*f - d*g))])/f^4
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

3.6.4 Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(f+\frac{g}{x}\right)^{3}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

3.6.5 Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="fricas"
)
```

output
```
integral((B*x^3*log(e*((b*x + a)/(d*x + c))^n) + A*x^3)/(f^3*x^3 + 3*f^2*g
*x^2 + 3*f*g^2*x + g^3), x)
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**3,x)
```

output
```
Timed out
```

3.6.7 Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="maxima"
)
```

output
```
-1/2*A*((6*f*g^2*x + 5*g^3)/(f^6*x^2 + 2*f^5*g*x + f^4*g^2) - 2*x/f^3 + 6*
g*log(f*x + g)/f^4) - B*integrate(-(x^3*log((b*x + a)^n) - x^3*log((d*x +
c)^n) + x^3*log(e))/(f^3*x^3 + 3*f^2*g*x^2 + 3*f*g^2*x + g^3), x)
```

3.6.8 Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^3, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx=\int \frac{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3, x)
```

3.7 $\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.7.1 Optimal result

Integrand size = 29, antiderivative size = 201

$$\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = -\frac{(bc-ad)^4 qrx}{5d^4} + \frac{(bc-ad)^3 qr(a+bx)^2}{10bd^3} - \frac{(bc-ad)^2 qr(a+bx)^3}{15bd^2} + \frac{(bc-ad)qr(a+bx)^4}{20bd} - \frac{pr(a+bx)^5}{25b} - \frac{qr(a+bx)^5}{25b} + \frac{(bc-ad)^5 qr \log(c+dx)}{5bd^5} + \frac{(a+bx)^5 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}$$

output
```
-1/5*(-a*d+b*c)^4*q*r*x/d^4+1/10*(-a*d+b*c)^3*q*r*(b*x+a)^2/b/d^3-1/15*(-a
*d+b*c)^2*q*r*(b*x+a)^3/b/d^2+1/20*(-a*d+b*c)*q*r*(b*x+a)^4/b/d-1/25*p*r*(
b*x+a)^5/b-1/25*q*r*(b*x+a)^5/b+1/5*(-a*d+b*c)^5*q*r*ln(d*x+c)/b/d^5+1/5*(
b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

3.7.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= \frac{-\frac{r(60bd(bc-ad)^4(p+5q)x-60b^2(bc-ad)^3(2p+5q)(c+dx)^2+40b^3(bc-ad)^2(3p+5q)(c+dx)^3-15b^4(bc-ad)(4p+5q)(c+dx)^4+12b^5(p+q)(c+dx)^5-}{60d^5}}{5b}$$

input
```
Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
(-1/60*(r*(60*b*d*(b*c - a*d)^4*(p + 5*q)*x - 60*b^2*(b*c - a*d)^3*(2*p +
5*q)*(c + d*x)^2 + 40*b^3*(b*c - a*d)^2*(3*p + 5*q)*(c + d*x)^3 - 15*b^4*(
b*c - a*d)*(4*p + 5*q)*(c + d*x)^4 + 12*b^5*(p + q)*(c + d*x)^5 - 60*(b*c
- a*d)^5*q*Log[c + d*x]))/d^5 + (a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r])/(5*b)
```

3.7.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2981, 17, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$\Big\downarrow \ 2981$$

$$-\frac{dqr\int \frac{(a+bx)^5}{c+dx}dx}{5b}-\frac{1}{5}pr\int (a+bx)^4dx+\frac{(a+bx)^5\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

$$\Big\downarrow \ 17$$

$$-\frac{dqr\int \frac{(a+bx)^5}{c+dx}dx}{5b}+\frac{(a+bx)^5\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}-\frac{pr(a+bx)^5}{25b}$$

$$\Big\downarrow \ 49$$

$$-\frac{dqr\int\left(\frac{(ad-bc)^5}{d^5(c+dx)}+\frac{b(bc-ad)^4}{d^5}+\frac{b(a+bx)^4}{d}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)^3(a+bx)}{d^4}\right)dx}{5b}+\frac{(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}-\frac{pr(a+bx)^5}{25b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{dqr\left(-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}\right)}{5b}+\frac{(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}-\frac{pr(a+bx)^5}{25b}$$

input
```
Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/25*(p*r*(a + b*x)^5)/b - (d*q*r*((b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)
^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*
d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/(5*d) - ((b*c - a*d)^5*Log[c + d*x])
/d^6))/(5*b) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b)
```

3.7.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.7.4 Maple [F]

$$\int (bx+a)^4 \ln\left(e(f(bx+a)^p (dx+c)^q)^r\right) dx$$

input
```
int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

output
```
int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(185) = 370.

Time = 0.32 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.10

$$\int (a+bx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx =$$
$$-\frac{12\,(b^5d^5p+b^5d^5q)rx^5+15\,(4\,ab^4d^5p-(b^5cd^4-5\,ab^4d^5)q)rx^4+20\,(6\,a^2b^3d^5p+(b^5c^2d^3-5\,ab^4cd^4+10}{}$$

input
```
integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas"
)
```

output
```
-1/300*(12*(b^5*d^5*p + b^5*d^5*q)*r*x^5 + 15*(4*a*b^4*d^5*p - (b^5*c*d^4
- 5*a*b^4*d^5)*q)*r*x^4 + 20*(6*a^2*b^3*d^5*p + (b^5*c^2*d^3 - 5*a*b^4*c*d
^4 + 10*a^2*b^3*d^5)*q)*r*x^3 + 30*(4*a^3*b^2*d^5*p - (b^5*c^3*d^2 - 5*a*b
^4*c^2*d^3 + 10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*q)*r*x^2 + 60*(a^4*b*d^5*p
 + (b^5*c^4*d - 5*a*b^4*c^3*d^2 + 10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 +
5*a^4*b*d^5)*q)*r*x - 60*(b^5*d^5*p*r*x^5 + 5*a*b^4*d^5*p*r*x^4 + 10*a^2*b
^3*d^5*p*r*x^3 + 10*a^3*b^2*d^5*p*r*x^2 + 5*a^4*b*d^5*p*r*x + a^5*d^5*p*r)
*log(b*x + a) - 60*(b^5*d^5*q*r*x^5 + 5*a*b^4*d^5*q*r*x^4 + 10*a^2*b^3*d^5
*q*r*x^3 + 10*a^3*b^2*d^5*q*r*x^2 + 5*a^4*b*d^5*q*r*x + (b^5*c^5 - 5*a*b^4
*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4)*q*r)*log
(d*x + c) - 60*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^
3*b^2*d^5*x^2 + 5*a^4*b*d^5*x)*log(e) - 60*(b^5*d^5*r*x^5 + 5*a*b^4*d^5*r*
x^4 + 10*a^2*b^3*d^5*r*x^3 + 10*a^3*b^2*d^5*r*x^2 + 5*a^4*b*d^5*r*x)*log(f
))/(b*d^5)
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = \text{Timed out}$$

input
```
integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Timed out
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(185) = 370$.

Time $= 0.20$ (sec) , antiderivative size $= 395$, normalized size of antiderivative $= 1.97$

$$\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{1}{5} \left(b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x\right) \log \left(\left(\left(bx+a\right)^p(dx+c)^q f\right)^r e\right)$$
$$+ \frac{\left(\frac{60\,a^5fp\log(bx+a)}{b} - \frac{12\,b^4d^4f(p+q)x^5+15\,\left(ab^3d^4f(4\,p+5\,q)-b^4cd^3fq\right)x^4+20\,\left(2\,a^2b^2d^4f(3\,p+5\,q)+b^4c^2d^2fq-5\,ab^3cd^3fq\right)x^3+30\,\left(2\,a^3bd\right.}{}\right.}{}$$

input
```
integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima"
)
```

output
```
1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log(
((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*(60*a^5*f*p*log(b*x + a)/b - (12*
b^4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 +
 20*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^
3 + 30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q -
10*a^2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^
3*c^3*d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^
4*c^5*f*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*
f*q + 5*a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r/f
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(185) = 370$.

Time = 10.72 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.84

$$
\begin{aligned}
&\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx \\
&= \frac{a^5pr\log (bx+a)}{5\,b} - \frac{1}{25}\left(b^4pr + b^4qr - 5\,b^4r\log (f) - 5\,b^4\log (e)\right)x^5 \\
&\quad - \frac{(4\,ab^3dpr - b^4cqr + 5\,ab^3dqr - 20\,ab^3dr\log (f) - 20\,ab^3d\log (e))x^4}{20\,d} \\
&\quad - \frac{(6\,a^2b^2d^2pr + b^4c^2qr - 5\,ab^3cdqr + 10\,a^2b^2d^2qr - 30\,a^2b^2d^2r\log (f) - 30\,a^2b^2d^2\log (e))x^3}{15\,d^2} \\
&\quad + \frac{1}{5}\left(b^4prx^5 + 5\,ab^3prx^4 + 10\,a^2b^2prx^3 + 10\,a^3bprx^2 + 5\,a^4prx\right)\log (bx+a) \\
&\quad + \frac{1}{5}\left(b^4qrx^5 + 5\,ab^3qrx^4 + 10\,a^2b^2qrx^3 + 10\,a^3bqrx^2 + 5\,a^4qrx\right)\log (dx+c) \\
&\quad - \frac{(4\,a^3bd^3pr - b^4c^3qr + 5\,ab^3c^2dqr - 10\,a^2b^2cd^2qr + 10\,a^3bd^3qr - 20\,a^3bd^3r\log (f) - 20\,a^3bd^3\log (e))x^2}{10\,d^3} \\
&\quad - \frac{(a^4d^4pr + b^4c^4qr - 5\,ab^3c^3dqr + 10\,a^2b^2c^2d^2qr - 10\,a^3bcd^3qr + 5\,a^4d^4qr - 5\,a^4d^4r\log (f) - 5\,a^4d^4\log (}{5\,d^4} \\
&\quad + \frac{(b^4c^5qr - 5\,ab^3c^4dqr + 10\,a^2b^2c^3d^2qr - 10\,a^3bc^2d^3qr + 5\,a^4cd^4qr)\log (-dx-c)}{5\,d^5}
\end{aligned}
$$

input
```
integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
1/5*a^5*p*r*log(b*x + a)/b - 1/25*(b^4*p*r + b^4*q*r - 5*b^4*r*log(f) - 5*
b^4*log(e))*x^5 - 1/20*(4*a*b^3*d*p*r - b^4*c*q*r + 5*a*b^3*d*q*r - 20*a*b
^3*d*r*log(f) - 20*a*b^3*d*log(e))*x^4/d - 1/15*(6*a^2*b^2*d^2*p*r + b^4*c
^2*q*r - 5*a*b^3*c*d*q*r + 10*a^2*b^2*d^2*q*r - 30*a^2*b^2*d^2*r*log(f) -
30*a^2*b^2*d^2*log(e))*x^3/d^2 + 1/5*(b^4*p*r*x^5 + 5*a*b^3*p*r*x^4 + 10*a
^2*b^2*p*r*x^3 + 10*a^3*b*p*r*x^2 + 5*a^4*p*r*x)*log(b*x + a) + 1/5*(b^4*q
*r*x^5 + 5*a*b^3*q*r*x^4 + 10*a^2*b^2*q*r*x^3 + 10*a^3*b*q*r*x^2 + 5*a^4*q
*r*x)*log(d*x + c) - 1/10*(4*a^3*b*d^3*p*r - b^4*c^3*q*r + 5*a*b^3*c^2*d*q
*r - 10*a^2*b^2*c*d^2*q*r + 10*a^3*b*d^3*q*r - 20*a^3*b*d^3*r*log(f) - 20*
a^3*b*d^3*log(e))*x^2/d^3 - 1/5*(a^4*d^4*p*r + b^4*c^4*q*r - 5*a*b^3*c^3*d
*q*r + 10*a^2*b^2*c^2*d^2*q*r - 10*a^3*b*c*d^3*q*r + 5*a^4*d^4*q*r - 5*a^4
*d^4*r*log(f) - 5*a^4*d^4*log(e))*x/d^4 + 1/5*(b^4*c^5*q*r - 5*a*b^3*c^4*d
*q*r + 10*a^2*b^2*c^3*d^2*q*r - 10*a^3*b*c^2*d^3*q*r + 5*a^4*c*d^4*q*r)*lo
g(-d*x - c)/d^5
```

3.7.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 886, normalized size of antiderivative = 4.41

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$= \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\left(a^4\,x+2\,a^3\,b\,x^2+2\,a^2\,b^2\,x^3+a\,b^3\,x^4+\frac{b^4\,x^5}{5}\right)$$

$$-x^4\left(\frac{b^3\,r\,\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{20\,d}-\frac{b^3\,r\,\left(p+q\right)\,\left(5\,a\,d+5\,b\,c\right)}{100\,d}\right)$$

$$+x^3\left(\frac{\left(\frac{b^3\,r\,\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{5\,d}-\frac{b^3\,r\,\left(p+q\right)\,\left(5\,a\,d+5\,b\,c\right)}{25\,d}\right)\,\left(5\,a\,d+5\,b\,c\right)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,r\,\left(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q\right)}{3\,d}+\frac{a\,b^3\,c\,r\,\left(p+q\right)}{15\,d}\right)-x\left(\frac{a^3\,r\,\left(a\,d\,p+2\,b\,c\,p+3\,a\,d\,q\right)}{d}\right.$$

$$-\frac{\left(5\,a\,d+5\,b\,c\right)\left(\frac{\left(5\,a\,d+5\,b\,c\right)\left(\frac{\left(\frac{b^3\,r\,\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{5\,d}-\frac{b^3\,r\,\left(p+q\right)\,\left(5\,a\,d+5\,b\,c\right)}{25\,d}\right)\,\left(5\,a\,d+5\,b\,c\right)}{5\,b\,d}-\frac{a\,b^2\,r\,\left(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q\right)}{d}+\frac{a\,b^3\,c\,r\,\left(p+q\right)}{5\,d}\right)}{5\,b\,d}\right)}{5\,b\,d}$$

$$\left.+\frac{a\,c\left(\frac{\left(\frac{b^3\,r\,\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{5\,d}-\frac{b^3\,r\,\left(p+q\right)\,\left(5\,a\,d+5\,b\,c\right)}{25\,d}\right)\,\left(5\,a\,d+5\,b\,c\right)}{5\,b\,d}-\frac{a\,b^2\,r\,\left(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q\right)}{d}+\frac{a\,b^3\,c\,r\,\left(p+q\right)}{5\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(5\,a\,d+5\,b\,c\right)\left(\frac{\left(\frac{b^3\,r\,\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{5\,d}-\frac{b^3\,r\,\left(p+q\right)\,\left(5\,a\,d+5\,b\,c\right)}{25\,d}\right)\,\left(5\,a\,d+5\,b\,c\right)}{5\,b\,d}-\frac{a\,b^2\,r\,\left(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q\right)}{d}+\frac{a\,b^3\,c\,r\,\left(p+q\right)}{5\,d}\right)}{10\,b\,d}\right.$$

$$\left.-\frac{a\,c\left(\frac{b^3\,r\,\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{5\,d}-\frac{b^3\,r\,\left(p+q\right)\,\left(5\,a\,d+5\,b\,c\right)}{25\,d}\right)}{2\,b\,d}+\frac{a^2\,b\,r\,\left(a\,d\,p+b\,c\,p+2\,a\,d\,q\right)}{d}\right)$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^4,x)
```

output
```
log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 +
a*b^3*x^4 + 2*a^2*b^2*x^3) - x^4*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(20*
d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(100*d)) + x^3*((((b^3*r*(5*a*d*p + b
*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d +
5*b*c))/(15*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/(3*d) + (a*b^3*c*
r*(p + q))/(15*d)) - x*((a^3*r*(a*d*p + 2*b*c*p + 3*a*d*q))/d - ((5*a*d +
5*b*c)*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b
^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*
(2*a*d*p + b*c*p + 3*a*d*q))/d + (a*b^3*c*r*(p + q))/(5*d)))/(5*b*d) - (a*
c*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b
*c))/(25*d)))/(b*d) + (2*a^2*b*r*(a*d*p + b*c*p + 2*a*d*q))/d))/(5*b*d) +
(a*c*((((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d
+ 5*b*c))/(25*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3
*a*d*q))/d + (a*b^3*c*r*(p + q))/(5*d)))/(b*d)) - x^2*(((5*a*d + 5*b*c)*((
((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c
))/(25*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q)
)/d + (a*b^3*c*r*(p + q))/(5*d)))/(10*b*d) - (a*c*((b^3*r*(5*a*d*p + b*c*p
 + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d)))/(2*b*d) + (a
^2*b*r*(a*d*p + b*c*p + 2*a*d*q))/d) + (log(c + d*x)*((b^4*c^5*q*r)/5 + a^
4*c*d^4*q*r + 2*a^2*b^2*c^3*d^2*q*r - a*b^3*c^4*d*q*r - 2*a^3*b*c^2*d^3...
```

3.7. $\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.8 $\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.8.1 Optimal result

Integrand size = 29, antiderivative size = 172

$$\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = \frac{(bc-ad)^3qrx}{4d^3} - \frac{(bc-ad)^2qr(a+bx)^2}{8bd^2} + \frac{(bc-ad)qr(a+bx)^3}{12bd} - \frac{pr(a+bx)^4}{16b} - \frac{qr(a+bx)^4}{16b} - \frac{(bc-ad)^4qr\log(c+dx)}{4bd^4} + \frac{(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b}$$

output
```
1/4*(-a*d+b*c)^3*q*r*x/d^3-1/8*(-a*d+b*c)^2*q*r*(b*x+a)^2/b/d^2+1/12*(-a*d
+b*c)*q*r*(b*x+a)^3/b/d-1/16*p*r*(b*x+a)^4/b-1/16*q*r*(b*x+a)^4/b-1/4*(-a*
d+b*c)^4*q*r*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)
/b
```

3.8.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

$$\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{\frac{r\left(12bd(bc-ad)^3(p+4q)x-18b^2(bc-ad)^2(p+2q)(c+dx)^2+4b^3(bc-ad)(3p+4q)(c+dx)^3-3b^4(p+q)(c+dx)^4-12(bc-ad)^4q\log(c+dx)\right)}{12d^4} + (a+b}{4b}$$

input
```
Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
((r*(12*b*d*(b*c - a*d)^3*(p + 4*q)*x - 18*b^2*(b*c - a*d)^2*(p + 2*q)*(c
+ d*x)^2 + 4*b^3*(b*c - a*d)*(3*p + 4*q)*(c + d*x)^3 - 3*b^4*(p + q)*(c +
d*x)^4 - 12*(b*c - a*d)^4*q*Log[c + d*x]))/(12*d^4) + (a + b*x)^4*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

3.8.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2981, 17, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$

$$\Big\downarrow \text{ 2981}$$

$$-\frac{dqr\int \frac{(a+bx)^4}{c+dx}dx}{4b}-\frac{1}{4}pr\int (a+bx)^3dx+\frac{(a+bx)^4\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{dqr\int \frac{(a+bx)^4}{c+dx}dx}{4b}+\frac{(a+bx)^4\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}-\frac{pr(a+bx)^4}{16b}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{dqr\int \left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{4b}+\frac{(a+bx)^4\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}-\frac{pr(a+bx)^4}{16b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{dqr\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{4b}+\frac{(a+bx)^4\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}-\frac{pr(a+bx)^4}{16b}$$

input
```
Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/16*(p*r*(a + b*x)^4)/b - (d*q*r*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a
*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)
^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x])/d^5))/(4*b) + ((a + b*x)^4*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

3.8.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.8.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(158) = 316.

Time = 301.79 (sec) , antiderivative size = 604, normalized size of antiderivative = 3.51

method	result
parallelrisch	$\frac{12a^4d^4pr+30a^3bc\,d^3pr-48a^2b^2c^2d^2qr+42a\,b^3c^3dqr+24x^2a\,b^3c\,d^3qr+72x\,a^2b^2c\,d^3qr-48xa\,b^3c^2d^2qr+120\ln(bx+a)a^3bc\,d^3pr}{}$

input
```
int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
1/48*(12*a^4*d^4*p*r+30*a^3*b*c*d^3*p*r-48*a^2*b^2*c^2*d^2*q*r+42*a*b^3*c^
3*d*q*r+24*x^2*a*b^3*c*d^3*q*r+72*x*a^2*b^2*c*d^3*q*r-48*x*a*b^3*c^2*d^2*q
*r+120*ln(b*x+a)*a^3*b*c*d^3*p*r+168*ln(d*x+c)*a^3*b*c*d^3*q*r-72*ln(d*x+c
)*a^2*b^2*c^2*d^2*q*r+48*ln(d*x+c)*a*b^3*c^3*d*q*r-48*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)*a^4*d^4+48*a^4*d^4*q*r-12*b^4*c^4*q*r+72*x^2*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)*a^2*b^2*d^4+48*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b*d^4-
120*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b*c*d^3+60*ln(b*x+a)*a^4*d^4*p*r+4
8*ln(d*x+c)*a^4*d^4*q*r-12*ln(d*x+c)*b^4*c^4*q*r-3*x^4*b^4*d^4*p*r-3*x^4*b
^4*d^4*q*r+48*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^3*d^4+12*x^4*ln(e*(f
*(b*x+a)^p*(d*x+c)^q)^r)*b^4*d^4-12*x^3*a*b^3*d^4*p*r-16*x^3*a*b^3*d^4*q*r
+4*x^3*b^4*c*d^3*q*r-18*x^2*a^2*b^2*d^4*p*r-36*x^2*a^2*b^2*d^4*q*r-6*x^2*b
^4*c^2*d^2*q*r-12*x*a^3*b*d^4*p*r-48*x*a^3*b*d^4*q*r+12*x*b^4*c^3*d*q*r+12
*a^3*b*c*d^3*q*r)/b/d^4
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(158) = 316$.

Time = 0.30 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.73

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx =$$
$$-\frac{3\,(b^4d^4p+b^4d^4q)rx^4+4\,(3\,ab^3d^4p-(b^4cd^3-4\,ab^3d^4)q)rx^3+6\,(3\,a^2b^2d^4p+(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2}{}$$

input
```
integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas"
)
```

output
```
-1/48*(3*(b^4*d^4*p + b^4*d^4*q)*r*x^4 + 4*(3*a*b^3*d^4*p - (b^4*c*d^3 - 4
*a*b^3*d^4)*q)*r*x^3 + 6*(3*a^2*b^2*d^4*p + (b^4*c^2*d^2 - 4*a*b^3*c*d^3 +
 6*a^2*b^2*d^4)*q)*r*x^2 + 12*(a^3*b*d^4*p - (b^4*c^3*d - 4*a*b^3*c^2*d^2
+ 6*a^2*b^2*c*d^3 - 4*a^3*b*d^4)*q)*r*x - 12*(b^4*d^4*p*r*x^4 + 4*a*b^3*d^
4*p*r*x^3 + 6*a^2*b^2*d^4*p*r*x^2 + 4*a^3*b*d^4*p*r*x + a^4*d^4*p*r)*log(b
*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^
2 + 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3)*q*r)*log(d*x + c) - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*
b^2*d^4*x^2 + 4*a^3*b*d^4*x)*log(e) - 12*(b^4*d^4*r*x^4 + 4*a*b^3*d^4*r*x^
3 + 6*a^2*b^2*d^4*r*x^2 + 4*a^3*b*d^4*r*x)*log(f))/(b*d^4)
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Timed out
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.66

$$\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{1}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)$$
$$+ \frac{\left(\frac{12\,a^4fp\log(bx+a)}{b} - \frac{3\,b^3d^3f(p+q)x^4+4\,(ab^2d^3f(3\,p+4\,q)-b^3cd^2fq)x^3+6\,(3\,a^2bd^3f(p+2\,q)+b^3c^2dfq-4\,ab^2cd^2fq)x^2+12\,(a^3d^3f(p+4\,q)}{d^3}\right.}{48\,f}$$

input
```
integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima"
)
```

output
```
1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x
+ c)^q*f)^r*e) + 1/48*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^
4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p
+ 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q)
- b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^
4*f*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x
 + c)/d^4)*r/f
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(158) = 316$.

Time = 3.74 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.41

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= \frac{a^4pr\log(bx+a)}{4b} - \frac{1}{16}\left(b^3pr + b^3qr - 4b^3r\log(f) - 4b^3\log(e)\right)x^4 - \frac{(3ab^2dpr - b^3cqr + 4ab^2dqr - 12ab^2dr\log(f) - 12ab^2d\log(e))x^3}{12d} + \frac{1}{4}\left(b^3prx^4 + 4ab^2prx^3 + 6a^2bprx^2 + 4a^3prx\right)\log(bx+a) + \frac{1}{4}\left(b^3qrx^4 + 4ab^2qrx^3 + 6a^2bqrx^2 + 4a^3qrx\right)\log(dx+c) - \frac{(3a^2bd^2pr + b^3c^2qr - 4ab^2cdqr + 6a^2bd^2qr - 12a^2bd^2r\log(f) - 12a^2bd^2\log(e))x^2}{8d^2} - \frac{(a^3d^3pr - b^3c^3qr + 4ab^2c^2dqr - 6a^2bcd^2qr + 4a^3d^3qr - 4a^3d^3r\log(f) - 4a^3d^3\log(e))x}{4d^3} - \frac{(b^3c^4qr - 4ab^2c^3dqr + 6a^2bc^2d^2qr - 4a^3cd^3qr)\log(-dx-c)}{4d^4}$$

input
```
integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
1/4*a^4*p*r*log(b*x + a)/b - 1/16*(b^3*p*r + b^3*q*r - 4*b^3*r*log(f) - 4*
b^3*log(e))*x^4 - 1/12*(3*a*b^2*d*p*r - b^3*c*q*r + 4*a*b^2*d*q*r - 12*a*b
^2*d*r*log(f) - 12*a*b^2*d*log(e))*x^3/d + 1/4*(b^3*p*r*x^4 + 4*a*b^2*p*r*
x^3 + 6*a^2*b*p*r*x^2 + 4*a^3*p*r*x)*log(b*x + a) + 1/4*(b^3*q*r*x^4 + 4*a
*b^2*q*r*x^3 + 6*a^2*b*q*r*x^2 + 4*a^3*q*r*x)*log(d*x + c) - 1/8*(3*a^2*b*
d^2*p*r + b^3*c^2*q*r - 4*a*b^2*c*d*q*r + 6*a^2*b*d^2*q*r - 12*a^2*b*d^2*r
*log(f) - 12*a^2*b*d^2*log(e))*x^2/d^2 - 1/4*(a^3*d^3*p*r - b^3*c^3*q*r +
4*a*b^2*c^2*d*q*r - 6*a^2*b*c*d^2*q*r + 4*a^3*d^3*q*r - 4*a^3*d^3*r*log(f)
 - 4*a^3*d^3*log(e))*x/d^3 - 1/4*(b^3*c^4*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*
b*c^2*d^2*q*r - 4*a^3*c*d^3*q*r)*log(-d*x - c)/d^4
```

3.8.9 Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.91

$$\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$$

$$= x^2 \left(\frac{\left(\frac{b^2 r (4 a d p + b c p + 5 a d q)}{4 d} - \frac{b^2 r (p+q) (4 a d + 4 b c)}{16 d}\right) (4 a d + 4 b c)}{8 b d} - \frac{a b r (3 a d p + 2 b c p + 5 a d q)}{4 d} + \frac{a b^2 c r (p+q)}{8 d} \right)$$
$$- x^3 \left(\frac{b^2 r (4 a d p + b c p + 5 a d q)}{12 d} - \frac{b^2 r (p+q) (4 a d + 4 b c)}{48 d} \right)$$
$$+ \ln \left(e \left(f (a+b x)^p (c+d x)^q\right)^r\right) \left(a^3 x + \frac{3 a^2 b x^2}{2} + a b^2 x^3 + \frac{b^3 x^4}{4}\right)$$
$$- x \left(\frac{(4 a d + 4 b c) \left(\frac{\left(\frac{b^2 r (4 a d p + b c p + 5 a d q)}{4 d} - \frac{b^2 r (p+q) (4 a d + 4 b c)}{16 d}\right) (4 a d + 4 b c)}{4 b d} - \frac{a b r (3 a d p + 2 b c p + 5 a d q)}{2 d} + \frac{a b^2 c r (p+q)}{4 d} \right)}{4 b d} + \frac{a^2 r (2 a d p + 3 b c p + 5 a d q)}{2 d} - \frac{a c \left(\frac{b^2 r (4 a d p + b c p + 5 a d q)}{4 d} - \frac{b^2 r (p+q) (4 a d + 4 b c)}{16 d}\right)}{b d} \right)$$
$$- \frac{\ln (c + d x) \left(-4 q r a^3 c d^3 + 6 q r a^2 b c^2 d^2 - 4 q r a b^2 c^3 d + q r b^3 c^4\right)}{4 d^4}$$
$$- \frac{b^3 r x^4 (p+q)}{16} + \frac{a^4 p r \ln (a + b x)}{4 b}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^3,x)
```

output
```
x^2*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d +
 4*b*c))/(16*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*r*(3*a*d*p + 2*b*c*p + 5*
a*d*q))/(4*d) + (a*b^2*c*r*(p + q))/(8*d)) - x^3*((b^2*r*(4*a*d*p + b*c*p
+ 5*a*d*q))/(12*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(48*d)) + log(e*(f*(a
 + b*x)^p*(c + d*x)^q)^r)*(a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x
^3) - x*(((4*a*d + 4*b*c)*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (
b^2*r*(p + q)*(4*a*d + 4*b*c))/(16*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*r*(
3*a*d*p + 2*b*c*p + 5*a*d*q))/(2*d) + (a*b^2*c*r*(p + q))/(4*d)))/(4*b*d)
+ (a^2*r*(2*a*d*p + 3*b*c*p + 5*a*d*q))/(2*d) - (a*c*((b^2*r*(4*a*d*p + b*
c*p + 5*a*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(16*d)))/(b*d)) -
(log(c + d*x)*(b^3*c^4*q*r - 4*a^3*c*d^3*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b
*c^2*d^2*q*r))/(4*d^4) - (b^3*r*x^4*(p + q))/16 + (a^4*p*r*log(a + b*x))/(
4*b)
```

3.8. $\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

3.9 $\int (a+bx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.9.1 Optimal result

Integrand size = 29, antiderivative size = 143

$$\int (a+bx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = -\frac{(bc-ad)^2qrx}{3d^2}+\frac{(bc-ad)qr(a+bx)^2}{6bd} -\frac{pr(a+bx)^3}{9b}-\frac{qr(a+bx)^3}{9b} +\frac{(bc-ad)^3qr\log(c+dx)}{3bd^3} +\frac{(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}$$

output
```
-1/3*(-a*d+b*c)^2*q*r*x/d^2+1/6*(-a*d+b*c)*q*r*(b*x+a)^2/b/d-1/9*p*r*(b*x+
a)^3/b-1/9*q*r*(b*x+a)^3/b+1/3*(-a*d+b*c)^3*q*r*ln(d*x+c)/b/d^3+1/3*(b*x+a
)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

3.9.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (a+bx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{-\frac{r\left(6bd(bc-ad)^2(p+3q)x-3b^2(bc-ad)(2p+3q)(c+dx)^2+2b^3(p+q)(c+dx)^3-6(bc-ad)^3q\log(c+dx)\right)}{6d^3}+(a+bx)^3\log\left(e\left(f(a+bx)^p(c\right.\right.}{3b}$$

input
```
Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
(-1/6*(r*(6*b*d*(b*c - a*d)^2*(p + 3*q)*x - 3*b^2*(b*c - a*d)*(2*p + 3*q)*
(c + d*x)^2 + 2*b^3*(p + q)*(c + d*x)^3 - 6*(b*c - a*d)^3*q*Log[c + d*x]))
/d^3 + (a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)
```

3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2981, 17, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$\Big\downarrow \; 2981$$

$$-\frac{dqr\int \frac{(a+bx)^3}{c+dx}dx}{3b}-\frac{1}{3}pr\int (a+bx)^2dx+\frac{(a+bx)^3\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \; 17$$

$$-\frac{dqr\int \frac{(a+bx)^3}{c+dx}dx}{3b}+\frac{(a+bx)^3\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}-\frac{pr(a+bx)^3}{9b}$$

$$\Big\downarrow \; 49$$

$$-\frac{dqr\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{3b}+\frac{(a+bx)^3\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}-\frac{pr(a+bx)^3}{9b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{dqr\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{3b}+\frac{(a+bx)^3\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}-\frac{pr(a+bx)^3}{9b}$$

input
```
Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/9*(p*r*(a + b*x)^3)/b - (d*q*r*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*
(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^
4))/(3*b) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)
```

3.9.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(131) = 262$.

Time = 70.99 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.15

method	result
parallelrisch	$\frac{12a^2bc\,d^2pr+9a^2bc\,d^2qr+6x^3\ln(e(f(bx+a)^p(dx+c)^q)^r)b^3d^3-18\ln(e(f(bx+a)^p(dx+c)^q)^r)a^3d^3+6c^3qr\,b^3+6a^3d^3pr+18a^3d^3}{}$

input
```
int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
1/18*(12*a^2*b*c*d^2*p*r+9*a^2*b*c*d^2*q*r+6*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)
^q)^r)*b^3*d^3-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*d^3+6*c^3*q*r*b^3+6*
a^3*d^3*p*r+18*a^3*d^3*q*r-15*a*c^2*d*q*r*b^2-2*x^3*b^3*d^3*p*r-2*x^3*b^3*
d^3*q*r+18*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^2*d^3+18*x*ln(e*(f*(b*x
+a)^p*(d*x+c)^q)^r)*a^2*b*d^3-36*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*b*c*d
^2+24*ln(b*x+a)*a^3*d^3*p*r+18*ln(d*x+c)*a^3*d^3*q*r+6*ln(d*x+c)*b^3*c^3*q
*r-6*x^2*a*b^2*d^3*p*r-9*x^2*a*b^2*d^3*q*r+3*x^2*b^3*c*d^2*q*r-6*x*a^2*b*d
^3*p*r-18*x*a^2*b*d^3*q*r-6*x*b^3*c^2*d*q*r+18*x*a*b^2*c*d^2*q*r+36*ln(b*x
+a)*a^2*b*c*d^2*p*r+54*ln(d*x+c)*a^2*b*c*d^2*q*r-18*ln(d*x+c)*a*b^2*c^2*d*
q*r)/b/d^3
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(131) = 262$.

Time = 0.29 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.27

$$\int (a+bx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\, dx = -\frac{2\,(b^3d^3p+b^3d^3q)rx^3+3\,(2\,ab^2d^3p-(b^3cd^2-3\,ab^2d^3)q)rx^2+6\,(a^2bd^3p+(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3)q}{}$$

input
```
integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas"
)
```

output
```
-1/18*(2*(b^3*d^3*p + b^3*d^3*q)*r*x^3 + 3*(2*a*b^2*d^3*p - (b^3*c*d^2 - 3
*a*b^2*d^3)*q)*r*x^2 + 6*(a^2*b*d^3*p + (b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2
*b*d^3)*q)*r*x - 6*(b^3*d^3*p*r*x^3 + 3*a*b^2*d^3*p*r*x^2 + 3*a^2*b*d^3*p*
r*x + a^3*d^3*p*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2
 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(
d*x + c) - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x)*log(e) - 6*(b
^3*d^3*r*x^3 + 3*a*b^2*d^3*r*x^2 + 3*a^2*b*d^3*r*x)*log(f))/(b*d^3)
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(124) = 248$.

Time = 132.78 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.41

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= \begin{cases} a^2x\log\left(e(a^pc^qf)^r\right) \\ a^2\left(\frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d} - qrx + x\log\left(e(a^pf(c+dx)^q)^r\right)\right) \\ \frac{a^3\log\left(e(c^qf(a+bx)^p)^r\right)}{3b} - \frac{a^2prx}{3} + a^2x\log\left(e(c^qf(a+bx)^p)^r\right) - \frac{abprx^2}{3} + abx^2\log\left(e(c^qf(a+bx)^p)^r\right) - \frac{b^2prx^3}{9} + \\ -\frac{a^3qr\log\left(\frac{c}{d}+x\right)}{3b} + \frac{a^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} + \frac{a^2cqr\log\left(\frac{c}{d}+x\right)}{d} - \frac{a^2prx}{3} - a^2qrx + a^2x\log\left(e(f(a+bx)^p(c+dx\right. \end{cases}$$

input
```
integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Piecewise((a**2*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a**2*(c*
log(e*(a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)*
*r)), Eq(b, 0)), (a**3*log(e*(c**q*f*(a + b*x)**p)**r)/(3*b) - a**2*p*r*x/
3 + a**2*x*log(e*(c**q*f*(a + b*x)**p)**r) - a*b*p*r*x**2/3 + a*b*x**2*log
(e*(c**q*f*(a + b*x)**p)**r) - b**2*p*r*x**3/9 + b**2*x**3*log(e*(c**q*f*(
a + b*x)**p)**r)/3, Eq(d, 0)), (-a**3*q*r*log(c/d + x)/(3*b) + a**3*log(e*
(f*(a + b*x)**p*(c + d*x)**q)**r)/(3*b) + a**2*c*q*r*log(c/d + x)/d - a**2
*p*r*x/3 - a**2*q*r*x + a**2*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - a
*b*c**2*q*r*log(c/d + x)/d**2 + a*b*c*q*r*x/d - a*b*p*r*x**2/3 - a*b*q*r*x
**2/2 + a*b*x**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) + b**2*c**3*q*r*l
og(c/d + x)/(3*d**3) - b**2*c**2*q*r*x/(3*d**2) + b**2*c*q*r*x**2/(6*d) -
b**2*p*r*x**3/9 - b**2*q*r*x**3/9 + b**2*x**3*log(e*(f*(a + b*x)**p*(c + d
*x)**q)**r)/3, True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.36

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= \frac{1}{3}\left(b^2x^3 + 3\,abx^2 + 3\,a^2x\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)$$
$$+ \frac{\left(\frac{6\,a^3fp\log(bx+a)}{b} - \frac{2\,b^2d^2f(p+q)x^3+3\,(abd^2f(2\,p+3\,q)-b^2cdfq)x^2+6\,(a^2d^2f(p+3\,q)+b^2c^2fq-3\,abcdfq)x}{d^2} + \frac{6\,(b^2c^3fq-3\,abc^2dfq+3\,a^2}{d^3}\right.}{18\,f}$$

input
```
integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima"
)
```

output
```
1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
 1/18*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*
(2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*
a*b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*
log(d*x + c)/d^3)*r/f
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(131) = 262$.

Time = 1.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.92

$$
\begin{aligned}
&\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx \\
&= \frac{a^3 pr \log (bx+a)}{3\,b} - \frac{1}{9}\left(b^2pr + b^2qr - 3\,b^2 r\log (f) - 3\,b^2\log (e)\right)x^3 \\
&\quad - \frac{(2\,abdpr - b^2cqr + 3\,abdqr - 6\,abdr\log (f) - 6\,abd\log (e))x^2}{6\,d} \\
&\quad + \frac{1}{3}\left(b^2prx^3 + 3\,abprx^2 + 3\,a^2prx\right)\log (bx+a) \\
&\quad + \frac{1}{3}\left(b^2qrx^3 + 3\,abqrx^2 + 3\,a^2qrx\right)\log (dx+c) \\
&\quad - \frac{(a^2d^2pr + b^2c^2qr - 3\,abcdqr + 3\,a^2d^2qr - 3\,a^2d^2r\log (f) - 3\,a^2d^2\log (e))x}{3\,d^2} \\
&\quad + \frac{(b^2c^3qr - 3\,abc^2dqr + 3\,a^2cd^2qr)\log (-dx-c)}{3\,d^3}
\end{aligned}
$$

input
```
integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
1/3*a^3*p*r*log(b*x + a)/b - 1/9*(b^2*p*r + b^2*q*r - 3*b^2*r*log(f) - 3*b
^2*log(e))*x^3 - 1/6*(2*a*b*d*p*r - b^2*c*q*r + 3*a*b*d*q*r - 6*a*b*d*r*lo
g(f) - 6*a*b*d*log(e))*x^2/d + 1/3*(b^2*p*r*x^3 + 3*a*b*p*r*x^2 + 3*a^2*p*
r*x)*log(b*x + a) + 1/3*(b^2*q*r*x^3 + 3*a*b*q*r*x^2 + 3*a^2*q*r*x)*log(d*
x + c) - 1/3*(a^2*d^2*p*r + b^2*c^2*q*r - 3*a*b*c*d*q*r + 3*a^2*d^2*q*r -
3*a^2*d^2*r*log(f) - 3*a^2*d^2*log(e))*x/d^2 + 1/3*(b^2*c^3*q*r - 3*a*b*c^
2*d*q*r + 3*a^2*c*d^2*q*r)*log(-d*x - c)/d^3
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.78

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= x\left(\frac{\left(\frac{b\,r\,(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q)}{3\,d}-\frac{b\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{9\,d}\right)(3\,a\,d+3\,b\,c)}{3\,b\,d}-\frac{a\,r\,(a\,d\,p+b\,c\,p+2\,a\,d\,q)}{d}+\frac{a\,b\,c\,r\,(p+q)}{3\,d}\right)-x^2\left(\frac{b\,r\,(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q)}{6\,d}-\frac{b\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{18\,d}\right)+\ln\left(e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}\right)+\frac{\ln(c+d\,x)\,\left(3\,q\,r\,a^2\,c\,d^2-3\,q\,r\,a\,b\,c^2\,d+q\,r\,b^2\,c^3\right)}{3\,d^3}-\frac{b^2\,r\,x^3\,(p+q)}{9}+\frac{a^3\,p\,r\,\ln(a+b\,x)}{3\,b}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^2,x)
```

output
```
x*((((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(3*d) - (b*r*(p + q)*(3*a*d + 3*b*c
))/(9*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*r*(a*d*p + b*c*p + 2*a*d*q))/d + (
a*b*c*r*(p + q))/(3*d)) - x^2*((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(6*d) - (
b*r*(p + q)*(3*a*d + 3*b*c))/(18*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r
)*(a^2*x + (b^2*x^3)/3 + a*b*x^2) + (log(c + d*x)*(b^2*c^3*q*r + 3*a^2*c*d
^2*q*r - 3*a*b*c^2*d*q*r))/(3*d^3) - (b^2*r*x^3*(p + q))/9 + (a^3*p*r*log(
a + b*x))/(3*b)
```

3.10 $\int (a+bx)\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

3.10.1 Optimal result

Integrand size = 27, antiderivative size = 116

$$\int (a+bx)\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = -\frac{1}{2}aprx+\frac{(bc-ad)qrx}{2d}-\frac{1}{4}bprx^2 -\frac{qr(a+bx)^2}{4b}-\frac{(bc-ad)^2qr\log(c+dx)}{2bd^2} +\frac{(a+bx)^2\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$

output
```
-1/2*a*p*r*x+1/2*(-a*d+b*c)*q*r*x/d-1/4*b*p*r*x^2-1/4*q*r*(b*x+a)^2/b-1/2*
(-a*d+b*c)^2*q*r*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b
```

3.10.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int (a+bx)\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \frac{a^2pr\log(a+bx)}{2b} -\frac{2c(bc-2ad)qr\log(c+dx)+dx(r(-2bcq+2ad(p+2q)+bd(p+q)x)-2d(2a+bx)\log \left(e(f(a+bx)^p\right.}{4d^2}$$

input
```
Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
(a^2*p*r*Log[a + b*x])/(2*b) - (2*c*(b*c - 2*a*d)*q*r*Log[c + d*x] + d*x*(
r*(-2*b*c*q + 2*a*d*(p + 2*q) + b*d*(p + q)*x) - 2*d*(2*a + b*x)*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]))/(4*d^2)
```

3.10.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2981, 17, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$$

$$\Big\downarrow \; 2981$$

$$-\frac{dqr\int\frac{(a+bx)^2}{c+dx}dx}{2b}-\frac{1}{2}pr\int(a+bx)dx+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{dqr\int\frac{(a+bx)^2}{c+dx}dx}{2b}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}$$

$$\Big\downarrow \; 49$$

$$-\frac{dqr\int\left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{2b}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{dqr\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{2b}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}$$

input
```
Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/4*(p*r*(a + b*x)^2)/b - (d*q*r*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/
(2*d) + ((b*c - a*d)^2*Log[c + d*x])/d^3))/(2*b) + ((a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(2*b)
```

3.10.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(104) = 208$.

Time = 12.56 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.63

method	result
parallelrisch	$\frac{-x^{2} b^{2} d^{2} p r - x^{2} b^{2} d^{2} q r + 6 \ln(bx+a) a^{2} d^{2} p r + 6 \ln(bx+a) a b c d p r + 4 \ln(dx+c) a^{2} d^{2} q r + 10 \ln(dx+c) a b c d q r - 2 \ln(dx+c) b^{2} c^{2} q r + 2x^{2}}{}$

input
```
int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-x^2*b^2*d^2*p*r-x^2*b^2*d^2*q*r+6*ln(b*x+a)*a^2*d^2*p*r+6*ln(b*x+a)*
a*b*c*d*p*r+4*ln(d*x+c)*a^2*d^2*q*r+10*ln(d*x+c)*a*b*c*d*q*r-2*ln(d*x+c)*b
^2*c^2*q*r+2*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^2*d^2-2*x*a*b*d^2*p*r-4
*x*a*b*d^2*q*r+2*x*b^2*c*d*q*r+4*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*d^2
+2*a^2*d^2*p*r+4*a^2*q*r*d^2+3*a*b*c*d*p*r+3*a*b*c*d*q*r-2*b^2*c^2*q*r-4*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*d^2-6*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a
*b*c*d)/b/d^2
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int (a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{(b^2d^2p+b^2d^2q)rx^2+2\,(abd^2p-(b^2cd-2\,abd^2)q)rx-2\,(b^2d^2prx^2+2\,abd^2prx+a^2d^2pr)\log{(bx+a)}}{}$$

input
```
integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

output
```
-1/4*((b^2*d^2*p + b^2*d^2*q)*r*x^2 + 2*(a*b*d^2*p - (b^2*c*d - 2*a*b*d^2)
*q)*r*x - 2*(b^2*d^2*p*r*x^2 + 2*a*b*d^2*p*r*x + a^2*d^2*p*r)*log(b*x + a)
 - 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2 - 2*a*b*c*d)*q*r)*log(d
*x + c) - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x)*log(e) - 2*(b^2*d^2*r*x^2 + 2*a*b*
d^2*r*x)*log(f))/(b*d^2)
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(104) = 208.

Time = 20.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.80

$$\int (a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \begin{cases} ax\log\left(e(a^pc^qf)^r\right) \\ a\left(\frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d}-qrx+x\log\left(e(a^pf(c+dx)^q)^r\right)\right) \\ \frac{a^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2b}-\frac{aprx}{2}+ax\log\left(e(c^qf(a+bx)^p)^r\right)-\frac{bprx^2}{4}+\frac{bx^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2} \\ -\frac{a^2qr\log\left(\frac{c}{d}+x\right)}{2b}+\frac{a^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}+\frac{acqr\log\left(\frac{c}{d}+x\right)}{d}-\frac{aprx}{2}-aqrx+ax\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right) \end{cases}$$

input
```
integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Piecewise((a*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a*(c*log(e*
(a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)**r)),
Eq(b, 0)), (a**2*log(e*(c**q*f*(a + b*x)**p)**r)/(2*b) - a*p*r*x/2 + a*x*l
og(e*(c**q*f*(a + b*x)**p)**r) - b*p*r*x**2/4 + b*x**2*log(e*(c**q*f*(a +
b*x)**p)**r)/2, Eq(d, 0)), (-a**2*q*r*log(c/d + x)/(2*b) + a**2*log(e*(f*(
a + b*x)**p*(c + d*x)**q)**r)/(2*b) + a*c*q*r*log(c/d + x)/d - a*p*r*x/2 -
 a*q*r*x + a*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - b*c**2*q*r*log(c/
d + x)/(2*d**2) + b*c*q*r*x/(2*d) - b*p*r*x**2/4 - b*q*r*x**2/4 + b*x**2*l
og(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/2, True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$=\frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left(\left(bx+a\right)^p(dx+c)^qf\right)^re\right)$$
$$+\frac{\left(\frac{2\,a^2fp\log(bx+a)}{b}-\frac{bdf(p+q)x^2+2\,(adf(p+2\,q)-bcfq)x}{d}-\frac{2\,(bc^2fq-2\,acdfq)\log(dx+c)}{d^2}\right)r}{4\,f}$$

input
```
integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*(2*a^2*f*p*
log(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d -
 2*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r/f
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$=\frac{a^2pr\log(bx+a)}{2\,b}-\frac{1}{4}\left(bpr+bqr-2\,br\log(f)-2\,b\log(e)\right)x^2$$
$$+\frac{1}{2}\left(bprx^2+2\,aprx\right)\log(bx+a)+\frac{1}{2}\left(bqrx^2+2\,aqrx\right)\log(dx+c)$$
$$-\frac{(adpr-bcqr+2\,adqr-2\,adr\log(f)-2\,ad\log(e))x}{2\,d}$$
$$-\frac{(bc^2qr-2\,acdqr)\log(-dx-c)}{2\,d^2}$$

input
```
integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
1/2*a^2*p*r*log(b*x + a)/b - 1/4*(b*p*r + b*q*r - 2*b*r*log(f) - 2*b*log(e
))*x^2 + 1/2*(b*p*r*x^2 + 2*a*p*r*x)*log(b*x + a) + 1/2*(b*q*r*x^2 + 2*a*q
*r*x)*log(d*x + c) - 1/2*(a*d*p*r - b*c*q*r + 2*a*d*q*r - 2*a*d*r*log(f) -
 2*a*d*log(e))*x/d - 1/2*(b*c^2*q*r - 2*a*c*d*q*r)*log(-d*x - c)/d^2
```

3.10.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(\frac{b\,x^2}{2}+a\,x\right) - x\left(\frac{r\,(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q)}{2\,d} - \frac{r\,(p+q)\,(2\,a\,d+2\,b\,c)}{4\,d}\right) - \frac{\ln(c+d\,x)\,(b\,c^2\,q\,r-2\,a\,c\,d\,q\,r)}{2\,d^2} - \frac{b\,r\,x^2\,(p+q)}{4} + \frac{a^2\,p\,r\,\ln(a+b\,x)}{2\,b}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x),x)
```

output
```
log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a*x + (b*x^2)/2) - x*((r*(2*a*d*p +
b*c*p + 3*a*d*q))/(2*d) - (r*(p + q)*(2*a*d + 2*b*c))/(4*d)) - (log(c + d*
x)*(b*c^2*q*r - 2*a*c*d*q*r))/(2*d^2) - (b*r*x^2*(p + q))/4 + (a^2*p*r*log
(a + b*x))/(2*b)
```

3.11 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

3.11.1 Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} - \frac{qr\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}$$

output
```
-1/2*p*r*ln(b*x+a)^2/b-q*r*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b+ln(b*x+a)*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-q*r*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b
```

3.11.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = -\frac{\log(a+bx)\left(pr\log(a+bx) + 2qr\log\left(\frac{b(c+dx)}{bc-ad}\right) - 2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right) + 2qr\,\mathrm{PolyLog}\left(2, \ldots\right.}{2b}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

output
```
-1/2*(Log[a + b*x]*(p*r*Log[a + b*x] + 2*q*r*Log[(b*(c + d*x))/(b*c - a*d)
] - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*q*r*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)])/b
```

3.11.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2980, 2837, 2738, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx} \, dx$$

$$\Big\downarrow \text{ 2980}$$

$$-\frac{dqr\int \frac{\log(a+bx)}{c+dx}dx}{b}-pr\int \frac{\log(a+bx)}{a+bx}dx+\frac{\log(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

$$\Big\downarrow \text{ 2837}$$

$$-\frac{dqr\int \frac{\log(a+bx)}{c+dx}dx}{b}-\frac{pr\int \frac{\log(a+bx)}{a+bx}d(a+bx)}{b}+\frac{\log(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

$$\Big\downarrow \text{ 2738}$$

$$-\frac{dqr\int \frac{\log(a+bx)}{c+dx}dx}{b}+\frac{\log(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 2841}$$

$$-\frac{dqr\left(\frac{\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d}-\frac{b\int \frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}dx}{d}\right)}{b}+\frac{\log(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 2840}$$

$$-\frac{dqr\left(\frac{\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d}-\frac{\int \frac{\log\left(\frac{d(a+bx)}{bc-ad}+1\right)}{a+bx}d(a+bx)}{d}\right)}{b}+\frac{\log(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

$\downarrow$ 2838

$$\frac{\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{dqr\left(\frac{\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{d}+\frac{\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d}\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

output
```
-1/2*(p*r*Log[a + b*x]^2)/b + (Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r])/b - (d*q*r*((Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d + Poly
Log[2, -((d*(a + b*x))/(b*c - a*d))]/d))/b
```

3.11.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2980
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Simp[b*p*(r/h)   Int[Log[g + h*x]/(a + b
*x), x], x] - Simp[d*q*(r/h)   Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

3.11.4 Maple [A] (verified)

Time = 9.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

method	result	size
parts	$\frac{\ln(bx+a)\ln\left(e\left(f(bx+a)^{p}(dx+c)^{q}\right)^{r}\right)}{b}-\frac{r\left(\frac{bp\ln(bx+a)^{2}}{2}+bdq\left(\frac{\operatorname{dilog}\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{d}+\frac{\ln(bx+a)\ln\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{d}\right)\right)}{b^{2}}$	125

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/b^2*r*(1/2*b*p*ln(b*x+a)^2+b
*d*q*(dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))/d+ln(b*x+a)*ln((-a*d+c*b+d*(b
*x+a))/(-a*d+b*c))/d))
```

3.11.5 Fricas [F]

$$\int \frac{\log\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)}{a+bx}\,dx=\int \frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{bx+a}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

3.11.6 Sympy [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p\left(c+dx\right)^q\right)^r\right)}{a+bx}\,dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a),x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(a + b*x), x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx \\
&= -\frac{\left(\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)fq}{b} - \frac{fp\log(bx+a)^2+2\,fq\log(bx+a)\log(dx+c)}{b}\right)r}{2\,f} \\
&\quad - \frac{(fp\log\left(bx+a\right)+fq\log\left(dx+c\right))r\log\left(bx+a\right)}{bf} \\
&\quad + \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(bx+a\right)}{b}
\end{aligned}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
 a*d)/(b*c - a*d)))*f*q/b - (f*p*log(b*x + a)^2 + 2*f*q*log(b*x + a)*log(d
*x + c))/b)*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(b*x + a)/(b*
f) + log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(b*x + a)/b
```

3.11.8 Giac [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{bx+a}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{a+b\,x}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x), x)
```

3.12 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2} dx$

3.12.1 Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{pr}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

output
```
-p*r/b/(b*x+a)+d*q*r*ln(b*x+a)/b/(-a*d+b*c)-d*q*r*ln(d*x+c)/b/(-a*d+b*c)-l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)
```

3.12.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{r\left(-\frac{p}{a+bx}+\frac{dq\log(a+bx)}{bc-ad}-\frac{dq\log(c+dx)}{bc-ad}\right)}{b} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

output
```
(r*(-(p/(a + b*x)) + (d*q*Log[a + b*x])/(b*c - a*d) - (d*q*Log[c + d*x])/(
b*c - a*d)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

3.12.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2981, 17, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 2981}$$

$$\frac{dqr\int \frac{1}{(a+bx)(c+dx)}dx}{b}+pr\int \frac{1}{(a+bx)^2}dx-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{dqr\int \frac{1}{(a+bx)(c+dx)}dx}{b}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}-\frac{pr}{b(a+bx)}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{dqr\left(\frac{b\int \frac{1}{a+bx}dx}{bc-ad}-\frac{d\int \frac{1}{c+dx}dx}{bc-ad}\right)}{b}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}-\frac{pr}{b(a+bx)}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+\frac{dqr\left(\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\right)}{b}-\frac{pr}{b(a+bx)}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

output
```
-((p*r)/(b*(a + b*x))) + (d*q*r*(Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(
b*c - a*d)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

3.12.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.12.4 Maple [A] (verified)

Time = 31.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.78

method	result
parallelrisch	$-\frac{\ln(bx+a)x\,b^3d^2qr-\ln(dx+c)x\,b^3d^2qr+\ln(bx+a)a\,b^2d^2qr-\ln(dx+c)a\,b^2d^2qr+a\,b^2d^2pr-b^3cdpr+\ln(e(f(bx+a)^p(dx+c)^q)^r)}{(ad-cb)(bx+a)b^3d}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(ln(b*x+a)*x*b^3*d^2*q*r-ln(d*x+c)*x*b^3*d^2*q*r+ln(b*x+a)*a*b^2*d^2*q*r-
ln(d*x+c)*a*b^2*d^2*q*r+a*b^2*d^2*p*r-b^3*c*d*p*r+ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r)*a*b^2*d^2-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*c*d)/(a*d-b*c)/(b*x+
a)/b^3/d
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{(bc-ad)pr+(bc-ad)r\log{(f)}-(bdqrx+(adq-(bc-ad)p)r)\log{(bx+a)}+(bdqrx+bcqr)\log{(dx}}{ab^2c-a^2bd+(b^3c-ab^2d)x}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="fricas"
)
```

output
```
-((b*c - a*d)*p*r + (b*c - a*d)*r*log(f) - (b*d*q*r*x + (a*d*q - (b*c - a*
d)*p)*r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c) + (b*c - a*d)*l
og(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

3.12.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**2,x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{\left(dfq\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)-\frac{bfp}{b^2x+ab}\right)r}{bf} - \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(bx+a)b}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="maxima"
)
```

output
```
(d*f*q*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - b*f*p/(b^2*
x + a*b))*r/(b*f) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)*b)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{dqr\log(bx+a)}{b^2c-abd} - \frac{dqr\log(dx+c)}{b^2c-abd} - \frac{pr\log(bx+a)}{b^2x+ab} - \frac{qr\log(dx+c)}{b^2x+ab} - \frac{pr+r\log(f)+\log(e)}{b^2x+ab}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="giac")
```

output
```
d*q*r*log(b*x + a)/(b^2*c - a*b*d) - d*q*r*log(d*x + c)/(b^2*c - a*b*d) -
p*r*log(b*x + a)/(b^2*x + a*b) - q*r*log(d*x + c)/(b^2*x + a*b) - (p*r + r
*log(f) + log(e))/(b^2*x + a*b)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\left(x+\frac{a}{b}\right)}{\left(a+b\,x\right)^2} - \frac{p\,r}{x\,b^2+a\,b} + \frac{d\,q\,r\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^2,x)
```

output
```
(d*q*r*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c)) - (p
*r)/(a*b + b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x + a/b))/(a +
b*x)^2
```

3.13 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}\,dx$

3.13.1 Optimal result

Integrand size = 29, antiderivative size = 135

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{pr}{4b(a+bx)^2}-\frac{dqr}{2b(bc-ad)(a+bx)}-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}$$

output
```
-1/4*p*r/b/(b*x+a)^2-1/2*d*q*r/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*q*r*ln(b*x+a)/
b/(-a*d+b*c)^2+1/2*d^2*q*r*ln(d*x+c)/b/(-a*d+b*c)^2-1/2*ln(e*(f*(b*x+a)^p*
(d*x+c)^q)^r)/b/(b*x+a)^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \frac{r\left(-\frac{p-\frac{2dq(a+bx)}{-bc+ad}}{2(a+bx)^2}-\frac{d^2q\log(a+bx)}{(bc-ad)^2}+\frac{d^2q\log(c+dx)}{(bc-ad)^2}\right)-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}}{2b}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

output
```
(r*(-1/2*(p - (2*d*q*(a + b*x))/(-(b*c) + a*d))/(a + b*x)^2 - (d^2*q*Log[a
 + b*x])/(b*c - a*d)^2 + (d^2*q*Log[c + d*x])/(b*c - a*d)^2) - Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2)/(2*b)
```

3.13.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2981, 17, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2981}$$

$$\frac{dqr\int \frac{1}{(a+bx)^2(c+dx)}dx}{2b}+\frac{1}{2}pr\int \frac{1}{(a+bx)^3}dx-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{dqr\int \frac{1}{(a+bx)^2(c+dx)}dx}{2b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{pr}{4b(a+bx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{dqr\int \left(\frac{d^2}{(bc-ad)^2(c+dx)}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{b}{(bc-ad)(a+bx)^2}\right)dx}{2b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{pr}{4b(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}+\frac{dqr\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{2b}-\frac{pr}{4b(a+bx)^2}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

output
```
-1/4*(p*r)/(b*(a + b*x)^2) + (d*q*r*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log
[a + b*x])/(b*c - a*d)^2 + (d*Log[c + d*x])/(b*c - a*d)^2))/(2*b) - Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)
```

3.13.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.13.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(125) = 250$.

Time = 150.01 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.68

method	result
parallelrisch	$-\frac{2\ln(bx+a)x^2a^4b^2c\,d^2pr+2\ln(bx+a)x^2a^4b^2c\,d^2qr-4\ln(bx+a)x^2a^3b^3c^2dpr-4\ln(dx+c)x^2a^3b^3c^2dqr+4\ln(bx+a)x\,a^5bc\,d^2p}{}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*ln(b*x+a)*x^2*a^4*b^2*c*d^2*p*r+2*ln(b*x+a)*x^2*a^4*b^2*c*d^2*q*r-
4*ln(b*x+a)*x^2*a^3*b^3*c^2*d*p*r-4*ln(d*x+c)*x^2*a^3*b^3*c^2*d*q*r+4*ln(b
*x+a)*x*a^5*b*c*d^2*p*r+4*ln(b*x+a)*x*a^5*b*c*d^2*q*r-8*ln(b*x+a)*x*a^4*b^
2*c^2*d*p*r-8*ln(d*x+c)*x*a^4*b^2*c^2*d*q*r-4*ln(b*x+a)*a^5*b*c^2*d*p*r-4*
ln(d*x+c)*a^5*b*c^2*d*q*r-x^2*a^4*b^2*c*d^2*p*r+2*x^2*a^4*b^2*c*d^2*q*r+2*
x^2*a^3*b^3*c^2*d*p*r-2*x^2*a^3*b^3*c^2*d*q*r-2*x*a^5*b*c*d^2*p*r+2*x*a^5*
b*c*d^2*q*r+4*x*a^4*b^2*c^2*d*p*r-2*x*a^4*b^2*c^2*d*q*r-2*x^2*ln(e*(f*(b*x
+a)^p*(d*x+c)^q)^r)*a^2*b^4*c^3-4*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b^
3*c^3+2*ln(b*x+a)*x^2*a^2*b^4*c^3*p*r+2*ln(d*x+c)*x^2*a^2*b^4*c^3*q*r+4*ln
(b*x+a)*x*a^3*b^3*c^3*p*r+4*ln(d*x+c)*x*a^3*b^3*c^3*q*r+2*ln(b*x+a)*a^6*c*
d^2*p*r+2*ln(b*x+a)*a^6*c*d^2*q*r+2*ln(b*x+a)*a^4*b^2*c^3*p*r+2*ln(d*x+c)*
a^4*b^2*c^3*q*r-x^2*a^2*b^4*c^3*p*r-2*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*
a^4*b^2*c*d^2+4*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b^3*c^2*d-2*x*a^3*
b^3*c^3*p*r-4*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^5*b*c*d^2+8*x*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)*a^4*b^2*c^2*d)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/(b*x+a)
^2/c/a^4
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(125) = 250$.

Time = 0.32 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.39

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{2\left(b^2cd-abd^2\right)qrx+2\left(b^2c^2-2\,abcd+a^2d^2\right)r\log\left(f\right)+\left(\left(b^2c^2-2\,abcd+a^2d^2\right)p+2\left(abcd-a^2d^2\right)q\right)r\ \cdots}{4\left(a^2b^3c^2-2\ \cdots\right.}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="fricas"
)
```

output
```
-1/4*(2*(b^2*c*d - a*b*d^2)*q*r*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*r*lo
g(f) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p + 2*(a*b*c*d - a^2*d^2)*q)*r + 2
*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x + (a^2*d^2*q + (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*p)*r)*log(b*x + a) - 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*
c^2 - 2*a*b*c*d)*q*r)*log(d*x + c) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a
^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

3.13.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**3,x)
```

output
```
Timed out
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.22

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{\left(2\,dfq\left(\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{abc-a^2d+(b^2c-abd)x}\right)+\frac{bfp}{b^3x^2+2\,ab^2x+a^2b}\right)r}{4\,bf} - \frac{\log\left(((bx+a)^p(dx+c)^qf)^r\,e\right)}{2\,(bx+a)^2b}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="maxima"
)
```

output
```
-1/4*(2*d*f*q*(d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - d*log(d*x
+ c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*
x)) + b*f*p/(b^3*x^2 + 2*a*b^2*x + a^2*b))*r/(b*f) - 1/2*log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)/((b*x + a)^2*b)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.85

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{d^2qr\log(bx+a)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)} + \frac{d^2qr\log(dx+c)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)} - \frac{pr\log(bx+a)}{2\,(b^3x^2+2\,ab^2x+a^2b)} - \frac{qr\log(dx+c)}{2\,(b^3x^2+2\,ab^2x+a^2b)} - \frac{2\,bdqrx+bcpr-adpr+2\,adqr+2\,bcr\log(f)-2\,adr\log(f)+2\,bc\log(e)-2\,ad\log(e)}{4\,(b^4cx^2-ab^3dx^2+2\,ab^3cx-2\,a^2b^2dx+a^2b^2c-a^3bd)}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*d^2*q*r*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 1/2*d^2*q*
r*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*p*r*log(b*x + a)/
(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*q*r*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x
+ a^2*b) - 1/4*(2*b*d*q*r*x + b*c*p*r - a*d*p*r + 2*a*d*q*r + 2*b*c*r*log(
f) - 2*a*d*r*log(f) + 2*b*c*log(e) - 2*a*d*log(e))/(b^4*c*x^2 - a*b^3*d*x^
2 + 2*a*b^3*c*x - 2*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \frac{\frac{b\,c\,p\,r-a\,d\,p\,r+2\,a\,d\,q\,r}{2\,(a\,d-b\,c)}+\frac{b\,d\,q\,r\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2} - \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(\frac{x}{2}+\frac{a}{2\,b}\right)}{(a+b\,x)^3} + \frac{d^2\,q\,r\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,(a\,d-b\,c)^2}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^3,x)
```

output
```
((b*c*p*r - a*d*p*r + 2*a*d*q*r)/(2*(a*d - b*c)) + (b*d*q*r*x)/(a*d - b*c)
)/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r
)*(x/2 + a/(2*b)))/(a + b*x)^3 + (d^2*q*r*atanh((2*b^3*c^2 - 2*a^2*b*d^2)/
(2*b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

3.14 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} dx$

3.14.1 Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4} dx = -\frac{pr}{9b(a+bx)^3} - \frac{dqr}{6b(bc-ad)(a+bx)^2} + \frac{d^2qr}{3b(bc-ad)^2(a+bx)} + \frac{d^3qr\log(a+bx)}{3b(bc-ad)^3} - \frac{d^3qr\log(c+dx)}{3b(bc-ad)^3} - \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

output
```
-1/9*p*r/b/(b*x+a)^3-1/6*d*q*r/b/(-a*d+b*c)/(b*x+a)^2+1/3*d^2*q*r/b/(-a*d+
b*c)^2/(b*x+a)+1/3*d^3*q*r*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d^3*q*r*ln(d*x+c)/
b/(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3
```

3.14.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4} dx = \frac{r\left(\frac{-2p+\frac{3dq(a+bx)}{-bc+ad}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}}{6(a+bx)^3} + \frac{d^3q\log(a+bx)}{(bc-ad)^3} - \frac{d^3q\log(c+dx)}{(bc-ad)^3}\right) - \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}}{3b}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

output
```
(r*((-2*p + (3*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c
- a*d)^2)/(6*(a + b*x)^3) + (d^3*q*Log[a + b*x])/(b*c - a*d)^3 - (d^3*q*Lo
g[c + d*x])/(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x
)^3)/(3*b)
```

3.14.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2981, 17, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} \, dx$$

$$\Big\downarrow \ 2981$$

$$\frac{dqr\int \frac{1}{(a+bx)^3(c+dx)}dx}{3b}+\frac{1}{3}pr\int \frac{1}{(a+bx)^4}dx-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}$$

$$\Big\downarrow \ 17$$

$$\frac{dqr\int \frac{1}{(a+bx)^3(c+dx)}dx}{3b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+bx)^3}$$

$$\Big\downarrow \ 54$$

$$\frac{dqr\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{3b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+bx)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{dqr\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{3b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+bx)^3}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

output
```
-1/9*(p*r)/(b*(a + b*x)^3) + (d*q*r*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/
((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c
+ d*x])/(b*c - a*d)^3))/(3*b) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*b*
(a + b*x)^3)
```

3.14.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.14.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^{p}(dx+c)^{q}\right)^{r}\right)}{(bx+a)^{4}} dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(152) = 304.

Time = 0.34 (sec) , antiderivative size = 580, normalized size of antiderivative = 3.54

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx$$
$$= \frac{6\,(b^3cd^2-ab^2d^3)qrx^2-3\,(b^3c^2d-6\,ab^2cd^2+5\,a^2bd^3)qrx-6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)r\log{(f)}}{}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="fricas"
)
```

output
```
1/18*(6*(b^3*c*d^2 - a*b^2*d^3)*q*r*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5
*a^2*b*d^3)*q*r*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
r*log(f) - (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*p + 3*(a
*b^2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*q)*r + 6*(b^3*d^3*q*r*x^3 + 3*a*b^
2*d^3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (a^3*d^3*q - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*p)*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b
^2*d^3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2)*q*r)*log(d*x + c) - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3
+ (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6
*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**4,x)
```

output
```
Timed out
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.76

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx$$
$$= \frac{\left(3\left(\frac{2\,d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} - \frac{2\,d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} + \frac{2\,bdx-bc+3\,ad}{a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2 \cdots}\right.\right.}{18\,bf}$$
$$- \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{3\,(bx+a)^3 b}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="maxima"
)
```

output
```
1/18*(3*(2*d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3) - 2*d^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) + (2*b*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4
*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*x))*d*f*q - 2*b*f*p/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b))
*r/(b*f) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^3*b)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(152) = 304$.

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.90

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx$$
$$= \frac{d^3qr\log(bx+a)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)} - \frac{d^3qr\log(dx+c)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)}$$
$$- \frac{pr\log(bx+a)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{qr\log(dx+c)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$
$$+ \frac{6\,b^2d^2qrx^2-3\,b^2cdqrx+15\,abd^2qrx-2\,b^2c^2pr+4\,abcdpr-2\,a^2d^2pr-3\,abcdqr+9\,a^2d^2qr-6\,b^2c^2r\,l \cdots}{18\,(b^6c^2x^3-2\,ab^5cdx^3+a^2b^4d^2x^3+3\,ab^5c^2x^2-6\,a^2b^4cdx^2+3\,a^3b^3d^2x^2+3 \cdots}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/3*d^3*q*r*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3) - 1/3*d^3*q*r*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d
^2 - a^3*b*d^3) - 1/3*p*r*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*
x + a^3*b) - 1/3*q*r*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a
^3*b) + 1/18*(6*b^2*d^2*q*r*x^2 - 3*b^2*c*d*q*r*x + 15*a*b*d^2*q*r*x - 2*b
^2*c^2*p*r + 4*a*b*c*d*p*r - 2*a^2*d^2*p*r - 3*a*b*c*d*q*r + 9*a^2*d^2*q*r
 - 6*b^2*c^2*r*log(f) + 12*a*b*c*d*r*log(f) - 6*a^2*d^2*r*log(f) - 6*b^2*c
^2*log(e) + 12*a*b*c*d*log(e) - 6*a^2*d^2*log(e))/(b^6*c^2*x^3 - 2*a*b^5*c
*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3
*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c
^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.11

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx = \frac{\frac{x\,\left(5\,a\,b\,d^2\,q\,r-b^2\,c\,d\,q\,r\right)}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{2\,a^2\,d^2\,p\,r+2\,b^2\,c^2\,p\,r-9\,a^2\,d^2\,q\,r-4\,a\,b\,c\,d\,p\,r+3\,a\,b\,c\,d\,q\,r}{6\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b^2\,d^2\,q\,r\,x^2}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3} - \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(\frac{x}{3}+\frac{a}{3\,b}\right)}{{\left(a+b\,x\right)}^4} - \frac{2\,d^3\,q\,r\,\mathrm{atanh}\left(\frac{3\,a^3\,b\,d^3-3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+3\,b^4\,c^3}{3\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{3\,b\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^4,x)
```

output
```
((x*(5*a*b*d^2*q*r - b^2*c*d*q*r))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (
2*a^2*d^2*p*r + 2*b^2*c^2*p*r - 9*a^2*d^2*q*r - 4*a*b*c*d*p*r + 3*a*b*c*d*
q*r)/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b^2*d^2*q*r*x^2)/(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2) - (l
og(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/3 + a/(3*b)))/(a + b*x)^4 - (2*d^3*
q*r*atanh((3*b^4*c^3 + 3*a^3*b*d^3 - 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)/(3*b
*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3)
)/(3*b*(a*d - b*c)^3)
```

3.15 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5} dx$

3.15.1 Optimal result

Integrand size = 29, antiderivative size = 193

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5} dx = -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{d^3qr}{4b(bc-ad)^3(a+bx)} - \frac{d^4qr\log(a+bx)}{4b(bc-ad)^4} + \frac{d^4qr\log(c+dx)}{4b(bc-ad)^4} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

output
```
-1/16*p*r/b/(b*x+a)^4-1/12*d*q*r/b/(-a*d+b*c)/(b*x+a)^3+1/8*d^2*q*r/b/(-a*
d+b*c)^2/(b*x+a)^2-1/4*d^3*q*r/b/(-a*d+b*c)^3/(b*x+a)-1/4*d^4*q*r*ln(b*x+a
)/b/(-a*d+b*c)^4+1/4*d^4*q*r*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)/b/(b*x+a)^4
```

3.15.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5} dx = \frac{r\left(\frac{-3p+\frac{4dq(a+bx)}{-bc+ad}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}-\frac{12d^3q(a+bx)^3}{(bc-ad)^3}}{12(a+bx)^4}-\frac{d^4q\log(a+bx)}{(bc-ad)^4}+\frac{d^4q\log(c+dx)}{(bc-ad)^4}\right)-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4}}{4b}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

output
```
(r*((-3*p + (4*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c
- a*d)^2 - (12*d^3*q*(a + b*x)^3)/(b*c - a*d)^3)/(12*(a + b*x)^4) - (d^4*q
*Log[a + b*x])/(b*c - a*d)^4 + (d^4*q*Log[c + d*x])/(b*c - a*d)^4) - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4)/(4*b)
```

3.15.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2981, 17, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5} dx$$

$$\Big\downarrow \text{ 2981}$$

$$\frac{dqr\int \frac{1}{(a+bx)^4(c+dx)}dx}{4b}+\frac{1}{4}pr\int \frac{1}{(a+bx)^5}dx-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{dqr\int \frac{1}{(a+bx)^4(c+dx)}dx}{4b}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}-\frac{pr}{16b(a+bx)^4}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{dqr\int\left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{4b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}-\frac{pr}{16b(a+bx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{dqr\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{4b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}-\frac{pr}{16b(a+bx)^4}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

output
```
-1/16*(p*r)/(b*(a + b*x)^4) + (d*q*r*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d
/(2*(b*c - a*d)^2*(a + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[
a + b*x])/(b*c - a*d)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(4*b) - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4)
```

3.15.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.15.4 Maple [F]

$$\int \frac{\ln \left(e{\left(f{\left(bx+a\right)}^{p} {\left(dx+c\right)}^{q}\right)}^{r}\right)}{\left(bx+a\right)^{5}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(179) = 358.

Time = 0.35 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.46

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{12\,(b^4cd^3-ab^3d^4)qrx^3-6\,(b^4c^2d^2-8\,ab^3cd^3+7\,a^2b^2d^4)qrx^2+4\,(b^4c^3d-6\,ab^3c^2d^2+18\,a^2b^2cd^3-13}{}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="fricas"
)
```

output
```
-1/48*(12*(b^4*c*d^3 - a*b^3*d^4)*q*r*x^3 - 6*(b^4*c^2*d^2 - 8*a*b^3*c*d^3
 + 7*a^2*b^2*d^4)*q*r*x^2 + 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*
d^3 - 13*a^3*b*d^4)*q*r*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*r*log(f) + (3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p + 2*(2*a*b^3*c^3*d - 9*a^2*b^2*
c^2*d^2 + 18*a^3*b*c*d^3 - 11*a^4*d^4)*q)*r + 12*(b^4*d^4*q*r*x^4 + 4*a*b^
3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + (a^4*d^4*q + (
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p)*
r)*log(b*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^
4*q*r*x^2 + 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3)*q*r)*log(d*x + c) + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*
c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b
^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b
^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d
^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4
*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c
^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**5,x)
```

output
```
Timed out
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(179) = 358$.

Time = 0.20 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.38

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx =$$

$$-\frac{\left(2\left(\frac{6\,d^3\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{6\,d^3\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} + \frac{}{a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(}\right.\right.}{}$$

$$-\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{4\,(bx+a)^4 b}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="maxima"
)
```

output
```
-1/48*(2*(6*d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4) - 6*d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (6*b^2*d^2*x^2 + 2*b^2*c^2
 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^
4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b
^4*c*d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c
*d^2 - a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d
^2 - a^5*b*d^3)*x))*d*f*q + 3*b*f*p/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2
 + 4*a^3*b^2*x + a^4*b))*r/(b*f) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e
)/((b*x + a)^4*b)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(179) = 358$.

Time = 0.29 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.92

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{d^4qr\log (bx+a)}{4\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)} + \frac{d^4qr\log (dx+c)}{4\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)} - \frac{pr\log (bx+a)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)} - \frac{qr\log (dx+c)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)} - \frac{12\,b^3d^3qrx^3-6\,b^3cd^2qrx^2+42\,ab^2d^3qrx^2+4\,b^3c^2dqrx-20\,ab^2cd^2qrx+52\,a^2bd^3qrx+3\,b^3c^3pr-9\,ab}{48\,(b^8c^3x^4-3\,ab^7c^2dx^4+3\,a^2b^6cd^2x^4-a^3b^5d^3x^4+4\,ab^7c^3x^3-12\,a^2b^6}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/4*d^4*q*r*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4) + 1/4*d^4*q*r*log(d*x + c)/(b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*p*r*log(b*x
 + a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*
q*r*log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^
4*b) - 1/48*(12*b^3*d^3*q*r*x^3 - 6*b^3*c*d^2*q*r*x^2 + 42*a*b^2*d^3*q*r*x
^2 + 4*b^3*c^2*d*q*r*x - 20*a*b^2*c*d^2*q*r*x + 52*a^2*b*d^3*q*r*x + 3*b^3
*c^3*p*r - 9*a*b^2*c^2*d*p*r + 9*a^2*b*c*d^2*p*r - 3*a^3*d^3*p*r + 4*a*b^2
*c^2*d*q*r - 14*a^2*b*c*d^2*q*r + 22*a^3*d^3*q*r + 12*b^3*c^3*r*log(f) - 3
6*a*b^2*c^2*d*r*log(f) + 36*a^2*b*c*d^2*r*log(f) - 12*a^3*d^3*r*log(f) + 1
2*b^3*c^3*log(e) - 36*a*b^2*c^2*d*log(e) + 36*a^2*b*c*d^2*log(e) - 12*a^3*
d^3*log(e))/(b^8*c^3*x^4 - 3*a*b^7*c^2*d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b
^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3
 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b
^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^5*c^3*x - 12*a^4*b^4*c^2*d*x +
12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a
^6*b^2*c*d^2 - a^7*b*d^3)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.73

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx$$
$$= \frac{\frac{3\,b^3\,c^3\,p\,r-3\,a^3\,d^3\,p\,r+22\,a^3\,d^3\,q\,r-9\,a\,b^2\,c^2\,d\,p\,r+9\,a^2\,b\,c\,d^2\,p\,r+4\,a\,b^2\,c^2\,d\,q\,r-14\,a^2\,b\,c\,d^2\,q\,r}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x\,\left(13\,q\,r\,a^2\,b\,d^3-5\,q\,r\,a\,b^2\,c\,d^2+q\,r\,b^3\,c^2\,d\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\cdots}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x\cdots}$$
$$-\frac{\ln\left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,\left(\frac{x}{4}+\frac{a}{4\,b}\right)}{{\left(a+b\,x\right)}^5}$$
$$+\frac{d^4\,q\,r\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,{\left(a\,d-b\,c\right)}^4}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^5,x)
```

output
```
((3*b^3*c^3*p*r - 3*a^3*d^3*p*r + 22*a^3*d^3*q*r - 9*a*b^2*c^2*d*p*r + 9*a
^2*b*c*d^2*p*r + 4*a*b^2*c^2*d*q*r - 14*a^2*b*c*d^2*q*r)/(12*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(13*a^2*b*d^3*q*r + b^3*c^2*d
*q*r - 5*a*b^2*c*d^2*q*r))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)) + (d*x^2*(7*a*b^2*d^2*q*r - b^3*c*d*q*r))/(2*(a^3*d^3 - b^3*c^3 +
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b^3*d^3*q*r*x^3)/(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a
*b^4*x^3 + 24*a^2*b^3*x^2) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 +
a/(4*b)))/(a + b*x)^5 + (d^4*q*r*atanh((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^
2*c*d^3 - 8*a*b^4*c^3*d)/(4*b*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*(a*d - b*c)^4)
```

3.16 $\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

3.16.1 Optimal result

Integrand size = 31, antiderivative size = 920

$$\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3}+\frac{2(bc-ad)^4pqr^2x}{25d^4}+\frac{77(bc-ad)^4q^2r^2x}{150d^4}$$
$$+\frac{2(bc-ad)^4q(p+q)r^2x}{5d^4}-\frac{b(bc-ad)^3pqr^2x^2}{10d^3}-\frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3}$$
$$-\frac{77(bc-ad)^3q^2r^2(a+bx)^2}{300bd^3}+\frac{16(bc-ad)^2pqr^2(a+bx)^3}{225bd^2}+\frac{47(bc-ad)^2q^2r^2(a+bx)^3}{450bd^2}$$
$$-\frac{9(bc-ad)pqr^2(a+bx)^4}{200bd}-\frac{9(bc-ad)q^2r^2(a+bx)^4}{200bd}+\frac{2p^2r^2(a+bx)^5}{125b}$$
$$+\frac{4pqr^2(a+bx)^5}{125b}+\frac{2q^2r^2(a+bx)^5}{125b}-\frac{2(bc-ad)^5pqr^2\log(c+dx)}{25bd^5}$$
$$-\frac{137(bc-ad)^5q^2r^2\log(c+dx)}{150bd^5}-\frac{2(bc-ad)^5pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{5bd^5}$$
$$-\frac{(bc-ad)^5q^2r^2\log^2(c+dx)}{5bd^5}-\frac{2(bc-ad)^4qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^4}$$
$$+\frac{(bc-ad)^3qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^3}$$
$$-\frac{2(bc-ad)^2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{15bd^2}$$
$$+\frac{(bc-ad)qr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{10bd}$$
$$-\frac{2pr(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b}$$
$$-\frac{2qr(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b}$$
$$+\frac{2(bc-ad)^5qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^5}$$
$$+\frac{(a+bx)^5\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}-\frac{2(bc-ad)^5pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}$$

output
```
1/5*(b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+2/125*p^2*r^2*(b*x+a)^5/
b+2/125*q^2*r^2*(b*x+a)^5/b-2/5*(-a*d+b*c)^5*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b
*c))*ln(d*x+c)/b/d^5-137/150*(-a*d+b*c)^5*q^2*r^2*ln(d*x+c)/b/d^5-1/5*(-a*
d+b*c)^5*q^2*r^2*ln(d*x+c)^2/b/d^5+2/25*(-a*d+b*c)^4*p*q*r^2*x/d^4+2/5*(-a
*d+b*c)^4*q*(p+q)*r^2*x/d^4-77/300*(-a*d+b*c)^3*q^2*r^2*(b*x+a)^2/b/d^3+47
/450*(-a*d+b*c)^2*q^2*r^2*(b*x+a)^3/b/d^2-9/200*(-a*d+b*c)*q^2*r^2*(b*x+a)
^4/b/d-2/25*p*r*(b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-2/25*q*r*(b*x+
a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+77/150*(-a*d+b*c)^4*q^2*r^2*x/d^4+4
/125*p*q*r^2*(b*x+a)^5/b-2/25*(-a*d+b*c)^5*p*q*r^2*ln(d*x+c)/b/d^5-2/5*(-a
*d+b*c)^4*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^4+1/5*(-a*d+b*c)
^3*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-2/15*(-a*d+b*c)^2*q
*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/10*(-a*d+b*c)*q*r*(b*
x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+2/5*(-a*d+b*c)^5*q*r*ln(d*x+c)*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^5-9/200*(-a*d+b*c)*p*q*r^2*(b*x+a)^4/b
/d-2/5*(-a*d+b*c)^5*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^5-1/5*a*(-
a*d+b*c)^3*p*q*r^2*x/d^3-1/10*b*(-a*d+b*c)^3*p*q*r^2*x^2/d^3-1/25*(-a*d+b*
c)^3*p*q*r^2*(b*x+a)^2/b/d^3+16/225*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^3/b/d^2
```

3.16.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2508 vs. $2(920) = 1840$.

Time = 1.54 (sec) , antiderivative size = 2508, normalized size of antiderivative = 2.73

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(2*a^5*p*q*r^2)/b + (2*a*b^3*c^4*p*q*r^2)/(5*d^4) - (2*a^2*b^2*c^3*p*q*r^2
)/d^3 + (4*a^3*b*c^2*p*q*r^2)/d^2 - (4*a^4*c*p*q*r^2)/d + (2*a^4*p^2*r^2*x
)/25 + (197*a^4*p*q*r^2*x)/150 + (12*b^4*c^4*p*q*r^2*x)/(25*d^4) - (11*a*b
^3*c^3*p*q*r^2*x)/(5*d^3) + (59*a^2*b^2*c^2*p*q*r^2*x)/(15*d^2) - (101*a^3
*b*c*p*q*r^2*x)/(30*d) + 2*a^4*q^2*r^2*x + (137*b^4*c^4*q^2*r^2*x)/(150*d^
4) - (25*a*b^3*c^3*q^2*r^2*x)/(6*d^3) + (22*a^2*b^2*c^2*q^2*r^2*x)/(3*d^2)
 - (6*a^3*b*c*q^2*r^2*x)/d + (4*a^3*b*p^2*r^2*x^2)/25 + (283*a^3*b*p*q*r^2
*x^2)/300 - (7*b^4*c^3*p*q*r^2*x^2)/(50*d^3) + (19*a*b^3*c^2*p*q*r^2*x^2)/
(30*d^2) - (67*a^2*b^2*c*p*q*r^2*x^2)/(60*d) + a^3*b*q^2*r^2*x^2 - (77*b^4
*c^3*q^2*r^2*x^2)/(300*d^3) + (13*a*b^3*c^2*q^2*r^2*x^2)/(12*d^2) - (5*a^2
*b^2*c*q^2*r^2*x^2)/(3*d) + (4*a^2*b^2*p^2*r^2*x^3)/25 + (257*a^2*b^2*p*q*
r^2*x^3)/450 + (16*b^4*c^2*p*q*r^2*x^3)/(225*d^2) - (29*a*b^3*c*p*q*r^2*x^
3)/(90*d) + (4*a^2*b^2*q^2*r^2*x^3)/9 + (47*b^4*c^2*q^2*r^2*x^3)/(450*d^2)
 - (7*a*b^3*c*q^2*r^2*x^3)/(18*d) + (2*a*b^3*p^2*r^2*x^4)/25 + (41*a*b^3*p
*q*r^2*x^4)/200 - (9*b^4*c*p*q*r^2*x^4)/(200*d) + (a*b^3*q^2*r^2*x^4)/8 -
(9*b^4*c*q^2*r^2*x^4)/(200*d) + (2*b^4*p^2*r^2*x^5)/125 + (4*b^4*p*q*r^2*x
^5)/125 + (2*b^4*q^2*r^2*x^5)/125 - (a^5*p^2*r^2*Log[a + b*x]^2)/(5*b) + (
2*a^5*p*q*r^2*Log[c + d*x])/b - (2*b^4*c^5*p*q*r^2*Log[c + d*x])/(25*d^5)
+ (2*a*b^3*c^4*p*q*r^2*Log[c + d*x])/(5*d^4) - (4*a^2*b^2*c^3*p*q*r^2*Log[
c + d*x])/(5*d^3) + (4*a^3*b*c^2*p*q*r^2*Log[c + d*x])/(5*d^2) - (2*a^4...
```

3.16.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 843, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2984, 2981, 17, 49, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$$

$$\Big\downarrow \ 2984$$

$$-\frac{2}{5}pr\int (a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx-\frac{2dqr\int \frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}dx}{5b}+\frac{(a+bx)^5 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}$$

$$\Big\downarrow \ 2981$$

$$-\frac{2}{5}pr\left(-\frac{dqr\int\frac{(a+bx)^5}{c+dx}dx}{5b}-\frac{1}{5}pr\int(a+bx)^4dx+\frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}\right)-\frac{2dqr\int\frac{(a+bx)^5\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{5b}+\frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

$\downarrow$ 17

$$-\frac{2}{5}pr\left(-\frac{dqr\int\frac{(a+bx)^5}{c+dx}dx}{5b}+\frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}-\frac{pr(a+bx)^5}{25b}\right)-\frac{2dqr\int\frac{(a+bx)^5\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{5b}+\frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

$\downarrow$ 49

$$-\frac{2}{5}pr\left(-\frac{dqr\int\left(\frac{(ad-bc)^5}{d^5(c+dx)}+\frac{b(bc-ad)^4}{d^5}+\frac{b(a+bx)^4}{d}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)^3(a+bx)}{d^4}\right)dx}{5b}+\frac{(a+bx\cdots\right.$$
$$\frac{2dqr\int\frac{(a+bx)^5\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{5b}+\frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

$\downarrow$ 2009

$$-\frac{2dqr\int\frac{(a+bx)^5\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{5b}-$$
$$\frac{2}{5}pr\left(-\frac{dqr\left(-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}\right)}{5b}+\frac{(a+\cdots\right.$$
$$\frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

$\downarrow$ 2994

$$-\frac{2dqr\int\left(\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(ad-bc)^5}{d^5(c+dx)}+\frac{b(bc-ad)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^5}+\frac{b(a+bx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{b(bc-ad\cdots}{5\cdots}\right.}{}$$
$$\frac{2}{5}pr\left(-\frac{dqr\left(-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}\right)}{5b}+\frac{(a+\cdots\right.$$
$$\frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

$\downarrow$ 2009

3.16. $\int(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(a+bx)^5}{5b} - \frac{2}{5}pr\left(-\frac{pr(a+bx)^5}{25b} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)(a+bx)^5}{5b} - \frac{dqr\left(-\frac{\log(c+dx)(bc-ad)^5}{d^6} + \frac{bx(bc-ad)^4}{d^5} - \frac{(a+bx)^2(bc-}{2d^4}\right.}{5}\right.$$

$$\frac{2dqr\left(\frac{qr\log^2(c+dx)(bc-ad)^5}{2d^6} + \frac{137qr\log(c+dx)(bc-ad)^5}{60d^6} + \frac{pr\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bc-ad)^5}{d^6} - \frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r}{d^6}\right.}{}$$

input
```
Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(5*b) - (2*p*r*(-1/25
*(p*r*(a + b*x)^5)/b - (d*q*r*((b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a
 + b*x)^2)/(2*d^4) + ((b*c - a*d)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a
 + b*x)^4)/(4*d^2) + (a + b*x)^5/(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6)
)/(5*b) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b)))/5 - (
2*d*q*r*((-77*b*(b*c - a*d)^4*q*r*x)/(60*d^5) - (b*(b*c - a*d)^4*(p + q)*r
*x)/d^5 + ((b*c - a*d)^3*p*r*(a + b*x)^2)/(4*d^4) + (77*(b*c - a*d)^3*q*r*
(a + b*x)^2)/(120*d^4) - ((b*c - a*d)^2*p*r*(a + b*x)^3)/(9*d^3) - (47*(b*
c - a*d)^2*q*r*(a + b*x)^3)/(180*d^3) + ((b*c - a*d)*p*r*(a + b*x)^4)/(16*
d^2) + (9*(b*c - a*d)*q*r*(a + b*x)^4)/(80*d^2) - (p*r*(a + b*x)^5)/(25*d)
 - (q*r*(a + b*x)^5)/(25*d) + (137*(b*c - a*d)^5*q*r*Log[c + d*x])/(60*d^6
) + ((b*c - a*d)^5*p*r*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d^6
 + ((b*c - a*d)^5*q*r*Log[c + d*x]^2)/(2*d^6) + ((b*c - a*d)^4*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^5 - ((b*c - a*d)^3*(a + b*x)^2*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^4) + ((b*c - a*d)^2*(a + b*x)^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d^3) - ((b*c - a*d)*(a + b*x)^4*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*d^2) + ((a + b*x)^5*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/(5*d) - ((b*c - a*d)^5*Log[c + d*x]*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/d^6 + ((b*c - a*d)^5*p*r*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/d^6))/(5*b)
```

3.16. $\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

3.16.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.16.4 Maple [F]

$$\int (bx+a)^4 \ln\left(e(f(bx+a)^p (dx+c)^q)^r\right)^2 dx$$

input
```
int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.16.5 Fricas [F]

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (bx+a)^4 \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

input
```
integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="frica
s")
```

output
```
integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(((b
*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Timed out}$$

input
```
integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Timed out
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1421, normalized size of antiderivative = 1.54

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxim
a")
```

output
```
1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log(
((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/150*(60*a^5*f*p*log(b*x + a)/b - (1
2*b^4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4
 + 20*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*
x^3 + 30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q
- 10*a^2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*
b^3*c^3*d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(
b^4*c^5*f*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^
3*f*q + 5*a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r*log(((b*x + a)^p*(d*x + c)^q*
f)^r*e)/f - 1/9000*r^2*(60*((12*p*q + 137*q^2)*b^4*c^5*f^2 - 5*(12*p*q + 1
25*q^2)*a*b^3*c^4*d*f^2 + 20*(6*p*q + 55*q^2)*a^2*b^2*c^3*d^2*f^2 - 60*(2*
p*q + 15*q^2)*a^3*b*c^2*d^3*f^2 + 60*(p*q + 5*q^2)*a^4*c*d^4*f^2)*log(d*x
+ c)/d^5 - 3600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2*p*q + 10*a^2*b^3*c^3*
d^2*f^2*p*q - 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*d^4*f^2*p*q - a^5*d^5
*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))/(b*d^5) - (144*(p^2 + 2*p*q + q^2)*b^5*d^5*f^2*x^5 -
 1800*a^5*d^5*f^2*p^2*log(b*x + a)^2 - 45*(9*(p*q + q^2)*b^5*c*d^4*f^2 - (
16*p^2 + 41*p*q + 25*q^2)*a*b^4*d^5*f^2)*x^4 + 20*((32*p*q + 47*q^2)*b^5*c
^2*d^3*f^2 - 5*(29*p*q + 35*q^2)*a*b^4*c*d^4*f^2 + (72*p^2 + 257*p*q + 200
*q^2)*a^2*b^3*d^5*f^2)*x^3 - 30*(7*(6*p*q + 11*q^2)*b^5*c^3*d^2*f^2 - 5...
```

3.16.8 Giac [F]

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (bx+a)^4 \log\left(\left(((bx+a)^p(dx+c)^q f)^r e\right)^2 dx$$

input
```
integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)^4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2 (a+b\,x)^4\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4, x)
```

3.17 $\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

3.17.1 Optimal result

Integrand size = 31, antiderivative size = 805

$$\begin{aligned}
&\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx \\
&= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} \\
&\quad + \frac{b(bc-ad)^2pqr^2x^2}{8d^2} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} + \frac{13(bc-ad)^2q^2r^2(a+bx)^2}{48bd^2} \\
&\quad - \frac{7(bc-ad)pqr^2(a+bx)^3}{72bd} - \frac{7(bc-ad)q^2r^2(a+bx)^3}{72bd} + \frac{p^2r^2(a+bx)^4}{32b} \\
&\quad + \frac{pqr^2(a+bx)^4}{16b} + \frac{q^2r^2(a+bx)^4}{32b} + \frac{(bc-ad)^4pqr^2\log(c+dx)}{8bd^4} \\
&\quad + \frac{25(bc-ad)^4q^2r^2\log(c+dx)}{24bd^4} + \frac{(bc-ad)^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2bd^4} \\
&\quad + \frac{(bc-ad)^4q^2r^2\log^2(c+dx)}{4bd^4} + \frac{(bc-ad)^3qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2bd^3} \\
&\quad - \frac{(bc-ad)^2qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{6bd} \\
&\quad - \frac{pr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} - \frac{qr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} \\
&\quad - \frac{(bc-ad)^4qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2bd^4} \\
&\quad + \frac{(a+bx)^4\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} + \frac{(bc-ad)^4pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bd^4}
\end{aligned}$$

output
```
1/4*a*(-a*d+b*c)^2*p*q*r^2*x/d^2-1/8*(-a*d+b*c)^3*p*q*r^2*x/d^3-13/24*(-a*
d+b*c)^3*q^2*r^2*x/d^3-1/2*(-a*d+b*c)^3*q*(p+q)*r^2*x/d^3+1/8*b*(-a*d+b*c)
^2*p*q*r^2*x^2/d^2+1/16*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^2/b/d^2+13/48*(-a*d+b
*c)^2*q^2*r^2*(b*x+a)^2/b/d^2-7/72*(-a*d+b*c)*p*q*r^2*(b*x+a)^3/b/d-7/72*(
-a*d+b*c)*q^2*r^2*(b*x+a)^3/b/d+1/32*p^2*r^2*(b*x+a)^4/b+1/16*p*q*r^2*(b*x
+a)^4/b+1/32*q^2*r^2*(b*x+a)^4/b+1/8*(-a*d+b*c)^4*p*q*r^2*ln(d*x+c)/b/d^4+
25/24*(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)/b/d^4+1/2*(-a*d+b*c)^4*p*q*r^2*ln(-d*
(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^4+1/4*(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)^2/b
/d^4+1/2*(-a*d+b*c)^3*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-1/
4*(-a*d+b*c)^2*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/6*(-a
*d+b*c)*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/8*p*r*(b*x+a)^
4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/8*q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)/b-1/2*(-a*d+b*c)^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b/d^4+1/4*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+1/2*(-a*d+b*c)^
4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^4
```

3.17.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1853 vs. $2(805) = 1610$.

Time = 0.95 (sec) , antiderivative size = 1853, normalized size of antiderivative = 2.30

$$\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(2*a^4*p*q*r^2)/b - (a*b^2*c^3*p*q*r^2)/(2*d^3) + (2*a^2*b*c^2*p*q*r^2)/d^
2 - (3*a^3*c*p*q*r^2)/d + (a^3*p^2*r^2*x)/8 + (37*a^3*p*q*r^2*x)/24 - (5*b
^3*c^3*p*q*r^2*x)/(8*d^3) + (9*a*b^2*c^2*p*q*r^2*x)/(4*d^2) - (35*a^2*b*c*
p*q*r^2*x)/(12*d) + 2*a^3*q^2*r^2*x - (25*b^3*c^3*q^2*r^2*x)/(24*d^3) + (1
1*a*b^2*c^2*q^2*r^2*x)/(3*d^2) - (9*a^2*b*c*q^2*r^2*x)/(2*d) + (3*a^2*b*p^
2*r^2*x^2)/16 + (41*a^2*b*p*q*r^2*x^2)/48 + (3*b^3*c^2*p*q*r^2*x^2)/(16*d^
2) - (2*a*b^2*c*p*q*r^2*x^2)/(3*d) + (3*a^2*b*q^2*r^2*x^2)/4 + (13*b^3*c^2
*q^2*r^2*x^2)/(48*d^2) - (5*a*b^2*c*q^2*r^2*x^2)/(6*d) + (a*b^2*p^2*r^2*x^
3)/8 + (25*a*b^2*p*q*r^2*x^3)/72 - (7*b^3*c*p*q*r^2*x^3)/(72*d) + (2*a*b^2
*q^2*r^2*x^3)/9 - (7*b^3*c*q^2*r^2*x^3)/(72*d) + (b^3*p^2*r^2*x^4)/32 + (b
^3*p*q*r^2*x^4)/16 + (b^3*q^2*r^2*x^4)/32 - (a^4*p^2*r^2*Log[a + b*x]^2)/(
4*b) + (2*a^4*p*q*r^2*Log[c + d*x])/b + (b^3*c^4*p*q*r^2*Log[c + d*x])/(8*
d^4) - (a*b^2*c^3*p*q*r^2*Log[c + d*x])/(2*d^3) + (3*a^2*b*c^2*p*q*r^2*Log
[c + d*x])/(4*d^2) - (a^3*c*p*q*r^2*Log[c + d*x])/(2*d) + (25*b^3*c^4*q^2*
r^2*Log[c + d*x])/(24*d^4) - (11*a*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) +
 (9*a^2*b*c^2*q^2*r^2*Log[c + d*x])/(2*d^2) - (2*a^3*c*q^2*r^2*Log[c + d*x
])/d + (b^3*c^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4) - (a*b^2*c^3*q^2*r^2*Log[c
 + d*x]^2)/d^3 + (3*a^2*b*c^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2) - (a^3*c*q^2
*r^2*Log[c + d*x]^2)/d - (2*a^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
b - (a^3*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - 2*a^3*q*r*x*Lo...
```

3.17.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 723, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2984, 2981, 17, 49, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$

$$\Big\downarrow \text{2984}$$

$$-\frac{1}{2}pr\int (a+bx)^3 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx - \frac{dqr\int \frac{(a+bx)^4 \log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{2b} + \frac{(a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$$\Big\downarrow \text{2981}$$

$$-\frac{1}{2}pr\left(-\frac{dqr\int \frac{(a+bx)^4}{c+dx}dx}{4b}-\frac{1}{4}pr\int (a+bx)^3dx+\frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}\right)-\frac{dqr\int \frac{(a+bx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{2b}+\frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$\downarrow$ 17

$$-\frac{1}{2}pr\left(-\frac{dqr\int \frac{(a+bx)^4}{c+dx}dx}{4b}+\frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}-\frac{pr(a+bx)^4}{16b}\right)-\frac{dqr\int \frac{(a+bx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{2b}+\frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$\downarrow$ 49

$$-\frac{1}{2}pr\left(-\frac{dqr\int \left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{4b}+\frac{(a+bx)^4\log\left(e(f(a+bx\right.}{4b}\right.$$
$$\frac{dqr\int \frac{(a+bx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{2b}+\frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$\downarrow$ 2009

$$-\frac{dqr\int \frac{(a+bx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{2b}-$$
$$\frac{1}{2}pr\left(-\frac{dqr\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{4b}+\frac{(a+bx)^4\log\left(e(f(a+b\right.}{4b}\right.$$
$$\frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$\downarrow$ 2994

$$-\frac{dqr\int \left(\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^4}+\frac{b(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{b(bc-ad)(\right.}{2b}$$
$$\frac{1}{2}pr\left(-\frac{dqr\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{4b}+\frac{(a+bx)^4\log\left(e(f(a+b\right.}{4b}\right.$$
$$\frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

$\downarrow$ 2009

3.17. $\int (a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

$$-\frac{dqr\left(\frac{(bc-ad)^4\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^5}-\frac{pr(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d^5}-\frac{pr(bc-ad)^4\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d^5}-\frac{qr(bc}{}\right.}{}$$

$$\frac{1}{2}pr\left(-\frac{dqr\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{4b}+\frac{(a+bx)^4\log(e(f(a+bx}{4b}\right.$$

$$\frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

input
```
Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(4*b) - (p*r*(-1/16*(
p*r*(a + b*x)^4)/b - (d*q*r*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(
a + b*x)^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d
) + ((b*c - a*d)^4*Log[c + d*x])/d^5))/(4*b) + ((a + b*x)^4*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])/(4*b)))/2 - (d*q*r*((13*b*(b*c - a*d)^3*q*r*x)/(12
*d^4) + (b*(b*c - a*d)^3*(p + q)*r*x)/d^4 - ((b*c - a*d)^2*p*r*(a + b*x)^2
)/(4*d^3) - (13*(b*c - a*d)^2*q*r*(a + b*x)^2)/(24*d^3) + ((b*c - a*d)*p*r
*(a + b*x)^3)/(9*d^2) + (7*(b*c - a*d)*q*r*(a + b*x)^3)/(36*d^2) - (p*r*(a
 + b*x)^4)/(16*d) - (q*r*(a + b*x)^4)/(16*d) - (25*(b*c - a*d)^4*q*r*Log[c
 + d*x])/(12*d^5) - ((b*c - a*d)^4*p*r*Log[-((d*(a + b*x))/(b*c - a*d))]*L
og[c + d*x])/d^5 - ((b*c - a*d)^4*q*r*Log[c + d*x]^2)/(2*d^5) - ((b*c - a*
d)^3*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^4 + ((b*c - a*d)^2*
(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^3) - ((b*c - a*d)*(
a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d^2) + ((a + b*x)^4*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*d) + ((b*c - a*d)^4*Log[c + d*x]*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^5 - ((b*c - a*d)^4*p*r*PolyLog[2, (b
*(c + d*x))/(b*c - a*d)])/d^5))/(2*b)
```

3.17.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.17.4 Maple [F]

$$\int (bx+a)^3 \ln \left(e(f(bx+a)^p (dx+c)^q)^r\right)^2 dx$$

input
```
int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.17.5 Fricas [F]

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (bx+a)^3 \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

input
```
integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="frica
s")
```

output
```
integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2, x)
```

3.17.6 Sympy [F]

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (a+bx)^3 \log\left(e(f(a+bx)^p (c+dx)^q)^r\right)^2 dx$$

input
```
integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral((a + b*x)**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1071, normalized size of antiderivative = 1.33

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxim
a")
```

output
```
1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x
+ c)^q*f)^r*e)^2 + 1/24*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*
x^4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(
p + 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q
) - b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*
c^4*f*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d
*x + c)/d^4)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*((3*
p*q + 25*q^2)*b^3*c^4*f^2 - 4*(3*p*q + 22*q^2)*a*b^2*c^3*d*f^2 + 18*(p*q +
 6*q^2)*a^2*b*c^2*d^2*f^2 - 12*(p*q + 4*q^2)*a^3*c*d^3*f^2)*log(d*x + c)/d
^4 - 144*(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*
p*q - 4*a^3*b*c*d^3*f^2*p*q + a^4*d^4*f^2*p*q)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^4) + (9*(p
^2 + 2*p*q + q^2)*b^4*d^4*f^2*x^4 - 72*a^4*d^4*f^2*p^2*log(b*x + a)^2 - 4*
(7*(p*q + q^2)*b^4*c*d^3*f^2 - (9*p^2 + 25*p*q + 16*q^2)*a*b^3*d^4*f^2)*x^
3 + 6*((9*p*q + 13*q^2)*b^4*c^2*d^2*f^2 - 8*(4*p*q + 5*q^2)*a*b^3*c*d^3*f^
2 + (9*p^2 + 41*p*q + 36*q^2)*a^2*b^2*d^4*f^2)*x^2 + 144*(b^4*c^4*f^2*p*q
- 4*a*b^3*c^3*d*f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^3*b*c*d^3*f^2*p*
q)*log(b*x + a)*log(d*x + c) + 72*(b^4*c^4*f^2*q^2 - 4*a*b^3*c^3*d*f^2*q^2
 + 6*a^2*b^2*c^2*d^2*f^2*q^2 - 4*a^3*b*c*d^3*f^2*q^2)*log(d*x + c)^2 - 12*
(5*(3*p*q + 5*q^2)*b^4*c^3*d*f^2 - 2*(27*p*q + 44*q^2)*a*b^3*c^2*d^2*f^...
```

3.17.8 Giac [F]

$$\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \int (bx+a)^3 \log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

input
```
integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int \ln \left(e \left(f \left(a+b\,x\right)^p \left(c+d\,x\right)^q\right)^r\right)^2 \left(a+b\,x\right)^3 dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3, x)
```

3.18 $\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

3.18.1 Optimal result

Integrand size = 31, antiderivative size = 686

$$\begin{aligned}
&\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx \\
&= -\frac{a(bc-ad)pqr^2x}{3d}+\frac{2(bc-ad)^2pqr^2x}{9d^2}+\frac{5(bc-ad)^2q^2r^2x}{9d^2}+\frac{2(bc-ad)^2q(p+q)r^2x}{3d^2} \\
&\quad-\frac{b(bc-ad)pqr^2x^2}{6d}-\frac{(bc-ad)pqr^2(a+bx)^2}{9bd}-\frac{5(bc-ad)q^2r^2(a+bx)^2}{18bd} \\
&\quad+\frac{2p^2r^2(a+bx)^3}{27b}+\frac{4pqr^2(a+bx)^3}{27b}+\frac{2q^2r^2(a+bx)^3}{27b}-\frac{2(bc-ad)^3pqr^2\log(c+dx)}{9bd^3} \\
&\quad-\frac{11(bc-ad)^3q^2r^2\log(c+dx)}{9bd^3}-\frac{2(bc-ad)^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3bd^3} \\
&\quad-\frac{(bc-ad)^3q^2r^2\log^2(c+dx)}{3bd^3}-\frac{2(bc-ad)^2qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^2} \\
&\quad+\frac{(bc-ad)qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd} \\
&\quad-\frac{2pr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad-\frac{2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad+\frac{2(bc-ad)^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3} \\
&\quad+\frac{(a+bx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}-\frac{2(bc-ad)^3pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}
\end{aligned}$$

output
```
-1/3*a*(-a*d+b*c)*p*q*r^2*x/d+2/9*(-a*d+b*c)^2*p*q*r^2*x/d^2+5/9*(-a*d+b*c
)^2*q^2*r^2*x/d^2+2/3*(-a*d+b*c)^2*q*(p+q)*r^2*x/d^2-1/6*b*(-a*d+b*c)*p*q*
r^2*x^2/d-1/9*(-a*d+b*c)*p*q*r^2*(b*x+a)^2/b/d-5/18*(-a*d+b*c)*q^2*r^2*(b*
x+a)^2/b/d+2/27*p^2*r^2*(b*x+a)^3/b+4/27*p*q*r^2*(b*x+a)^3/b+2/27*q^2*r^2*
(b*x+a)^3/b-2/9*(-a*d+b*c)^3*p*q*r^2*ln(d*x+c)/b/d^3-11/9*(-a*d+b*c)^3*q^2
*r^2*ln(d*x+c)/b/d^3-2/3*(-a*d+b*c)^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln
(d*x+c)/b/d^3-1/3*(-a*d+b*c)^3*q^2*r^2*ln(d*x+c)^2/b/d^3-2/3*(-a*d+b*c)^2*
q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/3*(-a*d+b*c)*q*r*(b*x+
a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-2/9*p*r*(b*x+a)^3*ln(e*(f*(b*x+a)
^p*(d*x+c)^q)^r)/b-2/9*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2/3
*(-a*d+b*c)^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3+1/3*(b*x
+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b-2/3*(-a*d+b*c)^3*p*q*r^2*polylog
(2,b*(d*x+c)/(-a*d+b*c))/b/d^3
```

3.18. $\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

3.18.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 1211, normalized size of antiderivative = 1.77

$$
\begin{aligned}
\int (a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{1}{54}\Bigg(&\frac{108a^3pqr^2}{b}+\frac{36abc^2pqr^2}{d^2}-\frac{108a^2cpqr^2}{d} \\
&+12a^2p^2r^2x+102a^2pqr^2x+\frac{48b^2c^2pqr^2x}{d^2}-\frac{126abcpqr^2x}{d}+108a^2q^2r^2x+\frac{66b^2c^2q^2r^2x}{d^2} \\
&-\frac{162abcq^2r^2x}{d}+12abp^2r^2x^2+39abpqr^2x^2-\frac{15b^2cpqr^2x^2}{d}+27abq^2r^2x^2-\frac{15b^2cq^2r^2x^2}{d} \\
&+4b^2p^2r^2x^3+8b^2pqr^2x^3+4b^2q^2r^2x^3-\frac{18a^3p^2r^2\log^2(a+bx)}{b}+\frac{108a^3pqr^2\log(c+dx)}{b} \\
&-\frac{12b^2c^3pqr^2\log(c+dx)}{d^3}+\frac{36abc^2pqr^2\log(c+dx)}{d^2}-\frac{36a^2cpqr^2\log(c+dx)}{d} \\
&-\frac{66b^2c^3q^2r^2\log(c+dx)}{d^3}+\frac{162abc^2q^2r^2\log(c+dx)}{d^2}-\frac{108a^2cq^2r^2\log(c+dx)}{d} \\
&-\frac{18b^2c^3q^2r^2\log^2(c+dx)}{d^3}+\frac{54abc^2q^2r^2\log^2(c+dx)}{d^2}-\frac{54a^2cq^2r^2\log^2(c+dx)}{d} \\
&-\frac{108a^3pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-36a^2prx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&-108a^2qrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-\frac{36b^2c^2qrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d^2} \\
&+\frac{108abcqrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d}-36abprx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&-54abqrx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+\frac{18b^2cqrx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d} \\
&-12b^2prx^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-12b^2qrx^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&+\frac{36b^2c^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d^3} \\
&-\frac{108abc^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d^2} \\
&+\frac{108a^2cqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d}+54a^2x\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&+54abx^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)+18b^2x^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&+\frac{6pr\log(a+bx)\Big(ad(a^2d^2(16p-11q)-6b^2c^2q+15abcdq)\,r-6bc(b^2c^2-3abcd+3a^2d^2)\,qr\log(c+dx)}{bd^3} \\
&+\frac{36(bc-ad)^3pqr^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bd^3}\Bigg)
\end{aligned}
$$

input
```
Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((108*a^3*p*q*r^2)/b + (36*a*b*c^2*p*q*r^2)/d^2 - (108*a^2*c*p*q*r^2)/d +
12*a^2*p^2*r^2*x + 102*a^2*p*q*r^2*x + (48*b^2*c^2*p*q*r^2*x)/d^2 - (126*a
*b*c*p*q*r^2*x)/d + 108*a^2*q^2*r^2*x + (66*b^2*c^2*q^2*r^2*x)/d^2 - (162*
a*b*c*q^2*r^2*x)/d + 12*a*b*p^2*r^2*x^2 + 39*a*b*p*q*r^2*x^2 - (15*b^2*c*p
*q*r^2*x^2)/d + 27*a*b*q^2*r^2*x^2 - (15*b^2*c*q^2*r^2*x^2)/d + 4*b^2*p^2*
r^2*x^3 + 8*b^2*p*q*r^2*x^3 + 4*b^2*q^2*r^2*x^3 - (18*a^3*p^2*r^2*Log[a +
b*x]^2)/b + (108*a^3*p*q*r^2*Log[c + d*x])/b - (12*b^2*c^3*p*q*r^2*Log[c +
 d*x])/d^3 + (36*a*b*c^2*p*q*r^2*Log[c + d*x])/d^2 - (36*a^2*c*p*q*r^2*Log
[c + d*x])/d - (66*b^2*c^3*q^2*r^2*Log[c + d*x])/d^3 + (162*a*b*c^2*q^2*r^
2*Log[c + d*x])/d^2 - (108*a^2*c*q^2*r^2*Log[c + d*x])/d - (18*b^2*c^3*q^2
*r^2*Log[c + d*x]^2)/d^3 + (54*a*b*c^2*q^2*r^2*Log[c + d*x]^2)/d^2 - (54*a
^2*c*q^2*r^2*Log[c + d*x]^2)/d - (108*a^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/b - 36*a^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a^2*
q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (36*b^2*c^2*q*r*x*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r])/d^2 + (108*a*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/d - 36*a*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 54
*a*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (18*b^2*c*q*r*x^2*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - 12*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r] - 12*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3
6*b^2*c^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - ...
```

3.18.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2984, 2981, 17, 49, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$

$$\Big\downarrow \text{ 2984}$$

$$-\frac{2}{3}pr\int (a+bx)^2 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)dx - \frac{2dqr\int \frac{(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3b} + \frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \text{ 2981}$$

$$-\frac{2}{3}pr\left(-\frac{dqr\int\frac{(a+bx)^3}{c+dx}dx}{3b}-\frac{1}{3}pr\int(a+bx)^2dx+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}\right)-\frac{2dqr\int\frac{(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3b}+\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2}{3}pr\left(-\frac{dqr\int\frac{(a+bx)^3}{c+dx}dx}{3b}+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}-\frac{pr(a+bx)^3}{9b}\right)-\frac{2dqr\int\frac{(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3b}+\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{2}{3}pr\left(-\frac{dqr\int\left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{3b}+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}-\right.$$
$$\frac{2dqr\int\frac{(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3b}+\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2dqr\int\frac{(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3b}-$$
$$\frac{2}{3}pr\left(-\frac{dqr\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{3b}+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}\right.$$
$$\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \text{2994}$$

$$-\frac{2dqr\int\left(\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^3}+\frac{b(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{b(bc-ad}{}\right.}{3b}$$
$$\frac{2}{3}pr\left(-\frac{dqr\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{3b}+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}\right.$$
$$\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

$$\Big\downarrow \text{2009}$$

3.18. $\int(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

$$-\frac{2dqr\left(-\frac{(bc-ad)^3\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^4}+\frac{pr(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d^4}+\frac{pr(bc-ad)^3\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d^4}+\cdots\right.}{\cdots}$$

$$\frac{2}{3}pr\left(-\frac{dqr\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{3b}+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}\cdots\right.$$

$$\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

input
```
Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b) - (2*p*r*(-1/9*
(p*r*(a + b*x)^3)/b - (d*q*r*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a +
b*x)^2)/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(
3*b) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)))/3 - (2*d
*q*r*((-5*b*(b*c - a*d)^2*q*r*x)/(6*d^3) - (b*(b*c - a*d)^2*(p + q)*r*x)/d
^3 + ((b*c - a*d)*p*r*(a + b*x)^2)/(4*d^2) + (5*(b*c - a*d)*q*r*(a + b*x)^
2)/(12*d^2) - (p*r*(a + b*x)^3)/(9*d) - (q*r*(a + b*x)^3)/(9*d) + (11*(b*c
 - a*d)^3*q*r*Log[c + d*x])/(6*d^4) + ((b*c - a*d)^3*p*r*Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[c + d*x])/d^4 + ((b*c - a*d)^3*q*r*Log[c + d*x]^2)/(2
*d^4) + ((b*c - a*d)^2*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3
 - ((b*c - a*d)*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^2)
+ ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d) - ((b*c - a*d)^
3*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^4 + ((b*c - a*d)^3*
p*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/d^4))/(3*b)
```

3.18.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.18.4 Maple [F]

$$\int (bx+a)^2 \ln \left(e\left(f(bx+a)^p \, (dx+c)^q\right)^r\right)^2 dx$$

input
```
int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.18.5 Fricas [F]

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (bx+a)^2 \log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

input
```
integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="frica
s")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
 x)
```

3.18.6 Sympy [F]

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (a+bx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)^2\,dx$$

input
```
integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral((a + b*x)**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.12

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{1}{3}\left(b^2x^3 + 3\,abx^2 + 3\,a^2x\right)\log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2$$
$$+ \frac{\left(\frac{6\,a^3fp\log(bx+a)}{b} - \frac{2\,b^2d^2f(p+q)x^3+3\,\left(abd^2f(2\,p+3\,q)-b^2cdfq\right)x^2+6\,\left(a^2d^2f(p+3\,q)+b^2c^2fq-3\,abcdfq\right)x}{d^2} + \frac{6\,\left(b^2c^3fq-3\,abc^2dfq+3\,a^2\right.}{d^3}}{9\,f}$$
$$- \frac{r^2\left(\frac{6\,\left(\left(2\,pq+11\,q^2\right)b^2c^3f^2-3\,\left(2\,pq+9\,q^2\right)abc^2df^2+6\,\left(pq+3\,q^2\right)a^2cd^2f^2\right)\log(dx+c)}{d^3} - \frac{36\,\left(b^3c^3f^2pq-3\,ab^2c^2df^2pq+3\,a^2bcd^2f^2pq-a^3d^3f^2\right.}{bd^3}\right.}{}$$

input
```
integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxim
a")
```

output
```
1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
 + 1/9*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f
*(2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3
*a*b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)
*log(d*x + c)/d^3)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*
((2*p*q + 11*q^2)*b^2*c^3*f^2 - 3*(2*p*q + 9*q^2)*a*b*c^2*d*f^2 + 6*(p*q +
 3*q^2)*a^2*c*d^2*f^2)*log(d*x + c)/d^3 - 36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^
2*d*f^2*p*q + 3*a^2*b*c*d^2*f^2*p*q - a^3*d^3*f^2*p*q)*(log(b*x + a)*log((
b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^3)
 - (4*(p^2 + 2*p*q + q^2)*b^3*d^3*f^2*x^3 - 18*a^3*d^3*f^2*p^2*log(b*x + a
)^2 - 3*(5*(p*q + q^2)*b^3*c*d^2*f^2 - (4*p^2 + 13*p*q + 9*q^2)*a*b^2*d^3*
f^2)*x^2 - 36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2*p*q + 3*a^2*b*c*d^2*f^2
*p*q)*log(b*x + a)*log(d*x + c) - 18*(b^3*c^3*f^2*q^2 - 3*a*b^2*c^2*d*f^2*
q^2 + 3*a^2*b*c*d^2*f^2*q^2)*log(d*x + c)^2 + 6*((8*p*q + 11*q^2)*b^3*c^2*
d*f^2 - 3*(7*p*q + 9*q^2)*a*b^2*c*d^2*f^2 + (2*p^2 + 17*p*q + 18*q^2)*a^2*
b*d^3*f^2)*x - 6*(6*a*b^2*c^2*d*f^2*p*q - 15*a^2*b*c*d^2*f^2*p*q + (2*p^2
+ 11*p*q)*a^3*d^3*f^2)*log(b*x + a))/(b*d^3))/f^2
```

3.18.8 Giac [F]

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (bx+a)^2 \log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

input
```
integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2 (a+b\,x)^2\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2, x)
```

3.19 $\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

3.19.1 Optimal result

Integrand size = 29, antiderivative size = 540

$$\begin{aligned}
&\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx \\
&= \frac{1}{2}ap^2r^2x+\frac{1}{2}apqr^2x-\frac{(bc-ad)pqr^2x}{2d}-\frac{(bc-ad)q^2r^2x}{2d}-\frac{(bc-ad)q(p+q)r^2x}{d} \\
&+\frac{1}{4}bp^2r^2x^2+\frac{1}{4}bpqr^2x^2+\frac{pqr^2(a+bx)^2}{4b}+\frac{q^2r^2(a+bx)^2}{4b}+\frac{(bc-ad)^2pqr^2\log(c+dx)}{2bd^2} \\
&+\frac{3(bc-ad)^2q^2r^2\log(c+dx)}{2bd^2}+\frac{(bc-ad)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd^2} \\
&+\frac{(bc-ad)^2q^2r^2\log^2(c+dx)}{2bd^2}+\frac{(bc-ad)qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd} \\
&-\frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} \\
&-\frac{(bc-ad)^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd^2} \\
&+\frac{(a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{(bc-ad)^2pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}
\end{aligned}$$

output
```
1/2*a*p^2*r^2*x+1/2*a*p*q*r^2*x-1/2*(-a*d+b*c)*p*q*r^2*x/d-1/2*(-a*d+b*c)*
q^2*r^2*x/d-(-a*d+b*c)*q*(p+q)*r^2*x/d+1/4*b*p^2*r^2*x^2+1/4*b*p*q*r^2*x^2
+1/4*p*q*r^2*(b*x+a)^2/b+1/4*q^2*r^2*(b*x+a)^2/b+1/2*(-a*d+b*c)^2*p*q*r^2*
ln(d*x+c)/b/d^2+3/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c)/b/d^2+(-a*d+b*c)^2*p*q*
r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^2+1/2*(-a*d+b*c)^2*q^2*r^2*ln(
d*x+c)^2/b/d^2+(-a*d+b*c)*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-
1/2*p*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/2*q*r*(b*x+a)^2*ln(e
*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-(-a*d+b*c)^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+(-a
*d+b*c)^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^2
```

3.19.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.45

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \frac{-4abcdpqr^2 + 8a^2d^2pqr^2 + 2abd^2p^2r^2x - 6b^2cdpqr^2x + 10abd^2pqr^2x - 6b^2cdq^2r^2x + 8abd^2q^2r^2x + b^2d^2p^2}{}$$

input
```
Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(-4*a*b*c*d*p*q*r^2 + 8*a^2*d^2*p*q*r^2 + 2*a*b*d^2*p^2*r^2*x - 6*b^2*c*d*
p*q*r^2*x + 10*a*b*d^2*p*q*r^2*x - 6*b^2*c*d*q^2*r^2*x + 8*a*b*d^2*q^2*r^2
*x + b^2*d^2*p^2*r^2*x^2 + 2*b^2*d^2*p*q*r^2*x^2 + b^2*d^2*q^2*r^2*x^2 - 2
*a^2*d^2*p^2*r^2*Log[a + b*x]^2 + 2*b^2*c^2*p*q*r^2*Log[c + d*x] - 4*a*b*c
*d*p*q*r^2*Log[c + d*x] + 8*a^2*d^2*p*q*r^2*Log[c + d*x] + 6*b^2*c^2*q^2*r
^2*Log[c + d*x] - 8*a*b*c*d*q^2*r^2*Log[c + d*x] + 2*b^2*c^2*q^2*r^2*Log[c
 + d*x]^2 - 4*a*b*c*d*q^2*r^2*Log[c + d*x]^2 - 8*a^2*d^2*p*r*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r] - 4*a*b*d^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] + 4*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*
r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*p*r*x^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r] - 4*b^2*c^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 8*a*b*c*d*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*d
^2*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*b^2*d^2*x^2*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(2*b*c*(b*c - 2*a*d)*q*r*Log
[c + d*x] - 2*(b*c - a*d)^2*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(3*a*
d*(p - q)*r + 2*b*c*q*r + 2*a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 4
*(b*c - a*d)^2*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*d^2)
```

3.19.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2984, 2981, 17, 49, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)dx$$

$$\Big\downarrow \text{2984}$$

$$-pr\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)dx-\frac{dqr\int \frac{(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$$\Big\downarrow \text{2981}$$

$$-pr\left(-\frac{dqr\int \frac{(a+bx)^2}{c+dx}dx}{2b}-\frac{1}{2}pr\int (a+bx)dx+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}\right)-\frac{dqr\int \frac{(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-pr\left(-\frac{dqr\int \frac{(a+bx)^2}{c+dx}dx}{2b}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}\right)-\frac{dqr\int \frac{(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$-pr\left(-\frac{dqr\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{2b}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}\right)-\frac{dqr\int \frac{(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{dqr\int\frac{(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}-$$

$$pr\left(-\frac{dqr\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{2b}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}\right)+$$

$$\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$\downarrow$ 2994

$$-\frac{dqr\int\left(\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^2}+\frac{b(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}\right)dx}{b}-$$

$$pr\left(-\frac{dqr\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{2b}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}\right)+$$

$$\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$\downarrow$ 2009

$$-\frac{dqr\left(\frac{(bc-ad)^2\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^3}-\frac{pr(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d^3}-\frac{pr(bc-ad)^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d^3}-qr(bc\cdots\right.}{}$$

$$pr\left(-\frac{dqr\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{2b}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{pr(a+bx)^2}{4b}\right)+$$

$$\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

input
```
Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*b) - p*r*(-1/4*(p*
r*(a + b*x)^2)/b - (d*q*r*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) +
((b*c - a*d)^2*Log[c + d*x])/d^3))/(2*b) + ((a + b*x)^2*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r])/(2*b)) - (d*q*r*((b*(b*c - a*d)*q*r*x)/(2*d^2) + (b*(b
*c - a*d)*(p + q)*r*x)/d^2 - (p*r*(a + b*x)^2)/(4*d) - (q*r*(a + b*x)^2)/(
4*d) - (3*(b*c - a*d)^2*q*r*Log[c + d*x])/(2*d^3) - ((b*c - a*d)^2*p*r*Log
[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d^3 - ((b*c - a*d)^2*q*r*Log[
c + d*x]^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r])/d^2 + ((a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d) +
((b*c - a*d)^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - ((
b*c - a*d)^2*p*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/d^3))/b
```

3.19. $\int(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)dx$

3.19.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.19.4 Maple [F]

$$\int (bx+a)\ln\left(e\left(f(bx+a)^{p}(dx+c)^{q}\right)^{r}\right)^{2}dx$$

input
```
int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.19.5 Fricas [F]

$$\int (a+bx)\log^{2}\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)\,dx = \int (bx+a)\log\left(\left(((bx+a)^{p}(dx+c)^{q}f)^{r}\,e\right)^{2}\,dx\right.$$

input
```
integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas"
)
```

output
```
integral((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.19.6 Sympy [F]

$$\int (a+bx)\log^{2}\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)\,dx = \int (a+bx)\log\left(e\left(f(a+bx)^{p}(c+dx)^{q}\right)^{r}\right)^{2}dx$$

input
```
integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral((a + b*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.93

$$\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2$$
$$+\frac{\left(\frac{2\,a^2fp\log(bx+a)}{b}-\frac{bdf(p+q)x^2+2\,(adf(p+2\,q)-bcfq)x}{d}-\frac{2\,(bc^2fq-2\,acdfq)\log(dx+c)}{d^2}\right)r\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{2\,f}$$
$$+\frac{r^2\left(\frac{2\,((pq+3\,q^2)bc^2f^2-2\,(pq+2\,q^2)acdf^2)\log(dx+c)}{d^2}-\frac{4\,(b^2c^2f^2pq-2\,abcdf^2pq+a^2d^2f^2pq)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bd^2}\right.}{}$$

input
```
integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima"
)
```

output
```
1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*(2*a^2*f*
p*log(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d
 - 2*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)/f + 1/4*r^2*(2*((p*q + 3*q^2)*b*c^2*f^2 - 2*(p*q + 2*q^2)*a*
c*d*f^2)*log(d*x + c)/d^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q + a^2*d
^2*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d
*x + a*d)/(b*c - a*d)))/(b*d^2) - (2*a^2*d^2*f^2*p^2*log(b*x + a)^2 - (p^2
 + 2*p*q + q^2)*b^2*d^2*f^2*x^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q)*
log(b*x + a)*log(d*x + c) - 2*(b^2*c^2*f^2*q^2 - 2*a*b*c*d*f^2*q^2)*log(d*
x + c)^2 + 2*(3*(p*q + q^2)*b^2*c*d*f^2 - (p^2 + 5*p*q + 4*q^2)*a*b*d^2*f^
2)*x - 2*(2*a*b*c*d*f^2*p*q - (p^2 + 3*p*q)*a^2*d^2*f^2)*log(b*x + a))/(b*
d^2))/f^2
```

3.19.8 Giac [F]

$$\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \int (bx+a)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx$$

input
```
integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

output
```
integrate((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2\left(a+b\,x\right)\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x), x)
```

3.20 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

3.20.1 Optimal result

Integrand size = 31, antiderivative size = 431

$$\begin{aligned}
&\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx \\
&= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr} \\
&\quad + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} - \frac{2qr\log\left((a+bx)^{pr}\right)\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b} \\
&\quad + \frac{2qr\log\left((c+dx)^{qr}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b} - \frac{1}{4}\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right. \\
&\quad \left.-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\left(\frac{\left(\log\left((a+bx)^{pr}\right)-\log\left((c+dx)^{qr}\right)+\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)^2}{bpr}\right. \\
&\quad \left.+8\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b}+\frac{qr\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right) \\
&\quad + \frac{2pqr^2\operatorname{PolyLog}\left(3,-\frac{d(a+bx)}{bc-ad}\right)}{b} - \frac{2q^2r^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{bc-ad}\right)}{b}
\end{aligned}$$

output
```
1/3*ln((b*x+a)^(p*r))^3/b/p/r-q*ln((b*x+a)^(p*r))^2*ln(b*(d*x+c)/(-a*d+b*c
))/b/p+ln((b*x+a)^(p*r))^2*ln((d*x+c)^(q*r))/b/p/r+ln(-d*(b*x+a)/(-a*d+b*c
))*ln((d*x+c)^(q*r))^2/b-2*q*r*ln((b*x+a)^(p*r))*polylog(2,-d*(b*x+a)/(-a*
d+b*c))/b+2*q*r*ln((d*x+c)^(q*r))*polylog(2,b*(d*x+c)/(-a*d+b*c))/b-1/4*(l
n((b*x+a)^(p*r))+ln((d*x+c)^(q*r))-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*((ln((
b*x+a)^(p*r))-ln((d*x+c)^(q*r))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))^2/b/p/r+8
*ln(-d*(b*x+a)/(-a*d+b*c))*ln((d*x+c)^(q*r))/b+8*q*r*polylog(2,b*(d*x+c)/(
-a*d+b*c))/b)+2*p*q*r^2*polylog(3,-d*(b*x+a)/(-a*d+b*c))/b-2*q^2*r^2*polyl
og(3,b*(d*x+c)/(-a*d+b*c))/b
```

3.20.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.07

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \frac{p^2r^2\log^3(a+bx)+6pqr^2\log^2(a+bx)\log(c+dx)-6pqr^2\log(a+bx)\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)+3q^2r^2\log\cdots}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

output
```
(p^2*r^2*Log[a + b*x]^3 + 6*p*q*r^2*Log[a + b*x]^2*Log[c + d*x] - 6*p*q*r^
2*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 3*q^2*r^2*
Log[a + b*x]*Log[c + d*x]^2 - 3*q^2*r^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*
Log[c + d*x]^2 - 3*p*q*r^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] -
 3*p*r*Log[a + b*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 6*q*r*Log[a +
 b*x]*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 6*q*r*Log[(d*(a
+ b*x))/(-(b*c) + a*d)]*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
+ 3*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*p*q*r^2*Log[a
+ b*x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*q*r*(-(p*r*Log[a + b*x
]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)] + 6*p*p*q*r^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*q^2*r^2*Po
lyLog[3, (b*(c + d*x))/(b*c - a*d)])/(3*b)
```

3.20.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2982, 2841, 2840, 2838, 7237, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}dx$$

$$\Big\downarrow \text{2982}$$

$$\int \frac{(\log((a+bx)^{pr})+\log((c+dx)^{qr}))^2}{a+bx}dx - (-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\int \frac{\log((a+bx)^{pr})-\log((c+dx)^{qr})+\log\right.}{a+bx}$$

$$\Big\downarrow \text{2841}$$

$$\int \frac{(\log((a+bx)^{pr})+\log((c+dx)^{qr}))^2}{a+bx}dx - (-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\int \frac{\log((a+bx)^{pr})-\log((c+dx)^{qr})+\log\right.}{a+bx}$$

$$\Big\downarrow \text{2840}$$

$$\int \frac{(\log((a+bx)^{pr})+\log((c+dx)^{qr}))^2}{a+bx}dx - (-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\int \frac{\log((a+bx)^{pr})-\log((c+dx)^{qr})+\log\right.}{a+bx}$$

$$\Big\downarrow \text{2838}$$

$$\int \frac{(\log((a+bx)^{pr})+\log((c+dx)^{qr}))^2}{a+bx}dx - (-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\int \frac{\log((a+bx)^{pr})-\log((c+dx)^{qr})+\log\right.}{a+bx}$$

$$\Big\downarrow \text{7237}$$

$$\int \frac{(\log((a+bx)^{pr})+\log((c+dx)^{qr}))^2}{a+bx}dx -$$
$$(-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\frac{(\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+\right.}{4bpr}$$

$\Big\downarrow$ 7293

$$\int \left(\frac{\log^2((a+bx)^{pr})}{a+bx}+\frac{2\log((c+dx)^{qr})\log((a+bx)^{pr})}{a+bx}+\frac{\log^2((c+dx)^{qr})}{a+bx}\right)dx -$$
$$(-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\frac{(\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+\right.}{4bpr}$$

$\Big\downarrow$ 2009

$$-(-\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a+bx)^{pr})+\log((c+dx)^{qr}))\left(\frac{(\log(e(f(a+bx)^p(c+dx)^q)^r)+\log((a\right.}{4bpr}$$
$$\frac{2pqr^2\,\text{PolyLog}\left(3,-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{2qr\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\log((a+bx)^{pr})}{b}-$$
$$\frac{q\log\left(\frac{b(c+dx)}{bc-ad}\right)\log^2((a+bx)^{pr})}{bp}+\frac{\log^2((a+bx)^{pr})\log((c+dx)^{qr})}{bpr}-\frac{2q^2r^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{bc-ad}\right)}{b}+$$
$$\frac{2qr\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)\log((c+dx)^{qr})}{b}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2((c+dx)^{qr})}{b}+\frac{\log^3((a+bx)^{pr})}{3bpr}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

output
```
Log[(a + b*x)^(p*r)]^3/(3*b*p*r) - (q*Log[(a + b*x)^(p*r)]^2*Log[(b*(c + d
*x))/(b*c - a*d)])/(b*p) + (Log[(a + b*x)^(p*r)]^2*Log[(c + d*x)^(q*r)])/(
b*p*r) + (Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(q*r)]^2)/b - (2
*q*r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2
*q*r*Log[(c + d*x)^(q*r)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b - (Log[
(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])*((Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r])^2/(4*b*p*r) + 2*((Log[-((d*(a + b*x))/(b*c - a*d))]*Log[
(c + d*x)^(q*r)])/b + (q*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b)) + (2
*p*q*r^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*PolyLog[
3, (b*(c + d*x))/(b*c - a*d)])/b
```

3.20. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2982
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Int[(Log[(a + b*x)^(p*r)] + Lo
g[(c + d*x)^(q*r)])^2/(g + h*x), x] + Simp[(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r] - Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)])*(2   Int[Log[(c + d*x)
^(q*r)]/(g + h*x), x] + Int[(Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] +
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(g + h*x), x]), x] /; FreeQ[{a, b, c,
 d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0] && EqQ[b*g - a*h, 0]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.20.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^{p} \left(dx+c\right)^{q}\right)^{r}\right)^{2}}{bx+a}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

3.20.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right.}{bx+a}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="fricas"
)
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

3.20.6 Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p\left(c+dx\right)^q\right)^r\right)^2}{a+bx}\,dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a),x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x), x)
```

3.20.7 Maxima [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r\, e\right)^2}{bx+a}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="maxima"
)
```

output
```
log(b*x + a)*log(((d*x + c)^q)^r)^2/b + integrate(((r^2*log(f)^2 + 2*r*log
(e)*log(f) + log(e)^2)*b*d*x + (r^2*log(f)^2 + 2*r*log(e)*log(f) + log(e)^
2)*b*c + (b*d*x + b*c)*log(((b*x + a)^p)^r)^2 + 2*((r*log(f) + log(e))*b*d
*x + (r*log(f) + log(e))*b*c)*log(((b*x + a)^p)^r) + 2*((r*log(f) + log(e)
)*b*d*x + (r*log(f) + log(e))*b*c - (b*d*q*r*x + a*d*q*r)*log(b*x + a) + (
b*d*x + b*c)*log(((b*x + a)^p)^r))*log(((d*x + c)^q)^r))/(b^2*d*x^2 + a*b*
c + (b^2*c + a*b*d)*x), x)
```

3.20.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r\, e\right)^2}{bx+a}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{a+b\,x}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x), x)
```

3.21 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}\,dx$

3.21.1 Optimal result

Integrand size = 31, antiderivative size = 465

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{2p^2r^2}{b(a+bx)}+\frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)}-\frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)} + \frac{dq^2r^2\log^2(c+dx)}{b(bc-ad)} - \frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + \frac{2dqr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{2dqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} - \frac{2dq^2r^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} + \frac{2dpqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}$$

output
```
-2*p^2*r^2/b/(b*x+a)+2*d*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)-d*p*q*r^2*ln(b*x+a
)^2/b/(-a*d+b*c)-2*d*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)+2*d*p*q*r^2*ln(-d*(b*x
+a)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)+d*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)-
2*d*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)-2*p*r*ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)+2*d*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r)/b/(-a*d+b*c)-2*d*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
/(-a*d+b*c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)-2*d*q^2*r^2*polylo
g(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)+2*d*p*q*r^2*polylog(2,b*(d*x+c)/(-
a*d+b*c))/b/(-a*d+b*c)
```

3.21.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.88

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$$
$$= \frac{-2bcp^2r^2+2adp^2r^2-dpqr^2(a+bx)\log^2(a+bx)-2adpqr^2\log(c+dx)-2bdpqr^2x\log(c+dx)+adq^2r}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

output
```
(-2*b*c*p^2*r^2 + 2*a*d*p^2*r^2 - d*p*q*r^2*(a + b*x)*Log[a + b*x]^2 - 2*a
*d*p*q*r^2*Log[c + d*x] - 2*b*d*p*q*r^2*x*Log[c + d*x] + a*d*q^2*r^2*Log[c
 + d*x]^2 + b*d*q^2*r^2*x*Log[c + d*x]^2 - 2*b*c*p*r*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r] + 2*a*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*q*r*x*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - b*c*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^2 + a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*d*q*r*(a +
b*x)*Log[a + b*x]*(p*r + p*r*Log[c + d*x] - (p + q)*r*Log[(b*(c + d*x))/(b
*c - a*d)] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) - 2*d*q*(p + q)*r^2*(a
+ b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)*(a + b*x))
```

3.21.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2984, 2981, 17, 47, 16, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \text{2984}$$
$$2pr\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}dx+\frac{2dqr\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}$$

$$\Big\downarrow \text{2981}$$

$$2pr\left(\frac{dqr\int \frac{1}{(a+bx)(c+dx)}dx}{b}+pr\int \frac{1}{(a+bx)^2}dx-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}\right)+\frac{2dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(c+dx)}dx}{b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

$$\Big\downarrow \text{17}$$

$$2pr\left(\frac{dqr\int \frac{1}{(a+bx)(c+dx)}dx}{b}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}-\frac{pr}{b(a+bx)}\right)+\frac{2dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(c+dx)}dx}{b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

$$\Big\downarrow \text{47}$$

$$2pr\left(\frac{dqr\left(\frac{b\int \frac{1}{a+bx}dx}{bc-ad}-\frac{d\int \frac{1}{c+dx}dx}{bc-ad}\right)}{b}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}-\frac{pr}{b(a+bx)}\right)+\frac{2dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(c+dx)}dx}{b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{2dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(c+dx)}dx}{b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+2pr\left(-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+\frac{dqr\left(\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\right)}{b}-\frac{pr}{b(a+bx)}\right)$$

$$\Big\downarrow \text{2994}$$

$$\frac{2dqr\int \left(\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)(a+bx)}-\frac{d\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)(c+dx)}\right)dx}{b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+2pr\left(-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+\frac{dqr\left(\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\right)}{b}-\frac{pr}{b(a+bx)}\right)$$

$$\Big\downarrow \text{2009}$$

3.21. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}dx$

$$\frac{2dqr\left(\frac{\log(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{bc-ad} - \frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{bc-ad} + \frac{pr\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bc-ad} - \frac{pr\log^2(a+bx)}{2(bc-ad)} + \frac{pr\log(\cdots}{\cdots}\right.}{b}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + 2pr\left(-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + \frac{dqr\left(\frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}\right)}{b} - \frac{pr}{b(a+bx)}\right)$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

output
```
-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(b*(a + b*x))) + 2*p*r*(-((p*r)/(
b*(a + b*x))) + (d*q*r*(Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d
)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))) + (2*d*q*r*(-1
/2*(p*r*Log[a + b*x]^2)/(b*c - a*d) + (p*r*Log[-((d*(a + b*x))/(b*c - a*d)
)]*Log[c + d*x])/(b*c - a*d) + (q*r*Log[c + d*x]^2)/(2*(b*c - a*d)) - (q*r
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*c - a*d) + (Log[a + b*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d) - (Log[c + d*x]*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d) - (q*r*PolyLog[2, -((d*(a + b*x)
)/(b*c - a*d))])/(b*c - a*d) + (p*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
/(b*c - a*d)))/b
```

3.21.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.21.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^{p} (dx+c)^{q}\right)^{r}\right)^{2}}{(bx+a)^{2}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

3.21.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx=\int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^2*x^2 + 2*a*b*x + a^2),
 x)
```

3.21.6 Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)^2}{(a+bx)^2}\,dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**2,x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**2, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx\\&=\frac{2\left(\frac{dfq\log(bx+a)}{bc-ad}-\frac{dfq\log(dx+c)}{bc-ad}-\frac{fp}{bx+a}\right)r\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bf}\\&\quad-\frac{\left(\frac{2\,df^2pq\log(dx+c)}{bc-ad}+\frac{2\left(pq+q^2\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)df^2}{bc-ad}+\frac{2\,bcf^2p^2-2\,adf^2p^2+\left(bdf^2pqx+adf^2pq\right)\log(bx+a)^2\cdots}{\cdots}\cdots\right.}{bf^2}\\&\quad-\frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)b}\end{aligned}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="maxim
a")
```

output
```
2*(d*f*q*log(b*x + a)/(b*c - a*d) - d*f*q*log(d*x + c)/(b*c - a*d) - f*p/(
b*x + a))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - (2*d*f^2*p*q*log(
d*x + c)/(b*c - a*d) + 2*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*d*f^2/(b*c - a*d) + (2*b*
c*f^2*p^2 - 2*a*d*f^2*p^2 + (b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)^2 -
 2*(b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)*log(d*x + c) - (b*d*f^2*q^2*
x + a*d*f^2*q^2)*log(d*x + c)^2 - 2*(b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x
+ a))/(a*b*c - a^2*d + (b^2*c - a*b*d)*x))*r^2/(b*f^2) - log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)^2/((b*x + a)*b)
```

3.21.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^2, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^2}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2, x)
```

3.22 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}\,dx$

3.22.1 Optimal result

Integrand size = 31, antiderivative size = 632

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx = -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2} + \frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2} - \frac{d^2q^2r^2\log(c+dx)}{b(bc-ad)^2} - \frac{d^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)^2} - \frac{d^2q^2r^2\log^2(c+dx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2} + \frac{d^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{d^2q^2r^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{d^2pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}$$

output
```
-1/4*p^2*r^2/b/(b*x+a)^2-3/2*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*p*q*r^
2*ln(b*x+a)/b/(-a*d+b*c)^2+d^2*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^2+1/2*d^2*p*
q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^2+1/2*d^2*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-
d^2*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))
*ln(d*x+c)/b/(-a*d+b*c)^2-1/2*d^2*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^2+d^2*q
^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^2-1/2*p*r*ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^2-d*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
/(-a*d+b*c)/(b*x+a)-d^2*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-
a*d+b*c)^2+d^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^
2-1/2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^2+d^2*q^2*r^2*polylog(2,
-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^2-d^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*
d+b*c))/b/(-a*d+b*c)^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.38

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx =$$

$$-\frac{b^2c^2p^2r^2 - 2abcdp^2r^2 + a^2d^2p^2r^2 + 6abcdpqr^2 - 6a^2d^2pqr^2 + 6b^2cdpqr^2x - 6abd^2pqr^2x - 2d^2pqr^2(a+$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

output
```
-1/4*(b^2*c^2*p^2*r^2 - 2*a*b*c*d*p^2*r^2 + a^2*d^2*p^2*r^2 + 6*a*b*c*d*p*
q*r^2 - 6*a^2*d^2*p*q*r^2 + 6*b^2*c*d*p*q*r^2*x - 6*a*b*d^2*p*q*r^2*x - 2*
d^2*p*q*r^2*(a + b*x)^2*Log[a + b*x]^2 - 2*a^2*d^2*p*q*r^2*Log[c + d*x] +
4*a^2*d^2*q^2*r^2*Log[c + d*x] - 4*a*b*d^2*p*q*r^2*x*Log[c + d*x] + 8*a*b*
d^2*q^2*r^2*x*Log[c + d*x] - 2*b^2*d^2*p*q*r^2*x^2*Log[c + d*x] + 4*b^2*d^
2*q^2*r^2*x^2*Log[c + d*x] + 2*a^2*d^2*q^2*r^2*Log[c + d*x]^2 + 4*a*b*d^2*
q^2*r^2*x*Log[c + d*x]^2 + 2*b^2*d^2*q^2*r^2*x^2*Log[c + d*x]^2 - 2*d^2*q*
r*(a + b*x)^2*Log[a + b*x]*(-(p*r) + 2*q*r - 2*p*r*Log[c + d*x] + 2*(p + q
)*r*Log[(b*(c + d*x))/(b*c - a*d)] - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]) + 2*b^2*c^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*c*d*p*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] + 4*a*b*c*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2
*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*b^2*c*d*q*r*x*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r] - 4*a^2*d^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 8*a*b*d^2*q*r*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*b^
2*d^2*q*r*x^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^
2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*a*b*c*d*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]^2 + 2*a^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*d
^2*q*(p + q)*r^2*(a + b*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/...
```

3.22.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2984, 2981, 17, 54, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2984}$$

$$pr\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}dx+\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2(c+dx)}dx}{b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{2981}$$

$$pr\left(\frac{dqr\int \frac{1}{(a+bx)^2(c+dx)}dx}{2b}+\frac{1}{2}pr\int \frac{1}{(a+bx)^3}dx-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}\right)+\frac{dqr\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2(c+dx)}dx}{b}-\frac{\log ^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{17}$$

$$pr\left(\frac{dqr\int \frac{1}{(a+bx)^2(c+dx)}dx}{2b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{pr}{4b(a+bx)^2}\right)+\frac{dqr\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2(c+dx)}dx}{b}-\frac{\log ^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{54}$$

$$pr\left(\frac{dqr\int \left(\frac{d^2}{(bc-ad)^2(c+dx)}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{b}{(bc-ad)(a+bx)^2}\right)dx}{2b}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{pr}{4b(a+bx)^2}\right)+\frac{dqr\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2(c+dx)}dx}{b}-\frac{\log ^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{dqr\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2(c+dx)}dx}{b}-\frac{\log ^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}+pr\left(-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}+\frac{dqr\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log (a+bx)}{(bc-ad)^2}+\frac{d\log (c+dx)}{(bc-ad)^2}\right)}{2b}-\frac{pr}{4b(a+bx)^2}\right)$$

$$\Big\downarrow \text{2994}$$

$$\frac{dqr\int \left(\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)d^2}{(bc-ad)^2(c+dx)}-\frac{b\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)d}{(bc-ad)^2(a+bx)}+\frac{b\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)(a+bx)^2}\right)dx}{b}-\frac{\log ^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}+pr\left(-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}+\frac{dqr\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log (a+bx)}{(bc-ad)^2}+\frac{d\log (c+dx)}{(bc-ad)^2}\right)}{2b}-\frac{pr}{4b(a+bx)^2}\right)$$

$$\Big\downarrow \text{2009}$$

3.22. $\int \frac{\log ^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}dx$

$$\frac{dqr\left(-\frac{d\log(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)^2}+\frac{d\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)^2}-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(bc-ad)}-\frac{dpr\operatorname{PolyLog}\left(2,\frac{b(}{b}\right.}{(bc-ad)^2}\right.}{}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}+$$

$$pr\left(-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}+\frac{dqr\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{2b}-\frac{pr}{4b(a+bx)^2}\right)$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

output
```
-1/2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(b*(a + b*x)^2) + p*r*(-1/4*(p
*r)/(b*(a + b*x)^2) + (d*q*r*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*
x])/(b*c - a*d)^2 + (d*Log[c + d*x])/(b*c - a*d)^2))/(2*b) - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)) + (d*q*r*(-((p*r)/((b*c - a*d)*
(a + b*x))) + (d*q*r*Log[a + b*x])/(b*c - a*d)^2 + (d*p*r*Log[a + b*x]^2)/
(2*(b*c - a*d)^2) - (d*q*r*Log[c + d*x])/(b*c - a*d)^2 - (d*p*r*Log[-((d*(
a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*c - a*d)^2 - (d*q*r*Log[c + d*x]^
2)/(2*(b*c - a*d)^2) + (d*q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])
/(b*c - a*d)^2 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((b*c - a*d)*(a + b*
x)) - (d*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d)^2
+ (d*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d)^2 + (d
*q*r*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*c - a*d)^2 - (d*p*r*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^2))/b
```

3.22.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.22.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}}{\left(bx+a\right)^{3}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

3.22.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^3}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^3*x^3 + 3*a*b^2*x^2 + 3
*a^2*b*x + a^3), x)
```

3.22.6 Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)^2}{(a+bx)^3}\,dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**3,x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**3, x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.19

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx =$$

$$-\frac{\left(\frac{2\,d^2fq\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{2\,d^2fq\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{2\,bdfqx-adf(p-2\,q)+bcfp}{a^2bc-a^3d+(b^3c-ab^2d)x^2+2\,(ab^2c-a^2bd)x}\right)r\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{2\,bf}$$

$$+\frac{\left(\frac{4\,(pq+q^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)d^2f^2}{b^2c^2-2\,abcd+a^2d^2}+\frac{2\,(pq-2\,q^2)d^2f^2\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}-\frac{b^2c^2f^2p^2-2\,(p^2-3\,pq)abcdf^2+(p^2-6\,pq}{}\right.}{}$$

$$-\frac{\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{2\,(bx+a)^2b}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="maxim
a")
```

output
```
-1/2*(2*d^2*f*q*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 2*d^2*f*q*l
og(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (2*b*d*f*q*x - a*d*f*(p - 2*
q) + b*c*f*p)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2 + 2*(a*b^2*c - a^2*
b*d)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/4*(4*(p*q + q^2)
*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*d^2*f^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 2*(p*q - 2*q^2)*d^2
*f^2*log(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - (b^2*c^2*f^2*p^2 - 2*(
p^2 - 3*p*q)*a*b*c*d*f^2 + (p^2 - 6*p*q)*a^2*d^2*f^2 - 2*(b^2*d^2*f^2*p*q*
x^2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)^2 + 4*(b^2*d^2*f
^2*p*q*x^2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)*log(d*x +
 c) + 2*(b^2*d^2*f^2*q^2*x^2 + 2*a*b*d^2*f^2*q^2*x + a^2*d^2*f^2*q^2)*log(
d*x + c)^2 + 6*(b^2*c*d*f^2*p*q - a*b*d^2*f^2*p*q)*x + 2*((p*q - 2*q^2)*b^
2*d^2*f^2*x^2 + 2*(p*q - 2*q^2)*a*b*d^2*f^2*x + (p*q - 2*q^2)*a^2*d^2*f^2)
*log(b*x + a))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x))*r^2/
(b*f^2) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^2*b)
```

3.22.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \int \frac{\log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^3}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^3, x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2}{\left(a+b\,x\right)^3}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3, x)
```

3.23 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} \, dx$

3.23.1 Optimal result

Integrand size = 31, antiderivative size = 764

$$
\begin{aligned}
\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = & -\frac{2p^2r^2}{27b(a+bx)^3}-\frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} \\
& +\frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} \\
& -\frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)}+\frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} \\
& -\frac{d^3q^2r^2\log(a+bx)}{b(bc-ad)^3}-\frac{d^3pqr^2\log^2(a+bx)}{3b(bc-ad)^3} \\
& -\frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3}+\frac{d^3q^2r^2\log(c+dx)}{b(bc-ad)^3} \\
& +\frac{2d^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3b(bc-ad)^3} \\
& +\frac{d^3q^2r^2\log^2(c+dx)}{3b(bc-ad)^3} \\
& -\frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} \\
& -\frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3} \\
& -\frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)(a+bx)^2} \\
& +\frac{2d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^2(a+bx)} \\
& +\frac{2d^3qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} \\
& -\frac{2d^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} \\
& -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3} \\
& -\frac{2d^3q^2r^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} \\
& +\frac{2d^3pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}
\end{aligned}
$$

output
```
-2/27*p^2*r^2/b/(b*x+a)^3-5/18*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^2+8/9*d^2*p*
q*r^2/b/(-a*d+b*c)^2/(b*x+a)-1/3*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)+2/9*d^
3*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-d^3*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-1/
3*d^3*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^3-2/9*d^3*p*q*r^2*ln(d*x+c)/b/(-a*d
+b*c)^3+d^3*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*ln(-d*(b*x+a)
/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^3+1/3*d^3*q^2*r^2*ln(d*x+c)^2/b/(-a*d+
b*c)^3-2/3*d^3*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3-2
/9*p*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3-1/3*d*q*r*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^2+2/3*d^2*q*r*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)/b/(-a*d+b*c)^2/(b*x+a)+2/3*d^3*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/b/(-a*d+b*c)^3-2/3*d^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r)/b/(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^3
-2/3*d^3*q^2*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^3+2/3*d^3*p
*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3
```

3.23.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 1407, normalized size of antiderivative = 1.84

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx =$$

$$-\frac{4b^3c^3p^2r^2 - 12ab^2c^2dp^2r^2 + 12a^2bcd^2p^2r^2 - 4a^3d^3p^2r^2 + 15ab^2c^2dpqr^2 - 78a^2bcd^2pqr^2 + 63a^3d^3pqr^2 +}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

output
```
-1/54*(4*b^3*c^3*p^2*r^2 - 12*a*b^2*c^2*d*p^2*r^2 + 12*a^2*b*c*d^2*p^2*r^2
 - 4*a^3*d^3*p^2*r^2 + 15*a*b^2*c^2*d*p*q*r^2 - 78*a^2*b*c*d^2*p*q*r^2 + 6
3*a^3*d^3*p*q*r^2 + 18*a^2*b*c*d^2*q^2*r^2 - 18*a^3*d^3*q^2*r^2 + 15*b^3*c
^2*d*p*q*r^2*x - 126*a*b^2*c*d^2*p*q*r^2*x + 111*a^2*b*d^3*p*q*r^2*x + 36*
a*b^2*c*d^2*q^2*r^2*x - 36*a^2*b*d^3*q^2*r^2*x - 48*b^3*c*d^2*p*q*r^2*x^2
+ 48*a*b^2*d^3*p*q*r^2*x^2 + 18*b^3*c*d^2*q^2*r^2*x^2 - 18*a*b^2*d^3*q^2*r
^2*x^2 + 18*d^3*p*q*r^2*(a + b*x)^3*Log[a + b*x]^2 + 12*a^3*d^3*p*q*r^2*Lo
g[c + d*x] - 54*a^3*d^3*q^2*r^2*Log[c + d*x] + 36*a^2*b*d^3*p*q*r^2*x*Log[
c + d*x] - 162*a^2*b*d^3*q^2*r^2*x*Log[c + d*x] + 36*a*b^2*d^3*p*q*r^2*x^2
*Log[c + d*x] - 162*a*b^2*d^3*q^2*r^2*x^2*Log[c + d*x] + 12*b^3*d^3*p*q*r^
2*x^3*Log[c + d*x] - 54*b^3*d^3*q^2*r^2*x^3*Log[c + d*x] - 18*a^3*d^3*q^2*
r^2*Log[c + d*x]^2 - 54*a^2*b*d^3*q^2*r^2*x*Log[c + d*x]^2 - 54*a*b^2*d^3*
q^2*r^2*x^2*Log[c + d*x]^2 - 18*b^3*d^3*q^2*r^2*x^3*Log[c + d*x]^2 + 12*b^
3*c^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a*b^2*c^2*d*p*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*a^2*b*c*d^2*p*r*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r] - 12*a^3*d^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*
a*b^2*c^2*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*a^2*b*c*d^2*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 54*a^3*d^3*q*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] + 18*b^3*c^2*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 - 108*a*b^2*c*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 90*a^2*...
```

3.23.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2984, 2981, 17, 54, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 2984}$$

$$\frac{2}{3}pr\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}dx+\frac{2dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3(c+dx)}dx}{3b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$$\Big\downarrow \text{ 2981}$$

$$\frac{2}{3}pr\left(\frac{dqr\int\frac{1}{(a+bx)^3(c+dx)}dx}{3b}+\frac{1}{3}pr\int\frac{1}{(a+bx)^4}dx-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}\right)+\frac{2dqr\int\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3(c+dx)}dx}{3b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$\downarrow$ 17

$$\frac{2}{3}pr\left(\frac{dqr\int\frac{1}{(a+bx)^3(c+dx)}dx}{3b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+bx)^3}\right)+\frac{2dqr\int\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3(c+dx)}dx}{3b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$\downarrow$ 54

$$\frac{2}{3}pr\left(\frac{dqr\int\left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{3b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}\right.$$
$$\frac{2dqr\int\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3(c+dx)}dx}{3b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$\downarrow$ 2009

$$\frac{2dqr\int\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3(c+dx)}dx}{3b}+$$
$$\frac{2}{3}pr\left(\frac{dqr\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{3b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+\right.$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$\downarrow$ 2994

$$\frac{2dqr\int\left(-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)d^3}{(bc-ad)^3(c+dx)}+\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)d^2}{(bc-ad)^3(a+bx)}-\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)d}{(bc-ad)^2(a+bx)^2}+\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)(a+bx)^3}\right.}{3b}$$
$$\frac{2}{3}pr\left(\frac{dqr\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{3b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+\right.$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$\downarrow$ 2009

3.23. $\int\frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4}dx$

$$\frac{2dqr\left(\frac{d^2\log(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)^3}-\frac{d^2\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{(bc-ad)^3}+\frac{d^2pr\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^3}-\frac{d^2pr\log^2(a+bx)}{2(bc-ad)^3}\right.}{}$$

$$\frac{2}{3}pr\left(\frac{dqr\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{3b}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}-\frac{pr}{9b(a+\right.$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

output
```
-1/3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(b*(a + b*x)^3) + (2*p*r*(-1/9
*(p*r)/(b*(a + b*x)^3) + (d*q*r*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*
c - a*d)^2*(a + b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*
x])/(b*c - a*d)^3))/(3*b) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*b*(a +
 b*x)^3)))/3 + (2*d*q*r*(-1/4*(p*r)/((b*c - a*d)*(a + b*x)^2) + (d*p*r)/((
b*c - a*d)^2*(a + b*x)) - (d*q*r)/(2*(b*c - a*d)^2*(a + b*x)) - (3*d^2*q*r
*Log[a + b*x])/(2*(b*c - a*d)^3) - (d^2*p*r*Log[a + b*x]^2)/(2*(b*c - a*d)
^3) + (3*d^2*q*r*Log[c + d*x])/(2*(b*c - a*d)^3) + (d^2*p*r*Log[-((d*(a +
b*x))/(b*c - a*d))]*Log[c + d*x])/(b*c - a*d)^3 + (d^2*q*r*Log[c + d*x]^2)
/(2*(b*c - a*d)^3) - (d^2*q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])
/(b*c - a*d)^3 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*(b*c - a*d)*(a +
b*x)^2) + (d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/((b*c - a*d)^2*(a + b*x
)) + (d^2*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d)^3
 - (d^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d)^3 -
 (d^2*q*r*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*c - a*d)^3 + (d^2*p
*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^3))/(3*b)
```

3.23.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.23.4 Maple [F]

$$\int \frac{\ln \left(e{\left(f{\left(bx+a\right)}^{p} {\left(dx+c\right)}^{q}\right)}^{r}\right)^{2}}{\left(bx+a\right)^{4}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

3.23.5 Fricas [F]

$$\int \frac{\log^2 (e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} dx = \int \frac{\log \left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^4} dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^4*x^4 + 4*a*b^3*x^3 + 6
*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

3.23.6 Sympy [F]

$$\int \frac{\log^2 (e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} dx = \int \frac{\log \left(e\left(f(a+bx)^p (c+dx)^q\right)^r\right)^2}{(a+bx)^4} dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**4,x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**4, x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1252, normalized size of antiderivative = 1.64

$$\int \frac{\log^2 (e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} dx = \text{Too large to display}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="maxim
a")
```

output
```
1/9*(6*d^3*f*q*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3) - 6*d^3*f*q*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3) + (6*b^2*d^2*f*q*x^2 + a*b*c*d*f*(4*p - 3*q) - a^2*d^2*f*(2*p - 9
*q) - 2*b^2*c^2*f*p - 3*(b^2*c*d*f*q - 5*a*b*d^2*f*q)*x)/(a^3*b^2*c^2 - 2*
a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4
*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d +
 a^4*b*d^2)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - 1/54*(36*(p
*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))*d^3*f^2/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3) + 6*(2*p*q - 9*q^2)*d^3*f^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3) + (4*b^3*c^3*f^2*p^2 - 3*(4*p^2 - 5*p*q)*a*b^2*
c^2*d*f^2 + 6*(2*p^2 - 13*p*q + 3*q^2)*a^2*b*c*d^2*f^2 - (4*p^2 - 63*p*q +
 18*q^2)*a^3*d^3*f^2 - 6*((8*p*q - 3*q^2)*b^3*c*d^2*f^2 - (8*p*q - 3*q^2)*
a*b^2*d^3*f^2)*x^2 + 18*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^2*d^3*f^2*p*q*x^2 + 3
*a^2*b*d^3*f^2*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)^2 - 36*(b^3*d^3*f^2*p
*q*x^3 + 3*a*b^2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2*p*q*x + a^3*d^3*f^2*p*q
)*log(b*x + a)*log(d*x + c) - 18*(b^3*d^3*f^2*q^2*x^3 + 3*a*b^2*d^3*f^2*q^
2*x^2 + 3*a^2*b*d^3*f^2*q^2*x + a^3*d^3*f^2*q^2)*log(d*x + c)^2 + 3*(5*b^3
*c^2*d*f^2*p*q - 6*(7*p*q - 2*q^2)*a*b^2*c*d^2*f^2 + (37*p*q - 12*q^2)*a^2
*b*d^3*f^2)*x - 6*((2*p*q - 9*q^2)*b^3*d^3*f^2*x^3 + 3*(2*p*q - 9*q^2)*...
```

3.23.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \int \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)^2}{\left(bx+a\right)^4}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^4, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^4}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4, x)
```

3.24 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5}\,dx$

3.24.1 Optimal result

Integrand size = 31, antiderivative size = 884

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2q^2r^2}{12b(bc-ad)^2(a+bx)^2} - \frac{5d^3pqr^2}{8b(bc-ad)^3(a+bx)} + \frac{5d^3q^2r^2}{12b(bc-ad)^3(a+bx)} - \frac{d^4pqr^2\log(a+bx)}{8b(bc-ad)^4} + \frac{11d^4q^2r^2\log(a+bx)}{12b(bc-ad)^4} + \frac{d^4pqr^2\log^2(a+bx)}{4b(bc-ad)^4} + \frac{d^4pqr^2\log(c+dx)}{8b(bc-ad)^4} - \frac{11d^4q^2r^2\log(c+dx)}{12b(bc-ad)^4} - \frac{d^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2b(bc-ad)^4} - \frac{d^4q^2r^2\log^2(c+dx)}{4b(bc-ad)^4} + \frac{d^4q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{6b(bc-ad)(a+bx)^3} + \frac{d^2qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(bc-ad)^2(a+bx)^2} - \frac{d^3qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^3(a+bx)} - \frac{d^4qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^4} + \frac{d^4qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^4} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{d^4q^2r^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{2b(bc-ad)^4} - \frac{d^4pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4}$$

output
```
-1/32*p^2*r^2/b/(b*x+a)^4-7/72*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^3+3/16*d^2*p
*q*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-1/12*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-
5/8*d^3*p*q*r^2/b/(-a*d+b*c)^3/(b*x+a)+5/12*d^3*q^2*r^2/b/(-a*d+b*c)^3/(b*
x+a)-1/8*d^4*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^4+11/12*d^4*q^2*r^2*ln(b*x+a)/
b/(-a*d+b*c)^4+1/4*d^4*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^4+1/8*d^4*p*q*r^2*
ln(d*x+c)/b/(-a*d+b*c)^4-11/12*d^4*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^4-1/2*d^
4*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*d^4*q^2*r
^2*ln(d*x+c)^2/b/(-a*d+b*c)^4+1/2*d^4*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d
+b*c))/b/(-a*d+b*c)^4-1/8*p*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^4-
1/6*d*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^3+1/4*d^2*q
*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2/(b*x+a)^2-1/2*d^3*q*r*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^3/(b*x+a)-1/2*d^4*q*r*ln(b*x+a)
*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^4+1/2*d^4*q*r*ln(d*x+c)*ln(e
*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)^2/b/(b*x+a)^4+1/2*d^4*q^2*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*
d+b*c)^4-1/2*d^4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^4
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2003 vs. 2(884) = 1768.

Time = 1.32 (sec) , antiderivative size = 2003, normalized size of antiderivative = 2.27

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \text{Result too large to show}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

output
```
(-9*b^4*c^4*p^2*r^2 + 36*a*b^3*c^3*d*p^2*r^2 - 54*a^2*b^2*c^2*d^2*p^2*r^2
+ 36*a^3*b*c*d^3*p^2*r^2 - 9*a^4*d^4*p^2*r^2 - 28*a*b^3*c^3*d*p*q*r^2 + 13
8*a^2*b^2*c^2*d^2*p*q*r^2 - 372*a^3*b*c*d^3*p*q*r^2 + 262*a^4*d^4*p*q*r^2
- 24*a^2*b^2*c^2*d^2*q^2*r^2 + 168*a^3*b*c*d^3*q^2*r^2 - 144*a^4*d^4*q^2*r
^2 - 28*b^4*c^3*d*p*q*r^2*x + 192*a*b^3*c^2*d^2*p*q*r^2*x - 840*a^2*b^2*c*
d^3*p*q*r^2*x + 676*a^3*b*d^4*p*q*r^2*x - 48*a*b^3*c^2*d^2*q^2*r^2*x + 456
*a^2*b^2*c*d^3*q^2*r^2*x - 408*a^3*b*d^4*q^2*r^2*x + 54*b^4*c^2*d^2*p*q*r^
2*x^2 - 648*a*b^3*c*d^3*p*q*r^2*x^2 + 594*a^2*b^2*d^4*p*q*r^2*x^2 - 24*b^4
*c^2*d^2*q^2*r^2*x^2 + 408*a*b^3*c*d^3*q^2*r^2*x^2 - 384*a^2*b^2*d^4*q^2*r
^2*x^2 - 180*b^4*c*d^3*p*q*r^2*x^3 + 180*a*b^3*d^4*p*q*r^2*x^3 + 120*b^4*c
*d^3*q^2*r^2*x^3 - 120*a*b^3*d^4*q^2*r^2*x^3 + 72*d^4*p*q*r^2*(a + b*x)^4*
Log[a + b*x]^2 + 36*a^4*d^4*p*q*r^2*Log[c + d*x] - 264*a^4*d^4*q^2*r^2*Log
[c + d*x] + 144*a^3*b*d^4*p*q*r^2*x*Log[c + d*x] - 1056*a^3*b*d^4*q^2*r^2*
x*Log[c + d*x] + 216*a^2*b^2*d^4*p*q*r^2*x^2*Log[c + d*x] - 1584*a^2*b^2*d
^4*q^2*r^2*x^2*Log[c + d*x] + 144*a*b^3*d^4*p*q*r^2*x^3*Log[c + d*x] - 105
6*a*b^3*d^4*q^2*r^2*x^3*Log[c + d*x] + 36*b^4*d^4*p*q*r^2*x^4*Log[c + d*x]
 - 264*b^4*d^4*q^2*r^2*x^4*Log[c + d*x] - 72*a^4*d^4*q^2*r^2*Log[c + d*x]^
2 - 288*a^3*b*d^4*q^2*r^2*x*Log[c + d*x]^2 - 432*a^2*b^2*d^4*q^2*r^2*x^2*L
og[c + d*x]^2 - 288*a*b^3*d^4*q^2*r^2*x^3*Log[c + d*x]^2 - 72*b^4*d^4*q^2*
r^2*x^4*Log[c + d*x]^2 + 12*d^4*q*r*(a + b*x)^4*Log[a + b*x]*(-3*p*r + ...
```

3.24.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 814, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2984, 2981, 17, 54, 2009, 2994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2984}$$

$$\frac{1}{2}pr\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}dx+\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4(c+dx)}dx}{2b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$$\Big\downarrow \text{2981}$$

$$\frac{1}{2}pr\left(\frac{dqr\int \frac{1}{(a+bx)^4(c+dx)}dx}{4b}+\frac{1}{4}pr\int \frac{1}{(a+bx)^5}dx-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}\right)+$$
$$\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4(c+dx)}dx}{2b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$\downarrow$ 17

$$\frac{1}{2}pr\left(\frac{dqr\int \frac{1}{(a+bx)^4(c+dx)}dx}{4b}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}-\frac{pr}{16b(a+bx)^4}\right)+$$
$$\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4(c+dx)}dx}{2b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$\downarrow$ 54

$$\frac{1}{2}pr\left(\frac{dqr\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{4b}-\frac{\log \left(e(f(a+\right.}{4b(\right.}$$
$$\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4(c+dx)}dx}{2b}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$\downarrow$ 2009

$$\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4(c+dx)}dx}{2b}+$$
$$\frac{1}{2}pr\left(\frac{dqr\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{4b}-\frac{\log \left(e(f(a+bx)^p(c-\right.}{4b(a+bx)^4}\right.$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$\downarrow$ 2994

$$\frac{dqr\int \left(\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)d^4}{(bc-ad)^4(c+dx)}-\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)d^3}{(bc-ad)^4(a+bx)}+\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)d^2}{(bc-ad)^3(a+bx)^2}-\frac{b\log(e(f(a+bx)^p(c+dx)^q)^r)d}{(bc-ad)^2(a+bx)^3}+\right.}{2b}$$
$$\frac{1}{2}pr\left(\frac{dqr\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{4b}-\frac{\log \left(e(f(a+bx)^p(c-\right.}{4b(a+bx)^4}\right.$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

$\downarrow$ 2009

3.24. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5}\,dx$

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}+$$

$$\frac{1}{2}pr\left(\frac{dq\left(-\frac{\log(a+bx)d^3}{(bc-ad)^4}+\frac{\log(c+dx)d^3}{(bc-ad)^4}-\frac{d^2}{(bc-ad)^3(a+bx)}+\frac{d}{2(bc-ad)^2(a+bx)^2}-\frac{1}{3(bc-ad)(a+bx)^3}\right)r}{4b}-\frac{pr}{16b(a+bx)^4}-\frac{\log(\right.$$

$$dqr\left(\frac{pr\log^2(a+bx)d^3}{2(bc-ad)^4}-\frac{qr\log^2(c+dx)d^3}{2(bc-ad)^4}+\frac{11qr\log(a+bx)d^3}{6(bc-ad)^4}-\frac{11qr\log(c+dx)d^3}{6(bc-ad)^4}-\frac{pr\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)d^3}{(bc-ad)^4}+\frac{qr\log(a+bx)\log(}{(bc-ad}\right.$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

output
```
-1/4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(b*(a + b*x)^4) + (p*r*(-1/16*
(p*r)/(b*(a + b*x)^4) + (d*q*r*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b
*c - a*d)^2*(a + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*
x])/(b*c - a*d)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(4*b) - Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4)))/2 + (d*q*r*(-1/9*(p*r)/((b*c
 - a*d)*(a + b*x)^3) + (d*p*r)/(4*(b*c - a*d)^2*(a + b*x)^2) - (d*q*r)/(6*
(b*c - a*d)^2*(a + b*x)^2) - (d^2*p*r)/((b*c - a*d)^3*(a + b*x)) + (5*d^2*
q*r)/(6*(b*c - a*d)^3*(a + b*x)) + (11*d^3*q*r*Log[a + b*x])/(6*(b*c - a*d
)^4) + (d^3*p*r*Log[a + b*x]^2)/(2*(b*c - a*d)^4) - (11*d^3*q*r*Log[c + d*
x])/(6*(b*c - a*d)^4) - (d^3*p*r*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
 d*x])/(b*c - a*d)^4 - (d^3*q*r*Log[c + d*x]^2)/(2*(b*c - a*d)^4) + (d^3*q
*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^4 - Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r]/(3*(b*c - a*d)*(a + b*x)^3) + (d*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])/(2*(b*c - a*d)^2*(a + b*x)^2) - (d^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d)^4 + (d^3*Log[c + d*x]*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*c - a*d)^4 + (d^3*q*r*PolyLog[2, -((d*
(a + b*x))/(b*c - a*d))])/(b*c - a*d)^4 - (d^3*p*r*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)])/(b*c - a*d)^4))/(2*b)
```

3.24.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2994
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

3.24.4 Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2}{(bx+a)^5}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

3.24.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \int \frac{\log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^5}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^5*x^5 + 5*a*b^4*x^4 + 1
0*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

3.24.6 Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p\,(c+dx)^q\right)^r\right)^2}{(a+bx)^5}\,dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**5,x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**5, x)
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1816 vs. 2(836) = 1672.

Time = 0.35 (sec) , antiderivative size = 1816, normalized size of antiderivative = 2.05

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="maxim
a")
```

output
```
-1/24*(12*d^4*f*q*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4) - 12*d^4*f*q*log(d*x + c)/(b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (12*b^3*d^3*f*q*x^3
 - a*b^2*c^2*d*f*(9*p - 4*q) + a^2*b*c*d^2*f*(9*p - 14*q) - a^3*d^3*f*(3*p
 - 22*q) + 3*b^3*c^3*f*p - 6*(b^3*c*d^2*f*q - 7*a*b^2*d^3*f*q)*x^2 + 4*(b^
3*c^2*d*f*q - 5*a*b^2*c*d^2*f*q + 13*a^2*b*d^3*f*q)*x)/(a^4*b^3*c^3 - 3*a^
5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b
^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c
*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d
^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2
 - a^6*b*d^3)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/288*(14
4*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b
*d*x + a*d)/(b*c - a*d)))*d^4*f^2/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 12*(3*p*q - 22*q^2)*d^4*f^2*log(d*x + c)
/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) -
 (9*b^4*c^4*f^2*p^2 - 4*(9*p^2 - 7*p*q)*a*b^3*c^3*d*f^2 + 6*(9*p^2 - 23*p*
q + 4*q^2)*a^2*b^2*c^2*d^2*f^2 - 12*(3*p^2 - 31*p*q + 14*q^2)*a^3*b*c*d^3*
f^2 + (9*p^2 - 262*p*q + 144*q^2)*a^4*d^4*f^2 + 60*((3*p*q - 2*q^2)*b^4*c*
d^3*f^2 - (3*p*q - 2*q^2)*a*b^3*d^4*f^2)*x^3 - 6*((9*p*q - 4*q^2)*b^4*c^2*
d^2*f^2 - 4*(27*p*q - 17*q^2)*a*b^3*c*d^3*f^2 + (99*p*q - 64*q^2)*a^2*b...
```

3.24.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx=\int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)}{(bx+a)^5}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^5, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^5}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5, x)
```

3.25 $\int (g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.25.1 Optimal result

Integrand size = 29, antiderivative size = 334

$$\begin{aligned}
&\int (g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx \\
&= -\frac{(bg-ah)^4 prx}{5b^4} - \frac{(dg-ch)^4 qrx}{5d^4} - \frac{(bg-ah)^3 pr(g+hx)^2}{10b^3h} - \frac{(dg-ch)^3 qr(g+hx)^2}{10d^3h} \\
&\quad - \frac{(bg-ah)^2 pr(g+hx)^3}{15b^2h} - \frac{(dg-ch)^2 qr(g+hx)^3}{15d^2h} - \frac{(bg-ah)pr(g+hx)^4}{20bh} \\
&\quad - \frac{(dg-ch)qr(g+hx)^4}{20dh} - \frac{pr(g+hx)^5}{25h} - \frac{qr(g+hx)^5}{25h} - \frac{(bg-ah)^5 pr \log(a+bx)}{5b^5h} \\
&\quad - \frac{(dg-ch)^5 qr \log(c+dx)}{5d^5h} + \frac{(g+hx)^5 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5h}
\end{aligned}$$

output
```
-1/5*(-a*h+b*g)^4*p*r*x/b^4-1/5*(-c*h+d*g)^4*q*r*x/d^4-1/10*(-a*h+b*g)^3*p
*r*(h*x+g)^2/b^3/h-1/10*(-c*h+d*g)^3*q*r*(h*x+g)^2/d^3/h-1/15*(-a*h+b*g)^2
*p*r*(h*x+g)^3/b^2/h-1/15*(-c*h+d*g)^2*q*r*(h*x+g)^3/d^2/h-1/20*(-a*h+b*g)
*p*r*(h*x+g)^4/b/h-1/20*(-c*h+d*g)*q*r*(h*x+g)^4/d/h-1/25*p*r*(h*x+g)^5/h-
1/25*q*r*(h*x+g)^5/h-1/5*(-a*h+b*g)^5*p*r*ln(b*x+a)/b^5/h-1/5*(-c*h+d*g)^5
*q*r*ln(d*x+c)/d^5/h+1/5*(h*x+g)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

3.25.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.82

$$\int (g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{-\frac{pr\left(60bh(bg-ah)^4x+30b^2(bg-ah)^3(g+hx)^2+20b^3(bg-ah)^2(g+hx)^3+15b^4(bg-ah)(g+hx)^4+12b^5(g+hx)^5+60(bg-ah)^5\log(a+bx)\right)}{60b^5} - \frac{qr(6}{}}{}$$

input
```
Integrate[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
(-1/60*(p*r*(60*b*h*(b*g - a*h)^4*x + 30*b^2*(b*g - a*h)^3*(g + h*x)^2 + 2
0*b^3*(b*g - a*h)^2*(g + h*x)^3 + 15*b^4*(b*g - a*h)*(g + h*x)^4 + 12*b^5*
(g + h*x)^5 + 60*(b*g - a*h)^5*Log[a + b*x]))/b^5 - (q*r*(60*d*h*(d*g - c*
h)^4*x + 30*d^2*(d*g - c*h)^3*(g + h*x)^2 + 20*d^3*(d*g - c*h)^2*(g + h*x)
^3 + 15*d^4*(d*g - c*h)*(g + h*x)^4 + 12*d^5*(g + h*x)^5 + 60*(d*g - c*h)^
5*Log[c + d*x]))/(60*d^5) + (g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r])/(5*h)
```

3.25.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$\Big\downarrow \; 2981$$
$$-\frac{bpr\int \frac{(g+hx)^5}{a+bx}dx}{5h} - \frac{dqr\int \frac{(g+hx)^5}{c+dx}dx}{5h} + \frac{(g+hx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5h}$$
$$\Big\downarrow \; 49$$

$$-\frac{bpr\int\left(\frac{(bg-ah)^5}{b^5(a+bx)}+\frac{h(bg-ah)^4}{b^5}+\frac{h(g+hx)(bg-ah)^3}{b^4}+\frac{h(g+hx)^2(bg-ah)^2}{b^3}+\frac{h(g+hx)^3(bg-ah)}{b^2}+\frac{h(g+hx)^4}{b}\right)dx}{5h}-\frac{dqr\int\left(\frac{(dg-ch)^5}{d^5(c+dx)}+\frac{h(dg-ch)^4}{d^5}+\frac{h(g+hx)(dg-ch)^3}{d^4}+\frac{h(g+hx)^2(dg-ch)^2}{d^3}+\frac{h(g+hx)^3(dg-ch)}{d^2}+\frac{h(g+hx)^4}{d}\right)dx}{5h}+\frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bpr\left(\frac{(bg-ah)^5\log(a+bx)}{b^6}+\frac{hx(bg-ah)^4}{b^5}+\frac{(g+hx)^2(bg-ah)^3}{2b^4}+\frac{(g+hx)^3(bg-ah)^2}{3b^3}+\frac{(g+hx)^4(bg-ah)}{4b^2}+\frac{(g+hx)^5}{5b}\right)}{5h}+\frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h}-\frac{dqr\left(\frac{(dg-ch)^5\log(c+dx)}{d^6}+\frac{hx(dg-ch)^4}{d^5}+\frac{(g+hx)^2(dg-ch)^3}{2d^4}+\frac{(g+hx)^3(dg-ch)^2}{3d^3}+\frac{(g+hx)^4(dg-ch)}{4d^2}+\frac{(g+hx)^5}{5d}\right)}{5h}$$

input
```
Int[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/5*(b*p*r*((h*(b*g - a*h)^4*x)/b^5 + ((b*g - a*h)^3*(g + h*x)^2)/(2*b^4)
 + ((b*g - a*h)^2*(g + h*x)^3)/(3*b^3) + ((b*g - a*h)*(g + h*x)^4)/(4*b^2)
 + (g + h*x)^5/(5*b) + ((b*g - a*h)^5*Log[a + b*x])/b^6))/h - (d*q*r*((h*(
d*g - c*h)^4*x)/d^5 + ((d*g - c*h)^3*(g + h*x)^2)/(2*d^4) + ((d*g - c*h)^2
*(g + h*x)^3)/(3*d^3) + ((d*g - c*h)*(g + h*x)^4)/(4*d^2) + (g + h*x)^5/(5
*d) + ((d*g - c*h)^5*Log[c + d*x])/d^6))/(5*h) + ((g + h*x)^5*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r])/(5*h)
```

3.25.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.25.4 Maple [F]

$$\int (hx+g)^4 \ln\left(e\left(f(bx+a)^p (dx+c)^q\right)^r\right) dx$$

input
```
int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

output
```
int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(308) = 616$.

Time = 0.33 (sec) , antiderivative size = 945, normalized size of antiderivative = 2.83

$$\int (g+hx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx =$$
$$-\frac{12\,(b^5d^5h^4p + b^5d^5h^4q)rx^5 + 15\,((5\,b^5d^5gh^3 - ab^4d^5h^4)p + (5\,b^5d^5gh^3 - b^5cd^4h^4)q)rx^4 + 20\,((10\,b^5d^5g^2h \cdots}{\cdots}$$

input
```
integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas"
)
```

output
```
-1/300*(12*(b^5*d^5*h^4*p + b^5*d^5*h^4*q)*r*x^5 + 15*((5*b^5*d^5*g*h^3 -
a*b^4*d^5*h^4)*p + (5*b^5*d^5*g*h^3 - b^5*c*d^4*h^4)*q)*r*x^4 + 20*((10*b^
5*d^5*g^2*h^2 - 5*a*b^4*d^5*g*h^3 + a^2*b^3*d^5*h^4)*p + (10*b^5*d^5*g^2*h
^2 - 5*b^5*c*d^4*g*h^3 + b^5*c^2*d^3*h^4)*q)*r*x^3 + 30*((10*b^5*d^5*g^3*h
 - 10*a*b^4*d^5*g^2*h^2 + 5*a^2*b^3*d^5*g*h^3 - a^3*b^2*d^5*h^4)*p + (10*b
^5*d^5*g^3*h - 10*b^5*c*d^4*g^2*h^2 + 5*b^5*c^2*d^3*g*h^3 - b^5*c^3*d^2*h^
4)*q)*r*x^2 + 60*((5*b^5*d^5*g^4 - 10*a*b^4*d^5*g^3*h + 10*a^2*b^3*d^5*g^2
*h^2 - 5*a^3*b^2*d^5*g*h^3 + a^4*b*d^5*h^4)*p + (5*b^5*d^5*g^4 - 10*b^5*c*
d^4*g^3*h + 10*b^5*c^2*d^3*g^2*h^2 - 5*b^5*c^3*d^2*g*h^3 + b^5*c^4*d*h^4)*
q)*r*x - 60*(b^5*d^5*h^4*p*r*x^5 + 5*b^5*d^5*g*h^3*p*r*x^4 + 10*b^5*d^5*g^
2*h^2*p*r*x^3 + 10*b^5*d^5*g^3*h*p*r*x^2 + 5*b^5*d^5*g^4*p*r*x + (5*a*b^4*
d^5*g^4 - 10*a^2*b^3*d^5*g^3*h + 10*a^3*b^2*d^5*g^2*h^2 - 5*a^4*b*d^5*g*h^
3 + a^5*d^5*h^4)*p*r)*log(b*x + a) - 60*(b^5*d^5*h^4*q*r*x^5 + 5*b^5*d^5*g
*h^3*q*r*x^4 + 10*b^5*d^5*g^2*h^2*q*r*x^3 + 10*b^5*d^5*g^3*h*q*r*x^2 + 5*b
^5*d^5*g^4*q*r*x + (5*b^5*c*d^4*g^4 - 10*b^5*c^2*d^3*g^3*h + 10*b^5*c^3*d^
2*g^2*h^2 - 5*b^5*c^4*d*g*h^3 + b^5*c^5*h^4)*q*r)*log(d*x + c) - 60*(b^5*d
^5*h^4*x^5 + 5*b^5*d^5*g*h^3*x^4 + 10*b^5*d^5*g^2*h^2*x^3 + 10*b^5*d^5*g^3
*h*x^2 + 5*b^5*d^5*g^4*x)*log(e) - 60*(b^5*d^5*h^4*r*x^5 + 5*b^5*d^5*g*h^3
*r*x^4 + 10*b^5*d^5*g^2*h^2*r*x^3 + 10*b^5*d^5*g^3*h*r*x^2 + 5*b^5*d^5*g^4
*r*x)*log(f))/(b^5*d^5)
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \text{Timed out}$$

input
```
integrate((h*x+g)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Timed out
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(308) = 616$.

Time = 0.20 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.87

$$\int (g+hx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{1}{5}\left(h^4x^5 + 5\,gh^3x^4 + 10\,g^2h^2x^3 + 10\,g^3hx^2 + 5\,g^4x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)$$
$$+ \frac{r\left(\frac{60\left(5\,ab^4fg^4p - 10\,a^2b^3fg^3hp + 10\,a^3b^2fg^2h^2p - 5\,a^4bfgh^3p + a^5fh^4p\right)\log(bx+a)}{b^5} + \frac{60\left(5\,cd^4fg^4q - 10\,c^2d^3fg^3hq + 10\,c^3d^2fg^2h^2q - 5\,c^4df\right.}{d^5}\right.}{}$$

input
```
integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima"
)
```

output
```
1/5*(h^4*x^5 + 5*g*h^3*x^4 + 10*g^2*h^2*x^3 + 10*g^3*h*x^2 + 5*g^4*x)*log(
((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*r*(60*(5*a*b^4*f*g^4*p - 10*a^2*b
^3*f*g^3*h*p + 10*a^3*b^2*f*g^2*h^2*p - 5*a^4*b*f*g*h^3*p + a^5*f*h^4*p)*l
og(b*x + a)/b^5 + 60*(5*c*d^4*f*g^4*q - 10*c^2*d^3*f*g^3*h*q + 10*c^3*d^2*
f*g^2*h^2*q - 5*c^4*d*f*g*h^3*q + c^5*f*h^4*q)*log(d*x + c)/d^5 - (12*b^4*
d^4*f*h^4*(p + q)*x^5 - 15*(a*b^3*d^4*f*h^4*p - (5*d^4*f*g*h^3*(p + q) - c
*d^3*f*h^4*q)*b^4)*x^4 - 20*(5*a*b^3*d^4*f*g*h^3*p - a^2*b^2*d^4*f*h^4*p -
 (10*d^4*f*g^2*h^2*(p + q) - 5*c*d^3*f*g*h^3*q + c^2*d^2*f*h^4*q)*b^4)*x^3
 - 30*(10*a*b^3*d^4*f*g^2*h^2*p - 5*a^2*b^2*d^4*f*g*h^3*p + a^3*b*d^4*f*h^
4*p - (10*d^4*f*g^3*h*(p + q) - 10*c*d^3*f*g^2*h^2*q + 5*c^2*d^2*f*g*h^3*q
 - c^3*d*f*h^4*q)*b^4)*x^2 - 60*(10*a*b^3*d^4*f*g^3*h*p - 10*a^2*b^2*d^4*f
*g^2*h^2*p + 5*a^3*b*d^4*f*g*h^3*p - a^4*d^4*f*h^4*p - (5*d^4*f*g^4*(p + q
) - 10*c*d^3*f*g^3*h*q + 10*c^2*d^2*f*g^2*h^2*q - 5*c^3*d*f*g*h^3*q + c^4*
f*h^4*q)*b^4)*x)/(b^4*d^4))/f
```

3.25.8 Giac [F]

$$\int (g+hx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (hx+g)^4 \log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)\,dx$$

input
```
integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
sage0*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.38

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \text{Too large to display}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^4,x)
```

output
```
log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^4*x + (h^4*x^5)/5 + 2*g^3*h*x^2 +
g*h^3*x^4 + 2*g^2*h^2*x^3) - x^2*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3
*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(
5*a*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h
*q + 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(10*b*d) - (a*c*((h
^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)
*(5*a*d + 5*b*c))/(25*b*d)))/(2*b*d) + (g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h
*q + b*d*g*q))/(b*d)) - x^4*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d
*g*q))/(20*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(100*b*d)) - x*((a*c*(((
5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d
) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p
 + 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))
/(b*d) - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r*(b*c
*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d +
 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q + 2*
b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(5*b*d) - (a*c*((h^3*r*(b*
c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d
+ 5*b*c))/(25*b*d)))/(b*d) + (2*g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*d
*g*q))/(b*d)))/(5*b*d) + (g^3*r*(2*b*c*h*p + b*d*g*p + 2*a*d*h*q + b*d*g*q
))/(b*d)) + x^3*(((5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*...
```

3.26 $\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

3.26.1 Optimal result

Integrand size = 29, antiderivative size = 276

$$\begin{aligned}\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx &= -\frac{(bg-ah)^3prx}{4b^3}-\frac{(dg-ch)^3qrx}{4d^3}-\frac{(bg-ah)^2pr(g+hx)^2}{8b^2h} \\ &\quad -\frac{(dg-ch)^2qr(g+hx)^2}{8d^2h}-\frac{(bg-ah)pr(g+hx)^3}{12bh}-\frac{(dg-ch)qr(g+hx)^3}{12dh} \\ &\quad -\frac{pr(g+hx)^4}{16h}-\frac{qr(g+hx)^4}{16h}-\frac{(bg-ah)^4pr\log(a+bx)}{4b^4h} \\ &\quad -\frac{(dg-ch)^4qr\log(c+dx)}{4d^4h}+\frac{(g+hx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}\end{aligned}$$

output
```
-1/4*(-a*h+b*g)^3*p*r*x/b^3-1/4*(-c*h+d*g)^3*q*r*x/d^3-1/8*(-a*h+b*g)^2*p*
r*(h*x+g)^2/b^2/h-1/8*(-c*h+d*g)^2*q*r*(h*x+g)^2/d^2/h-1/12*(-a*h+b*g)*p*r
*(h*x+g)^3/b/h-1/12*(-c*h+d*g)*q*r*(h*x+g)^3/d/h-1/16*p*r*(h*x+g)^4/h-1/16
*q*r*(h*x+g)^4/h-1/4*(-a*h+b*g)^4*p*r*ln(b*x+a)/b^4/h-1/4*(-c*h+d*g)^4*q*r
*ln(d*x+c)/d^4/h+1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

3.26.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84

$$\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{\frac{1}{12} r \left(-\frac{p\left(12bh(bg-ah)^3x+6b^2(bg-ah)^2(g+hx)^2+4b^3(bg-ah)(g+hx)^3+3b^4(g+hx)^4+12(bg-ah)^4\log(a+bx)\right)}{b^4} - \frac{q(12dh(dg-ch)^3x+6d^2(dg-}{}\right.}{4h}$$

input
```
Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
((r*(-((p*(12*b*h*(b*g - a*h)^3*x + 6*b^2*(b*g - a*h)^2*(g + h*x)^2 + 4*b^
3*(b*g - a*h)*(g + h*x)^3 + 3*b^4*(g + h*x)^4 + 12*(b*g - a*h)^4*Log[a + b
*x]))/b^4) - (q*(12*d*h*(d*g - c*h)^3*x + 6*d^2*(d*g - c*h)^2*(g + h*x)^2
+ 4*d^3*(d*g - c*h)*(g + h*x)^3 + 3*d^4*(g + h*x)^4 + 12*(d*g - c*h)^4*Log
[c + d*x]))/d^4))/12 + (g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(
4*h)
```

3.26.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$\Big\downarrow \ 2981$$
$$-\frac{bpr\int \frac{(g+hx)^4}{a+bx}dx}{4h} - \frac{dqr\int \frac{(g+hx)^4}{c+dx}dx}{4h} + \frac{(g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}$$
$$\Big\downarrow \ 49$$
$$-\frac{bpr\int \left(\frac{(bg-ah)^4}{b^4(a+bx)}+\frac{h(bg-ah)^3}{b^4}+\frac{h(g+hx)(bg-ah)^2}{b^3}+\frac{h(g+hx)^2(bg-ah)}{b^2}+\frac{h(g+hx)^3}{b}\right)dx}{4h} - \frac{dqr\int \left(\frac{(dg-ch)^4}{d^4(c+dx)}+\frac{h(dg-ch)^3}{d^4}+\frac{h(g+hx)(dg-ch)^2}{d^3}+\frac{h(g+hx)^2(dg-ch)}{d^2}+\frac{h(g+hx)^3}{d}\right)dx}{4h} + \frac{(g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}$$

$$\downarrow 2009$$

$$-\frac{bpr\left(\frac{(bg-ah)^4\log(a+bx)}{b^5}+\frac{hx(bg-ah)^3}{b^4}+\frac{(g+hx)^2(bg-ah)^2}{2b^3}+\frac{(g+hx)^3(bg-ah)}{3b^2}+\frac{(g+hx)^4}{4b}\right)}{4h}+\frac{(g+hx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}-\frac{dqr\left(\frac{(dg-ch)^4\log(c+dx)}{d^5}+\frac{hx(dg-ch)^3}{d^4}+\frac{(g+hx)^2(dg-ch)^2}{2d^3}+\frac{(g+hx)^3(dg-ch)}{3d^2}+\frac{(g+hx)^4}{4d}\right)}{4h}$$

input
```
Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/4*(b*p*r*((h*(b*g - a*h)^3*x)/b^4 + ((b*g - a*h)^2*(g + h*x)^2)/(2*b^3)
 + ((b*g - a*h)*(g + h*x)^3)/(3*b^2) + (g + h*x)^4/(4*b) + ((b*g - a*h)^4*
Log[a + b*x])/b^5))/h - (d*q*r*((h*(d*g - c*h)^3*x)/d^4 + ((d*g - c*h)^2*(
g + h*x)^2)/(2*d^3) + ((d*g - c*h)*(g + h*x)^3)/(3*d^2) + (g + h*x)^4/(4*d
) + ((d*g - c*h)^4*Log[c + d*x])/d^5))/(4*h) + ((g + h*x)^4*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])/(4*h)
```

3.26.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(254) = 508$.

Time = 291.42 (sec) , antiderivative size = 1079, normalized size of antiderivative = 3.91

method	result	size
parallelrisch	Expression too large to display	1079

input
```
int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-24*x^2*b^4*c*d^3*g*h^2*q*r-48*a^3*g*h^2*p*r*d^4*b-48*b^4*c^3*d*g*h
^2*q*r+6*a^3*b*c*d^3*h^3*p*r+6*a*b^3*c^3*d*h^3*q*r+6*x^2*a^2*b^2*d^4*h^3*p
*r+6*x^2*b^4*c^2*d^2*h^3*q*r+36*x^2*b^4*d^4*g^2*h*p*r+36*x^2*b^4*d^4*g^2*h
*q*r-96*ln(b*x+a)*a*b^3*d^4*g^3*p*r-12*x*a^3*b*d^4*h^3*p*r-12*x*b^4*c^3*d*
h^3*q*r+72*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^3*c*d^3*g^2*h-48*ln(b*x+a)*
b^4*c*d^3*g^3*p*r-48*ln(d*x+c)*a*b^3*d^4*g^3*q*r-96*ln(d*x+c)*b^4*c*d^3*g^
3*q*r-72*ln(b*x+a)*a*b^3*c*d^3*g^2*h*p*r-72*ln(d*x+c)*a*b^3*c*d^3*g^2*h*q*
r-4*x^3*a*b^3*d^4*h^3*p*r-4*x^3*b^4*c*d^3*h^3*q*r+16*x^3*b^4*d^4*g*h^2*p*r
+16*x^3*b^4*d^4*g*h^2*q*r+72*a^2*b^2*d^4*p*r*g^2*h+72*b^4*c^2*d^2*q*r*g^2*
h-24*a^2*b^2*c*d^3*g*h^2*p*r-24*a*b^3*c^2*d^2*g*h^2*q*r+36*a*b^3*c*d^3*g^2
*h*p*r+36*a*b^3*c*d^3*g^2*h*q*r+12*a^4*h^3*p*r*d^4+12*b^4*c^4*h^3*q*r-12*x
^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^4*d^4*h^3-48*x*ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r)*b^4*d^4*g^3+48*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^3*d^4*g^3+48*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^4*c*d^3*g^3-48*a*b^3*d^4*p*r*g^3-48*a*b^
3*d^4*q*r*g^3-48*b^4*c*d^3*p*r*g^3-48*b^4*c*d^3*q*r*g^3-72*x^2*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)*b^4*d^4*g^2*h+48*x*b^4*d^4*g^3*p*r+48*x*b^4*d^4*g^3*q
*r+12*ln(b*x+a)*a^4*d^4*h^3*p*r+12*ln(d*x+c)*b^4*c^4*h^3*q*r+3*x^4*b^4*d^4
*h^3*p*r+3*x^4*b^4*d^4*h^3*q*r-48*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^4*
d^4*g*h^2+48*x*a^2*b^2*d^4*g*h^2*p*r-72*x*a*b^3*d^4*g^2*h*p*r+48*x*b^4*c^2
*d^2*g*h^2*q*r-72*x*b^4*c*d^3*g^2*h*q*r-48*ln(b*x+a)*a^3*b*d^4*g*h^2*p*...
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(254) = 508$.

Time = 0.30 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.46

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx =$$

$$-\frac{3\,(b^4d^4h^3p+b^4d^4h^3q)rx^4+4\,((4\,b^4d^4gh^2-ab^3d^4h^3)p+(4\,b^4d^4gh^2-b^4cd^3h^3)q)rx^3+6\,((6\,b^4d^4g^2h-4}{}$$

input
```
integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas"
)
```

output
```
-1/48*(3*(b^4*d^4*h^3*p + b^4*d^4*h^3*q)*r*x^4 + 4*((4*b^4*d^4*g*h^2 - a*b
^3*d^4*h^3)*p + (4*b^4*d^4*g*h^2 - b^4*c*d^3*h^3)*q)*r*x^3 + 6*((6*b^4*d^4
*g^2*h - 4*a*b^3*d^4*g*h^2 + a^2*b^2*d^4*h^3)*p + (6*b^4*d^4*g^2*h - 4*b^4
*c*d^3*g*h^2 + b^4*c^2*d^2*h^3)*q)*r*x^2 + 12*((4*b^4*d^4*g^3 - 6*a*b^3*d^
4*g^2*h + 4*a^2*b^2*d^4*g*h^2 - a^3*b*d^4*h^3)*p + (4*b^4*d^4*g^3 - 6*b^4*
c*d^3*g^2*h + 4*b^4*c^2*d^2*g*h^2 - b^4*c^3*d*h^3)*q)*r*x - 12*(b^4*d^4*h^
3*p*r*x^4 + 4*b^4*d^4*g*h^2*p*r*x^3 + 6*b^4*d^4*g^2*h*p*r*x^2 + 4*b^4*d^4*
g^3*p*r*x + (4*a*b^3*d^4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a
^4*d^4*h^3)*p*r)*log(b*x + a) - 12*(b^4*d^4*h^3*q*r*x^4 + 4*b^4*d^4*g*h^2*
q*r*x^3 + 6*b^4*d^4*g^2*h*q*r*x^2 + 4*b^4*d^4*g^3*q*r*x + (4*b^4*c*d^3*g^3
 - 6*b^4*c^2*d^2*g^2*h + 4*b^4*c^3*d*g*h^2 - b^4*c^4*h^3)*q*r)*log(d*x + c
) - 12*(b^4*d^4*h^3*x^4 + 4*b^4*d^4*g*h^2*x^3 + 6*b^4*d^4*g^2*h*x^2 + 4*b^
4*d^4*g^3*x)*log(e) - 12*(b^4*d^4*h^3*r*x^4 + 4*b^4*d^4*g*h^2*r*x^3 + 6*b^
4*d^4*g^2*h*r*x^2 + 4*b^4*d^4*g^3*r*x)*log(f))/(b^4*d^4)
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Timed out
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.56

$$\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$= \frac{1}{4}\left(h^3x^4+4\,gh^2x^3+6\,g^2hx^2+4\,g^3x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)$$
$$+ \frac{r\left(\frac{12\,\left(4\,ab^3fg^3p-6\,a^2b^2fg^2hp+4\,a^3bfgh^2p-a^4fh^3p\right)\log(bx+a)}{b^4}+\frac{12\,\left(4\,cd^3fg^3q-6\,c^2d^2fg^2hq+4\,c^3dfgh^2q-c^4fh^3q\right)\log(dx+c)}{d^4}-\frac{3\,b^3d\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima"
)
```

output
```
1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x
+ c)^q*f)^r*e) + 1/48*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3
*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2
*d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^
3*d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) -
c*d^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (
6*d^3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(
6*a*b^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f
*g^3*(p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x
)/(b^3*d^3))/f
```

3.26.8 Giac [F(-1)]

Timed out.

$$\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = \text{Timed out}$$

input
```
integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
Timed out
```

3.26.9 Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.32

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$= \ln \left(e \left(f \left(a+b\,x\right)^p \left(c+d\,x\right)^q\right)^r\right) \left(g^3\,x+\frac{3\,g^2\,h\,x^2}{2}+g\,h^2\,x^3+\frac{h^3\,x^4}{4}\right)$$

$$-x \left(\frac{\left(4\,a\,d+4\,b\,c\right) \left(\frac{\left(4\,a\,d+4\,b\,c\right)\left(\frac{h^2\,r\,\left(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q\right)}{4\,b\,d}-\frac{h^3\,r\,\left(p+q\right)\,\left(4\,a\,d+4\,b\,c\right)}{16\,b\,d}\right)}{4\,b\,d}-\frac{g\,h\,r\,\left(2\,b\,c\,h\,p+3\,b\,d\,g\,p+2\,a\,d\,h\,q+3\,b}{2\,b\,d}\right.}{4\,b\,d}\right.$$

$$\left. +\frac{g^2\,r\,\left(3\,b\,c\,h\,p+2\,b\,d\,g\,p+3\,a\,d\,h\,q+2\,b\,d\,g\,q\right)}{2\,b\,d} -\frac{a\,c\,\left(\frac{h^2\,r\,\left(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q\right)}{4\,b\,d}-\frac{h^3\,r\,\left(p+q\right)\,\left(4\,a\,d+4\,b\,c\right)}{16\,b\,d}\right)}{b\,d}\right)$$

$$-x^3\,\left(\frac{h^2\,r\,\left(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q\right)}{12\,b\,d}-\frac{h^3\,r\,\left(p+q\right)\,\left(4\,a\,d+4\,b\,c\right)}{48\,b\,d}\right)$$

$$+x^2\,\left(\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{h^2\,r\,\left(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q\right)}{4\,b\,d}-\frac{h^3\,r\,\left(p+q\right)\,\left(4\,a\,d+4\,b\,c\right)}{16\,b\,d}\right)}{8\,b\,d}\right.$$

$$\left. -\frac{g\,h\,r\,\left(2\,b\,c\,h\,p+3\,b\,d\,g\,p+2\,a\,d\,h\,q+3\,b\,d\,g\,q\right)}{4\,b\,d}+\frac{a\,c\,h^3\,r\,\left(p+q\right)}{8\,b\,d}\right)$$

$$-\frac{\ln\left(a+b\,x\right)\,\left(p\,r\,a^4\,h^3-4\,p\,r\,a^3\,b\,g\,h^2+6\,p\,r\,a^2\,b^2\,g^2\,h-4\,p\,r\,a\,b^3\,g^3\right)}{4\,b^4}$$

$$-\frac{\ln\left(c+d\,x\right)\,\left(q\,r\,c^4\,h^3-4\,q\,r\,c^3\,d\,g\,h^2+6\,q\,r\,c^2\,d^2\,g^2\,h-4\,q\,r\,c\,d^3\,g^3\right)}{4\,d^4}-\frac{h^3\,r\,x^4\,\left(p+q\right)}{16}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^3,x)
```

output
```
log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^3*x + (h^3*x^4)/4 + (3*g^2*h*x^2)/
2 + g*h^2*x^3) - x*(((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((h^2*r*(b*c*h*p +
4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c)
)/(16*b*d)))/(4*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*h*q + 3*b*d*g
*q))/(2*b*d) + (a*c*h^3*r*(p + q))/(4*b*d)))/(4*b*d) + (g^2*r*(3*b*c*h*p +
 2*b*d*g*p + 3*a*d*h*q + 2*b*d*g*q))/(2*b*d) - (a*c*((h^2*r*(b*c*h*p + 4*b
*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(
16*b*d)))/(b*d)) - x^3*((h^2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q)
)/(12*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(48*b*d)) + x^2*(((4*a*d + 4*
b*c)*((h^2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r
*(p + q)*(4*a*d + 4*b*c))/(16*b*d)))/(8*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g
*p + 2*a*d*h*q + 3*b*d*g*q))/(4*b*d) + (a*c*h^3*r*(p + q))/(8*b*d)) - (log
(a + b*x)*(a^4*h^3*p*r - 4*a*b^3*g^3*p*r - 4*a^3*b*g*h^2*p*r + 6*a^2*b^2*g
^2*h*p*r))/(4*b^4) - (log(c + d*x)*(c^4*h^3*q*r - 4*c*d^3*g^3*q*r - 4*c^3*
d*g*h^2*q*r + 6*c^2*d^2*g^2*h*q*r))/(4*d^4) - (h^3*r*x^4*(p + q))/16
```

3.26. $\int (g + hx)^3 \log \left(e(f(a + bx)^p (c + dx)^q)^r\right) \, dx$

3.27 $\int (g+hx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.27.1 Optimal result

Integrand size = 29, antiderivative size = 218

$$\int (g+hx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = -\frac{(bg-ah)^2prx}{3b^2} - \frac{(dg-ch)^2qrx}{3d^2} - \frac{(bg-ah)pr(g+hx)^2}{6bh} - \frac{(dg-ch)qr(g+hx)^2}{6dh} - \frac{pr(g+hx)^3}{9h} - \frac{qr(g+hx)^3}{9h} - \frac{(bg-ah)^3pr\log(a+bx)}{3b^3h} - \frac{(dg-ch)^3qr\log(c+dx)}{3d^3h} + \frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

output
```
-1/3*(-a*h+b*g)^2*p*r*x/b^2-1/3*(-c*h+d*g)^2*q*r*x/d^2-1/6*(-a*h+b*g)*p*r*
(h*x+g)^2/b/h-1/6*(-c*h+d*g)*q*r*(h*x+g)^2/d/h-1/9*p*r*(h*x+g)^3/h-1/9*q*r
*(h*x+g)^3/h-1/3*(-a*h+b*g)^3*p*r*ln(b*x+a)/b^3/h-1/3*(-c*h+d*g)^3*q*r*ln(
d*x+c)/d^3/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

3.27.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96

$$\int (g+hx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$
$$= \frac{-\frac{r\left(6d^3(bg-ah)^3p\log(a+bx)+b\left(6a^2d^3h^3px-3abd^3hp(g^2+6ghx+h^2x^2)+b^2d\left(6c^2h^3qx-3cdhq(g^2+6ghx+h^2x^2)+d^2(p+q)\left(5g^3+18g^2hx+9gh^2\right.\right.\right.\right.}{6b^3d^3}}{3h}$$

input
```
Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
(-1/6*(r*(6*d^3*(b*g - a*h)^3*p*Log[a + b*x] + b*(6*a^2*d^3*h^3*p*x - 3*a*
b*d^3*h*p*(g^2 + 6*g*h*x + h^2*x^2) + b^2*d*(6*c^2*h^3*q*x - 3*c*d*h*q*(g^
2 + 6*g*h*x + h^2*x^2) + d^2*(p + q)*(5*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2
*h^3*x^3)) + 6*b^2*(d*g - c*h)^3*q*Log[c + d*x])))/(b^3*d^3) + (g + h*x)^3
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

3.27.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$$

$$\Big\downarrow \; 2981$$

$$-\frac{bpr\int \frac{(g+hx)^3}{a+bx}dx}{3h}-\frac{dqr\int \frac{(g+hx)^3}{c+dx}dx}{3h}+\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

$$\Big\downarrow \; 49$$

$$-\frac{bpr\int \left(\frac{(bg-ah)^3}{b^3(a+bx)}+\frac{h(bg-ah)^2}{b^3}+\frac{h(g+hx)(bg-ah)}{b^2}+\frac{h(g+hx)^2}{b}\right)dx}{3h}-\frac{dqr\int \left(\frac{(dg-ch)^3}{d^3(c+dx)}+\frac{h(dg-ch)^2}{d^3}+\frac{h(g+hx)(dg-ch)}{d^2}+\frac{h(g+hx)^2}{d}\right)dx}{3h}+\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bpr\left(\frac{(bg-ah)^3\log(a+bx)}{b^4}+\frac{hx(bg-ah)^2}{b^3}+\frac{(g+hx)^2(bg-ah)}{2b^2}+\frac{(g+hx)^3}{3b}\right)}{3h}+\frac{(g+hx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}-\frac{dqr\left(\frac{(dg-ch)^3\log(c+dx)}{d^4}+\frac{hx(dg-ch)^2}{d^3}+\frac{(g+hx)^2(dg-ch)}{2d^2}+\frac{(g+hx)^3}{3d}\right)}{3h}$$

input
```
Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/3*(b*p*r*((h*(b*g - a*h)^2*x)/b^3 + ((b*g - a*h)*(g + h*x)^2)/(2*b^2) +
 (g + h*x)^3/(3*b) + ((b*g - a*h)^3*Log[a + b*x])/b^4))/h - (d*q*r*((h*(d*
g - c*h)^2*x)/d^3 + ((d*g - c*h)*(g + h*x)^2)/(2*d^2) + (g + h*x)^3/(3*d)
+ ((d*g - c*h)^3*Log[c + d*x])/d^4))/(3*h) + ((g + h*x)^3*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(3*h)
```

3.27.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeBQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(200) = 400$.

Time = 68.29 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.46

method	result
parallelrisch	$\frac{-18\ln(bx+a)a^2b\,d^3ghpr-18\ln(dx+c)b^3c^2dghqr+18xa\,b^2d^3ghpr+18x\,b^3c\,d^2ghqr-9ca\,b^2d^2ghpr-9ca\,b^2d^2ghqr+18\ln(bx+a)}{}$

input
```
int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-18*ln(b*x+a)*a^2*b*d^3*g*h*p*r-18*ln(d*x+c)*b^3*c^2*d*g*h*q*r+18*x*
a*b^2*d^3*g*h*p*r+18*x*b^3*c*d^2*g*h*q*r-9*c*a*b^2*d^2*g*h*p*r-9*c*a*b^2*d
^2*g*h*q*r+18*ln(b*x+a)*a*b^2*c*d^2*g*h*p*r+18*ln(d*x+c)*a*b^2*c*d^2*g*h*q
*r+6*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*d^3*h^2+18*x*ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)*b^3*d^3*g^2-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^2*d^3*g
^2-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*c*d^2*g^2-2*x^3*b^3*d^3*h^2*q*r+
18*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*d^3*g*h-18*x*b^3*d^3*g^2*p*r-18
*x*b^3*d^3*g^2*q*r+6*ln(b*x+a)*a^3*d^3*h^2*p*r+6*ln(d*x+c)*b^3*c^3*h^2*q*r
-2*x^3*b^3*d^3*h^2*p*r+6*a^3*h^2*p*r*d^3+6*b^3*c^3*h^2*q*r+18*a*b^2*d^3*g^
2*p*r+18*b^3*c*d^2*g^2*q*r+3*x^2*a*b^2*d^3*h^2*p*r+3*x^2*b^3*c*d^2*h^2*q*r
-9*x^2*b^3*d^3*g*h*p*r-9*x^2*b^3*d^3*g*h*q*r-6*x*a^2*b*d^3*h^2*p*r-6*x*b^3
*c^2*d*h^2*q*r-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^2*c*d^2*g*h+36*ln(b*
x+a)*a*b^2*d^3*g^2*p*r+18*ln(b*x+a)*b^3*c*d^2*g^2*p*r+18*ln(d*x+c)*a*b^2*d
^3*g^2*q*r+36*ln(d*x+c)*b^3*c*d^2*g^2*q*r+18*a*b^2*d^3*g^2*q*r+18*b^3*c*d^
2*g^2*p*r+3*c*a^2*b*d^2*h^2*p*r+3*c^2*a*b^2*d*h^2*q*r-18*a^2*g*h*p*r*d^3*b
-18*b^3*c^2*d*g*h*q*r)/b^3/d^3
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(200) = 400$.

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.02

$$\int (g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx =$$
$$-\frac{2\,(b^3d^3h^2p+b^3d^3h^2q)rx^3+3\,((3\,b^3d^3gh-ab^2d^3h^2)p+(3\,b^3d^3gh-b^3cd^2h^2)q)rx^2+6\,((3\,b^3d^3g^2-3\,ab^{}}{}$$

input
```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas"
)
```

output
```
-1/18*(2*(b^3*d^3*h^2*p + b^3*d^3*h^2*q)*r*x^3 + 3*((3*b^3*d^3*g*h - a*b^2
*d^3*h^2)*p + (3*b^3*d^3*g*h - b^3*c*d^2*h^2)*q)*r*x^2 + 6*((3*b^3*d^3*g^2
 - 3*a*b^2*d^3*g*h + a^2*b*d^3*h^2)*p + (3*b^3*d^3*g^2 - 3*b^3*c*d^2*g*h +
 b^3*c^2*d*h^2)*q)*r*x - 6*(b^3*d^3*h^2*p*r*x^3 + 3*b^3*d^3*g*h*p*r*x^2 +
3*b^3*d^3*g^2*p*r*x + (3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2)*p*
r)*log(b*x + a) - 6*(b^3*d^3*h^2*q*r*x^3 + 3*b^3*d^3*g*h*q*r*x^2 + 3*b^3*d
^3*g^2*q*r*x + (3*b^3*c*d^2*g^2 - 3*b^3*c^2*d*g*h + b^3*c^3*h^2)*q*r)*log(
d*x + c) - 6*(b^3*d^3*h^2*x^3 + 3*b^3*d^3*g*h*x^2 + 3*b^3*d^3*g^2*x)*log(e
) - 6*(b^3*d^3*h^2*r*x^3 + 3*b^3*d^3*g*h*r*x^2 + 3*b^3*d^3*g^2*r*x)*log(f)
)/(b^3*d^3)
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(192) = 384$.

Time = 166.57 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.27

$$\int (g + hx)^2 \log \left(e(f(a + bx)^p (c + dx)^q)^r\right) dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Piecewise(((g**2*x + g*h*x**2 + h**2*x**3/3)*log(e*(a**p*c**q*f)**r), Eq(b
, 0) & Eq(d, 0)), (c**3*h**2*log(e*(a**p*f*(c + d*x)**q)**r)/(3*d**3) - c*
*2*g*h*log(e*(a**p*f*(c + d*x)**q)**r)/d**2 - c**2*h**2*q*r*x/(3*d**2) + c
*g**2*log(e*(a**p*f*(c + d*x)**q)**r)/d + c*g*h*q*r*x/d + c*h**2*q*r*x**2/
(6*d) - g**2*q*r*x + g**2*x*log(e*(a**p*f*(c + d*x)**q)**r) - g*h*q*r*x**2
/2 + g*h*x**2*log(e*(a**p*f*(c + d*x)**q)**r) - h**2*q*r*x**3/9 + h**2*x**
3*log(e*(a**p*f*(c + d*x)**q)**r)/3, Eq(b, 0)), (a**3*h**2*log(e*(c**q*f*(
a + b*x)**p)**r)/(3*b**3) - a**2*g*h*log(e*(c**q*f*(a + b*x)**p)**r)/b**2
- a**2*h**2*p*r*x/(3*b**2) + a*g**2*log(e*(c**q*f*(a + b*x)**p)**r)/b + a*
g*h*p*r*x/b + a*h**2*p*r*x**2/(6*b) - g**2*p*r*x + g**2*x*log(e*(c**q*f*(a
 + b*x)**p)**r) - g*h*p*r*x**2/2 + g*h*x**2*log(e*(c**q*f*(a + b*x)**p)**r
) - h**2*p*r*x**3/9 + h**2*x**3*log(e*(c**q*f*(a + b*x)**p)**r)/3, Eq(d, 0
)), (-a**3*h**2*q*r*log(c/d + x)/(3*b**3) + a**3*h**2*log(e*(f*(a + b*x)**
p*(c + d*x)**q)**r)/(3*b**3) + a**2*g*h*q*r*log(c/d + x)/b**2 - a**2*g*h*l
og(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/b**2 - a**2*h**2*p*r*x/(3*b**2) - a
*g**2*q*r*log(c/d + x)/b + a*g**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/
b + a*g*h*p*r*x/b + a*h**2*p*r*x**2/(6*b) + c**3*h**2*q*r*log(c/d + x)/(3*
d**3) - c**2*g*h*q*r*log(c/d + x)/d**2 - c**2*h**2*q*r*x/(3*d**2) + c*g**2
*q*r*log(c/d + x)/d + c*g*h*q*r*x/d + c*h**2*q*r*x**2/(6*d) - g**2*p*r*x -
 g**2*q*r*x + g**2*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - g*h*p*r*...
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.23

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= \frac{1}{3} \left(h^2x^3 + 3\,ghx^2 + 3\,g^2x\right) \log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)$$
$$+ \frac{r\left(\frac{6\,(3\,ab^2fg^2p - 3\,a^2bfghp + a^3fh^2p)\log(bx+a)}{b^3} + \frac{6\,(3\,cd^2fg^2q - 3\,c^2dfghq + c^3fh^2q)\log(dx+c)}{d^3} - \frac{2\,b^2d^2fh^2(p+q)x^3 - 3\,(abd^2fh^2p - (3\,d\right.}{18\,f}$$

input
```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima"
)
```

output
```
1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
 1/18*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/
b^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3
 - (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q
) - c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^
2*f*g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))/f
```

3.27.8 Giac [A] (verification not implemented)

Time = 58.02 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.67

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= -\frac{1}{9}\left(h^2pr + h^2qr - 3h^2r\log(f) - 3h^2\log(e)\right)x^3 + \frac{1}{3}\left(h^2prx^3 + 3ghprx^2 + 3g^2prx\right)\log(bx+a) + \frac{1}{3}\left(h^2qrx^3 + 3ghqrx^2 + 3g^2qrx\right)\log(dx+c) - \frac{(3bdghpr - adh^2pr + 3bdghqr - bch^2qr - 6bdghr\log(f) - 6bdgh\log(e))x^2}{6bd} + \frac{(3ab^2g^2pr - 3a^2bghpr + a^3h^2pr)\log(bx+a)}{3b^3} + \frac{(3cd^2g^2qr - 3c^2dghqr + c^3h^2qr)\log(-dx-c)}{3d^3} - \frac{(3b^2d^2g^2pr - 3abd^2ghpr + a^2d^2h^2pr + 3b^2d^2g^2qr - 3b^2cdghqr + b^2c^2h^2qr - 3b^2d^2g^2r\log(f) - 3b^2d^2}{3b^2d^2}$$

input
```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
-1/9*(h^2*p*r + h^2*q*r - 3*h^2*r*log(f) - 3*h^2*log(e))*x^3 + 1/3*(h^2*p*
r*x^3 + 3*g*h*p*r*x^2 + 3*g^2*p*r*x)*log(b*x + a) + 1/3*(h^2*q*r*x^3 + 3*g
*h*q*r*x^2 + 3*g^2*q*r*x)*log(d*x + c) - 1/6*(3*b*d*g*h*p*r - a*d*h^2*p*r
+ 3*b*d*g*h*q*r - b*c*h^2*q*r - 6*b*d*g*h*r*log(f) - 6*b*d*g*h*log(e))*x^2
/(b*d) + 1/3*(3*a*b^2*g^2*p*r - 3*a^2*b*g*h*p*r + a^3*h^2*p*r)*log(b*x + a
)/b^3 + 1/3*(3*c*d^2*g^2*q*r - 3*c^2*d*g*h*q*r + c^3*h^2*q*r)*log(-d*x - c
)/d^3 - 1/3*(3*b^2*d^2*g^2*p*r - 3*a*b*d^2*g*h*p*r + a^2*d^2*h^2*p*r + 3*b
^2*d^2*g^2*q*r - 3*b^2*c*d*g*h*q*r + b^2*c^2*h^2*q*r - 3*b^2*d^2*g^2*r*log
(f) - 3*b^2*d^2*g^2*log(e))*x/(b^2*d^2)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= x\left(\frac{\left(\frac{h\,r\,(b\,c\,h\,p+3\,b\,d\,g\,p+a\,d\,h\,q+3\,b\,d\,g\,q)}{3\,b\,d}-\frac{h^2\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{9\,b\,d}\right)(3\,a\,d+3\,b\,c)}{3\,b\,d} - \frac{g\,r\,(b\,c\,h\,p+b\,d\,g\,p+a\,d\,h\,q+b\,d\,g\,q)}{b\,d}+\frac{a\,c\,h^2\,r\,(p+q)}{3\,b\,d}\right)$$
$$- x^2\left(\frac{h\,r\,(b\,c\,h\,p+3\,b\,d\,g\,p+a\,d\,h\,q+3\,b\,d\,g\,q)}{6\,b\,d}-\frac{h^2\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{18\,b\,d}\right)$$
$$+ \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(g^2\,x+g\,h\,x^2+\frac{h^2\,x^3}{3}\right)$$
$$+ \frac{\ln(a+b\,x)\,\left(p\,r\,a^3\,h^2-3\,p\,r\,a^2\,b\,g\,h+3\,p\,r\,a\,b^2\,g^2\right)}{3\,b^3}$$
$$+ \frac{\ln(c+d\,x)\,\left(q\,r\,c^3\,h^2-3\,q\,r\,c^2\,d\,g\,h+3\,q\,r\,c\,d^2\,g^2\right)}{3\,d^3} - \frac{h^2\,r\,x^3\,(p+q)}{9}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^2,x)
```

output
```
x*((((h*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(3*b*d) - (h^2*r*(p
 + q)*(3*a*d + 3*b*c))/(9*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (g*r*(b*c*h*p +
 b*d*g*p + a*d*h*q + b*d*g*q))/(b*d) + (a*c*h^2*r*(p + q))/(3*b*d)) - x^2*
((h*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(6*b*d) - (h^2*r*(p + q
)*(3*a*d + 3*b*c))/(18*b*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^2*x
 + (h^2*x^3)/3 + g*h*x^2) + (log(a + b*x)*(a^3*h^2*p*r + 3*a*b^2*g^2*p*r -
 3*a^2*b*g*h*p*r))/(3*b^3) + (log(c + d*x)*(c^3*h^2*q*r + 3*c*d^2*g^2*q*r
- 3*c^2*d*g*h*q*r))/(3*d^3) - (h^2*r*x^3*(p + q))/9
```

3.28 $\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

3.28.1 Optimal result

Integrand size = 27, antiderivative size = 160

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{(bg-ah)prx}{2b}-\frac{(dg-ch)qrx}{2d}-\frac{pr(g+hx)^2}{4h}-\frac{qr(g+hx)^2}{4h}-\frac{(bg-ah)^2pr\log(a+bx)}{2b^2h}-\frac{(dg-ch)^2qr\log(c+dx)}{2d^2h}+\frac{(g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h}$$

output
```
-1/2*(-a*h+b*g)*p*r*x/b-1/2*(-c*h+d*g)*q*r*x/d-1/4*p*r*(h*x+g)^2/h-1/4*q*r
*(h*x+g)^2/h-1/2*(-a*h+b*g)^2*p*r*ln(b*x+a)/b^2/h-1/2*(-c*h+d*g)^2*q*r*ln(
d*x+c)/d^2/h+1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

3.28.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{2ad^2(-2bg+ah)pr\log(a+bx)+b(2bc(-2dg+ch)qr\log(c+dx)+dx(r(-2adhp-2bchq+bd(p+q}{4b^2d^2}$$

input
```
Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/4*(2*a*d^2*(-2*b*g + a*h)*p*r*Log[a + b*x] + b*(2*b*c*(-2*d*g + c*h)*q*
r*Log[c + d*x] + d*x*(r*(-2*a*d*h*p - 2*b*c*h*q + b*d*(p + q)*(4*g + h*x))
 - 2*b*d*(2*g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])))/(b^2*d^2)
```

3.28.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2981, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$$

$$\Big\downarrow \text{2981}$$

$$-\frac{bpr\int \frac{(g+hx)^2}{a+bx}dx}{2h}-\frac{dqr\int \frac{(g+hx)^2}{c+dx}dx}{2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$$\Big\downarrow \text{49}$$

$$-\frac{bpr\int \left(\frac{(bg-ah)^2}{b^2(a+bx)}+\frac{h(bg-ah)}{b^2}+\frac{h(g+hx)}{b}\right)dx}{2h}-\frac{dqr\int \left(\frac{(dg-ch)^2}{d^2(c+dx)}+\frac{h(dg-ch)}{d^2}+\frac{h(g+hx)}{d}\right)dx}{2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bpr\left(\frac{(bg-ah)^2\log(a+bx)}{b^3}+\frac{hx(bg-ah)}{b^2}+\frac{(g+hx)^2}{2b}\right)}{2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{dqr\left(\frac{(dg-ch)^2\log(c+dx)}{d^3}+\frac{hx(dg-ch)}{d^2}+\frac{(g+hx)^2}{2d}\right)}{2h}$$

input
```
Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-1/2*(b*p*r*((h*(b*g - a*h)*x)/b^2 + (g + h*x)^2/(2*b) + ((b*g - a*h)^2*Lo
g[a + b*x])/b^3))/h - (d*q*r*((h*(d*g - c*h)*x)/d^2 + (g + h*x)^2/(2*d) +
((d*g - c*h)^2*Log[c + d*x])/d^3))/(2*h) + ((g + h*x)^2*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r])/(2*h)
```

3.28.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(146) = 292$.

Time = 11.89 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.74

method	result
parallelrisch	$-\frac{-2\ln(bx+a)abcdhpr-2\ln(dx+c)abcdhqr+abcdhpr+abcdhqr-2xab\,d^2hpr-2x\,b^2cdhqr-8\ln(bx+a)ab\,d^2gpr-4\ln(bx+a)b^2}{}$

input
```
int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*ln(b*x+a)*a*b*c*d*h*p*r-2*ln(d*x+c)*a*b*c*d*h*q*r+a*b*c*d*h*p*r+a
*b*c*d*h*q*r-2*x*a*b*d^2*h*p*r-2*x*b^2*c*d*h*q*r-8*ln(b*x+a)*a*b*d^2*g*p*r
-4*ln(b*x+a)*b^2*c*d*g*p*r-4*ln(d*x+c)*a*b*d^2*g*q*r-8*ln(d*x+c)*b^2*c*d*g
*q*r-2*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^2*d^2*h-4*x*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)*b^2*d^2*g+4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*d^2*g+4*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^2*c*d*g+x^2*b^2*d^2*h*p*r+x^2*b^2*d^2*h*q*r
+4*x*b^2*d^2*g*p*r+4*x*b^2*d^2*g*q*r+2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b
*c*d*h+2*ln(b*x+a)*a^2*d^2*h*p*r+2*ln(d*x+c)*b^2*c^2*h*q*r+2*a^2*h*p*r*d^2
+2*b^2*c^2*h*q*r-4*a*b*d^2*g*q*r-4*b^2*c*d*g*p*r-4*a*b*d^2*g*p*r-4*b^2*c*d
*g*q*r)/b^2/d^2
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.51

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx =$$
$$-\frac{(b^2d^2hp+b^2d^2hq)rx^2+2\left((2\,b^2d^2g-abd^2h)p+(2\,b^2d^2g-b^2cdh)q\right)rx-2\left(b^2d^2hprx^2+2\,b^2d^2gprx+\right.}{}$$

input
```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

output
```
-1/4*((b^2*d^2*h*p + b^2*d^2*h*q)*r*x^2 + 2*((2*b^2*d^2*g - a*b*d^2*h)*p +
 (2*b^2*d^2*g - b^2*c*d*h)*q)*r*x - 2*(b^2*d^2*h*p*r*x^2 + 2*b^2*d^2*g*p*r
*x + (2*a*b*d^2*g - a^2*d^2*h)*p*r)*log(b*x + a) - 2*(b^2*d^2*h*q*r*x^2 +
2*b^2*d^2*g*q*r*x + (2*b^2*c*d*g - b^2*c^2*h)*q*r)*log(d*x + c) - 2*(b^2*d
^2*h*x^2 + 2*b^2*d^2*g*x)*log(e) - 2*(b^2*d^2*h*r*x^2 + 2*b^2*d^2*g*r*x)*l
og(f))/(b^2*d^2)
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(141) = 282$.

Time = 25.80 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.14

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \begin{cases} \left(gx+\frac{hx^2}{2}\right)\log\left(e(a^pc^qf)^r\right) \\ -\frac{c^2h\log\left(e(a^pf(c+dx)^q)^r\right)}{2d^2}+\frac{cg\log\left(e(a^pf(c+dx)^q)^r\right)}{d}+\frac{chqrx}{2d}-gqrx+gx\log\left(e(a^pf(c+dx)^q)^r\right)-\frac{hqrx^2}{4}+\frac{hx^2\log(e \\ -\frac{a^2h\log\left(e(c^qf(a+bx)^p)^r\right)}{2b^2}+\frac{ag\log\left(e(c^qf(a+bx)^p)^r\right)}{b}+\frac{ahprx}{2b}-gprx+gx\log\left(e(c^qf(a+bx)^p)^r\right)-\frac{hprx^2}{4}+\frac{hx^2\log(e \\ \frac{a^2hqr\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{a^2h\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b^2}-\frac{agqr\log\left(\frac{c}{d}+x\right)}{b}+\frac{ag\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{ahprx}{2b}-\frac{c^2hqr\log\left(\frac{c}{d}+x\right)}{2d^2} \end{cases}$$

input
```
integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Piecewise(((g*x + h*x**2/2)*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)),
 (-c**2*h*log(e*(a**p*f*(c + d*x)**q)**r)/(2*d**2) + c*g*log(e*(a**p*f*(c
+ d*x)**q)**r)/d + c*h*q*r*x/(2*d) - g*q*r*x + g*x*log(e*(a**p*f*(c + d*x)
**q)**r) - h*q*r*x**2/4 + h*x**2*log(e*(a**p*f*(c + d*x)**q)**r)/2, Eq(b,
0)), (-a**2*h*log(e*(c**q*f*(a + b*x)**p)**r)/(2*b**2) + a*g*log(e*(c**q*f
*(a + b*x)**p)**r)/b + a*h*p*r*x/(2*b) - g*p*r*x + g*x*log(e*(c**q*f*(a +
b*x)**p)**r) - h*p*r*x**2/4 + h*x**2*log(e*(c**q*f*(a + b*x)**p)**r)/2, Eq
(d, 0)), (a**2*h*q*r*log(c/d + x)/(2*b**2) - a**2*h*log(e*(f*(a + b*x)**p*
(c + d*x)**q)**r)/(2*b**2) - a*g*q*r*log(c/d + x)/b + a*g*log(e*(f*(a + b*
x)**p*(c + d*x)**q)**r)/b + a*h*p*r*x/(2*b) - c**2*h*q*r*log(c/d + x)/(2*d
**2) + c*g*q*r*log(c/d + x)/d + c*h*q*r*x/(2*d) - g*p*r*x - g*q*r*x + g*x*
log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - h*p*r*x**2/4 - h*q*r*x**2/4 + h*
x**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/2, True))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{1}{2}\left(hx^2+2\,gx\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)\right) + \frac{r\left(\frac{2\left(2\,abfgp-a^2fhp\right)\log(bx+a)}{b^2}+\frac{2\left(2\,cdfgq-c^2fhq\right)\log(dx+c)}{d^2}-\frac{bdfh(p+q)x^2-2\left(adfhp-\left(2\,dfg(p+q)-cfhq\right)b\right)x}{bd}\right)}{4\,f}$$

input
```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

output
```
1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*r*(2*(2*a*b
*f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x
 + c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h
*q)*b)*x)/(b*d))/f
```

3.28.8 Giac [A] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int (g+hx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= -\frac{1}{4}\left(hpr+hqr-2\,hr\log\left(f\right)-2\,h\log\left(e\right)\right)x^2 + \frac{1}{2}\left(hprx^2+2\,gprx\right)\log\left(bx+a\right)+\frac{1}{2}\left(hqrx^2+2\,gqrx\right)\log\left(dx+c\right) - \frac{\left(2\,bdgpr-adhpr+2\,bdgqr-bchqr-2\,bdgr\log\left(f\right)-2\,bdg\log\left(e\right)\right)x}{2\,bd} + \frac{\left(2\,abgpr-a^2hpr\right)\log\left(-bx-a\right)}{2\,b^2}+\frac{\left(2\,cdgqr-c^2hqr\right)\log\left(dx+c\right)}{2\,d^2}$$

input
```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
-1/4*(h*p*r + h*q*r - 2*h*r*log(f) - 2*h*log(e))*x^2 + 1/2*(h*p*r*x^2 + 2*
g*p*r*x)*log(b*x + a) + 1/2*(h*q*r*x^2 + 2*g*q*r*x)*log(d*x + c) - 1/2*(2*
b*d*g*p*r - a*d*h*p*r + 2*b*d*g*q*r - b*c*h*q*r - 2*b*d*g*r*log(f) - 2*b*d
*g*log(e))*x/(b*d) + 1/2*(2*a*b*g*p*r - a^2*h*p*r)*log(-b*x - a)/b^2 + 1/2
*(2*c*d*g*q*r - c^2*h*q*r)*log(d*x + c)/d^2
```

3.28.9 Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

$$\int (g+hx)\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\, dx = \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\left(\frac{h\,x^2}{2}+g\,x\right) - x\left(\frac{r\left(b\,c\,h\,p+2\,b\,d\,g\,p+a\,d\,h\,q+2\,b\,d\,g\,q\right)}{2\,b\,d} - \frac{h\,r\left(p+q\right)\left(2\,a\,d+2\,b\,c\right)}{4\,b\,d}\right) - \frac{\ln\left(a+b\,x\right)\left(a^2\,h\,p\,r-2\,a\,b\,g\,p\,r\right)}{2\,b^2} - \frac{\ln\left(c+d\,x\right)\left(c^2\,h\,q\,r-2\,c\,d\,g\,q\,r\right)}{2\,d^2} - \frac{h\,r\,x^2\left(p+q\right)}{4}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x),x)
```

output
```
log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g*x + (h*x^2)/2) - x*((r*(b*c*h*p +
2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(2*b*d) - (h*r*(p + q)*(2*a*d + 2*b*c))/
(4*b*d)) - (log(a + b*x)*(a^2*h*p*r - 2*a*b*g*p*r))/(2*b^2) - (log(c + d*x
)*(c^2*h*q*r - 2*c*d*g*q*r))/(2*d^2) - (h*r*x^2*(p + q))/4
```

3.29 $\int \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = -((p+q)rx)+\frac{(bc-ad)qr\log(c+dx)}{bd}+\frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$

output
```
-(p+q)*r*x+(-a*d+b*c)*q*r*ln(d*x+c)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)/b
```

3.29.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = \frac{apr\log(a+bx)}{b}+\frac{cqr\log(c+dx)}{d}+x\left(-((p+q)r)+\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
(a*p*r*Log[a + b*x])/b + (c*q*r*Log[c + d*x])/d + x*(-((p + q)*r) + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r])
```

3.29.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2979, 16, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$\Big\downarrow \text{2979}$$

$$\frac{qr(bc-ad)\int \frac{1}{c+dx}dx}{b}-r(p+q)\int 1dx+\frac{(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

$$\Big\downarrow \text{16}$$

$$-r(p+q)\int 1dx+\frac{(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}$$

$$\Big\downarrow \text{24}$$

$$\frac{(a+bx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

output
```
-((p + q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/b
```

3.29.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2979
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Simp[q*r*s*((b*c - a*d)/b)   Int[Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Simp[r*s*(p + q)   Int[Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q
, r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4
]
```

3.29.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

method	result
default	$\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)x-r\left(xp+xq-\frac{cq\ln(dx+c)}{d}-\frac{ap\ln(bx+a)}{b}\right)$
parallelrisch	$\frac{\ln(bx+a)adpq\,r^{2}-\ln(bx+a)bcpq\,r^{2}-xbdpq\,r^{2}-xbd\,q^{2}r^{2}+x\ln(e(f(bx+a)^{p}(dx+c)^{q})^{r})bdqr+adpq\,r^{2}+ad\,q^{2}r^{2}+bcpq\,r^{2}+bc\,r^{2}q^{2}}{bdqr}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

output
```
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*x-r*(x*p+x*q-c*q/d*ln(d*x+c)-a*p/b*ln(b*x+
a))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \log\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)\,dx$$
$$=\frac{bdrx\log\left(f\right)+bdx\log\left(e\right)-(bdp+bdq)rx+(bdprx+adpr)\log\left(bx+a\right)+(bdqrx+bcqr)\log\left(dx+c\right)}{bd}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

output
```
(b*d*r*x*log(f) + b*d*x*log(e) - (b*d*p + b*d*q)*r*x + (b*d*p*r*x + a*d*p*
r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c))/(b*d)
```

3.29.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(54) = 108.

Time = 5.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.02

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= \begin{cases} x \log \left(e(a^p c^q f)^r\right) & \text{fo} \\ \frac{c \log \left(e(a^p f(c+dx)^q)^r\right)}{d} - qrx + x \log \left(e(a^p f(c+dx)^q)^r\right) & \text{fo} \\ \frac{a \log \left(e(c^q f(a+bx)^p)^r\right)}{b} - prx + x \log \left(e(c^q f(a+bx)^p)^r\right) & \text{fo} \\ -\frac{aqr \log (c+dx)}{b} + \frac{a \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{cqr \log (c+dx)}{d} - prx - qrx + x \log \left(e(f(a+bx)^p (c+dx)^q)^r\right) & \text{ot} \end{cases}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

output
```
Piecewise((x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (c*log(e*(a**p
*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)**r), Eq(b,
0)), (a*log(e*(c**q*f*(a + b*x)**p)**r)/b - p*r*x + x*log(e*(c**q*f*(a + b
*x)**p)**r), Eq(d, 0)), (-a*q*r*log(c + d*x)/b + a*log(e*(f*(a + b*x)**p*(
c + d*x)**q)**r)/b + c*q*r*log(c + d*x)/d - p*r*x - q*r*x + x*log(e*(f*(a
+ b*x)**p*(c + d*x)**q)**r), True))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = x \log \left(((bx+a)^p(dx+c)^q f)^r e\right) - \frac{\left(bfp\left(\frac{x}{b} - \frac{a \log(bx+a)}{b^2}\right) + dfq\left(\frac{x}{d} - \frac{c \log(dx+c)}{d^2}\right)\right)r}{f}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

output
```
x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) - (b*f*p*(x/b - a*log(b*x + a)/b^2)
 + d*f*q*(x/d - c*log(d*x + c)/d^2))*r/f
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = prx\log(bx+a) + qrx\log(dx+c) + \frac{apr\log(bx+a)}{b} + \frac{cqr\log(-dx-c)}{d} - (pr+qr-r\log(f)-\log(e))x$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

output
```
p*r*x*log(b*x + a) + q*r*x*log(d*x + c) + a*p*r*log(b*x + a)/b + c*q*r*log
(-d*x - c)/d - (p*r + q*r - r*log(f) - log(e))*x
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = x\,\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right) - p\,r\,x - q\,r\,x + \frac{a\,p\,r\,\ln(a+b\,x)}{b} + \frac{c\,q\,r\,\ln(c+d\,x)}{d}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r),x)
```

output
```
x*log(e*(f*(a + b*x)^p*(c + d*x)^q)^r) - p*r*x - q*r*x + (a*p*r*log(a + b*
x))/b + (c*q*r*log(c + d*x))/d
```

3.30 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

3.30.1 Optimal result

Integrand size = 29, antiderivative size = 148

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}$$

output

```
-p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h-q*r*ln(-h*(d*x+c)/(-c*h+d*g))*l
n(h*x+g)/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-p*r*polylog(2,b*(h*
x+g)/(-a*h+b*g))/h-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

3.30.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$$
$$= \frac{-pr\log(a+bx)\log(g+hx) - qr\log(c+dx)\log(g+hx) + \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx) + p}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

output
```
(-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g +
 h*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*
r*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/
(-(d*g) + c*h)])/h
```

3.30.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2980, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$$
$$\Big\downarrow \ 2980$$
$$-\frac{bpr\int \frac{\log(g+hx)}{a+bx}dx}{h} - \frac{dqr\int \frac{\log(g+hx)}{c+dx}dx}{h} + \frac{\log(g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$
$$\Big\downarrow \ 2841$$

$$-\frac{bpr\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{h\int\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{g+hx}dx}{b}\right)}{h}-\frac{dqr\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{h\int\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{g+hx}dx}{d}\right)}{h}+\frac{\log(g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$

$$\Big\downarrow \; 2840$$

$$-\frac{bpr\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)}{b}\right)}{h}-\frac{dqr\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)}{d}\right)}{h}+\frac{\log(g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$

$$\Big\downarrow \; 2838$$

$$\frac{\log(g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}-\frac{bpr\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}\right)}{h}-\frac{dqr\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}+\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}\right)}{h}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

output
```
(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x])/h - (b*p*r*((Log[-((h*
(a + b*x))/(b*g - a*h))]*Log[g + h*x])/b + PolyLog[2, (b*(g + h*x))/(b*g -
 a*h)]/b))/h - (d*q*r*((Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/d
+ PolyLog[2, (d*(g + h*x))/(d*g - c*h)]/d))/h
```

3.30. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

3.30.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2980
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Simp[b*p*(r/h)   Int[Log[g + h*x]/(a + b
*x), x], x] - Simp[d*q*(r/h)   Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

3.30.4 Maple [A] (verified)

Time = 18.98 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

method	result
parts	$\frac{\ln(e(f(bx+a)^p(dx+c)^q)^r)\ln(hx+g)}{h}-\frac{r\left(bph\left(\frac{\operatorname{dilog}\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}+\frac{\ln(hx+g)\ln\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}\right)+dqh\left(\frac{\operatorname{dilog}\left(\frac{d(hx+g)+ch}{ch-dg}\right)}{d}\right.\right.}{h^2}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-1/h^2*r*(b*p*h*(dilog(((h*x+g)
*b+a*h-b*g)/(a*h-b*g))/b+ln(h*x+g)*ln(((h*x+g)*b+a*h-b*g)/(a*h-b*g))/b)+d*
q*h*(dilog((d*(h*x+g)+c*h-d*g)/(c*h-d*g))/d+ln(h*x+g)*ln((d*(h*x+g)+c*h-d*
g)/(c*h-d*g))/d))
```

3.30.5 Fricas [F]

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \int \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{hx+g}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g),x)
```

output
```
Timed out
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{h}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{h}\right)r}{f} - \frac{(fp\log(bx+a)+fq\log(dx+c))r\log(hx+g)}{fh} + \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)\log(hx+g)}{h}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="maxima")
```

output
```
((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(
b*g - a*h)))*f*p/h + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + di
log(-(d*h*x + c*h)/(d*g - c*h)))*f*q/h)*r/f - (f*p*log(b*x + a) + f*q*log(
d*x + c))*r*log(h*x + g)/(f*h) + log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(
h*x + g)/h
```

3.30.8 Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{hx+g}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{g+h\,x}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x), x)
```

3.31 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2}\,dx$

3.31.1 Optimal result

Integrand size = 29, antiderivative size = 128

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \frac{bpr\log(a+bx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} - \frac{bpr\log(g+hx)}{h(bg-ah)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

output
```
b*p*r*ln(b*x+a)/h/(-a*h+b*g)+d*q*r*ln(d*x+c)/h/(-c*h+d*g)-ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)/h/(h*x+g)-b*p*r*ln(h*x+g)/h/(-a*h+b*g)-d*q*r*ln(h*x+g)/h/(
-c*h+d*g)
```

3.31.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \frac{-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}+\frac{bpr(\log(a+bx)-\log(g+hx))}{bg-ah}+\frac{dqr(\log(c+dx)-\log(g+hx))}{dg-ch}}{h}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

output
```
(-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)) + (b*p*r*(Log[a + b*x]
- Log[g + h*x]))/(b*g - a*h) + (d*q*r*(Log[c + d*x] - Log[g + h*x]))/(d*g
- c*h))/h
```

3.31.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2} \, dx$$

$$\Big\downarrow \text{2981}$$

$$\frac{bpr\int \frac{1}{(a+bx)(g+hx)}dx}{h}+\frac{dqr\int \frac{1}{(c+dx)(g+hx)}dx}{h}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}$$

$$\Big\downarrow \text{47}$$

$$\frac{bpr\left(\frac{b\int \frac{1}{a+bx}dx}{bg-ah}-\frac{h\int \frac{1}{g+hx}dx}{bg-ah}\right)}{h}+\frac{dqr\left(\frac{d\int \frac{1}{c+dx}dx}{dg-ch}-\frac{h\int \frac{1}{g+hx}dx}{dg-ch}\right)}{h}-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}+\frac{bpr\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)}{h}+\frac{dqr\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)}{h}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

output
```
-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x))) + (b*p*r*(Log[a + b*
x]/(b*g - a*h) - Log[g + h*x]/(b*g - a*h)))/h + (d*q*r*(Log[c + d*x]/(d*g
- c*h) - Log[g + h*x]/(d*g - c*h)))/h
```

3.31.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(128) = 256$.

Time = 84.84 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.74

method	result
parallelrisch	$-\frac{\ln(hx+g)xabcd\,g^2hpr+\ln(hx+g)xabcd\,g^2hqr-\ln(dx+c)ab\,c^2g^2hqr-\ln(hx+g)a^2cd\,g^2hqr-\ln(hx+g)ab\,c^2g^2hpr+\ln(hx+g)}{}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
-(ln(h*x+g)*x*a*b*c*d*g^2*h*p*r+ln(h*x+g)*x*a*b*c*d*g^2*h*q*r-ln(d*x+c)*a*
b*c^2*g^2*h*q*r-ln(h*x+g)*a^2*c*d*g^2*h*q*r-ln(h*x+g)*a*b*c^2*g^2*h*p*r+ln
(h*x+g)*a*b*c*d*g^3*p*r+ln(h*x+g)*a*b*c*d*g^3*q*r-x*ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r)*a*b*c*d*g^2*h-ln(b*x+a)*a^2*c*d*g^2*h*p*r-ln(b*x+a)*x*a^2*c*d*g*
h^2*p*r-ln(d*x+c)*x*a*b*c^2*g*h^2*q*r-ln(h*x+g)*x*a^2*c*d*g*h^2*q*r-ln(h*x
+g)*x*a*b*c^2*g*h^2*p*r-x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*c^2*h^3+ln(b
*x+a)*x*a^2*c^2*h^3*p*r+ln(d*x+c)*x*a^2*c^2*h^3*q*r+ln(b*x+a)*a^2*c^2*g*h^
2*p*r+ln(d*x+c)*a^2*c^2*g*h^2*q*r+x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*c*
d*g*h^2+x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*c^2*g*h^2)/(a*c*h^2-a*d*g*h-
b*c*g*h+b*d*g^2)/h/(h*x+g)/a/c/g
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(128) = 256$.

Time = 72.61 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.19

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = -\frac{(bdg^2+ach^2-(bc+ad)gh)r\log(f)-((bdgh-bch^2)prx+(adgh-ach^2)pr)\log(bx+a)-((bdgh-$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas"
)
```

output
```
-((b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*r*log(f) - ((b*d*g*h - b*c*h^2)*p*
r*x + (a*d*g*h - a*c*h^2)*p*r)*log(b*x + a) - ((b*d*g*h - a*d*h^2)*q*r*x +
 (b*c*g*h - a*c*h^2)*q*r)*log(d*x + c) + (((b*d*g*h - b*c*h^2)*p + (b*d*g*
h - a*d*h^2)*q)*r*x + ((b*d*g^2 - b*c*g*h)*p + (b*d*g^2 - a*d*g*h)*q)*r)*l
og(h*x + g) + (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(e))/(b*d*g^3*h + a
*c*g*h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h^2 + a*c*h^4 - (b*c + a*d)*g*h^
3)*x)
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**2,x)
```

output
```
Timed out
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx$$
$$= \frac{\left(bfp\left(\frac{\log(bx+a)}{bg-ah}-\frac{\log(hx+g)}{bg-ah}\right)+dfq\left(\frac{\log(dx+c)}{dg-ch}-\frac{\log(hx+g)}{dg-ch}\right)\right)r}{fh} - \frac{\log\left(((bx+a)^p(dx+c)^qf)^r e\right)}{(hx+g)h}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="maxima"
)
```

output
```
(b*f*p*(log(b*x + a)/(b*g - a*h) - log(h*x + g)/(b*g - a*h)) + d*f*q*(log(
d*x + c)/(d*g - c*h) - log(h*x + g)/(d*g - c*h)))*r/(f*h) - log(((b*x + a)
^p*(d*x + c)^q*f)^r*e)/((h*x + g)*h)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.49

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \frac{b^2pr\log\left(|-bx-a|\right)}{b^2gh-abh^2} + \frac{d^2qr\log\left(|dx+c|\right)}{d^2gh-cdh^2} - \frac{pr\log\left(bx+a\right)}{h^2x+gh} - \frac{qr\log\left(dx+c\right)}{h^2x+gh} - \frac{(bdgpr-bchpr+bdgqr-adhqr)\log\left(hx+g\right)}{bdg^2h-bcgh^2-adgh^2+ach^3} - \frac{r\log\left(f\right)+\log\left(e\right)}{h^2x+gh}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="giac")
```

output
```
b^2*p*r*log(abs(-b*x - a))/(b^2*g*h - a*b*h^2) + d^2*q*r*log(abs(d*x + c))
/(d^2*g*h - c*d*h^2) - p*r*log(b*x + a)/(h^2*x + g*h) - q*r*log(d*x + c)/(
h^2*x + g*h) - (b*d*g*p*r - b*c*h*p*r + b*d*g*q*r - a*d*h*q*r)*log(h*x + g
)/(b*d*g^2*h - b*c*g*h^2 - a*d*g*h^2 + a*c*h^3) - (r*log(f) + log(e))/(h^2
*x + g*h)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.19

$$\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2}\,dx = \frac{\ln(g+hx)\,(bchpr-g(bdpr+bdqr)+adhqr)}{ach^3-adgh^2-bcgh^2+bdg^2h} - \frac{\ln(e(f(a+bx)^p(c+dx)^q)^r)\,\left(x+\frac{g}{h}\right)}{(g+hx)^2} - \frac{bpr\,\ln(a+bx)}{ah^2-bgh} - \frac{dqr\,\ln(c+dx)}{ch^2-dgh}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^2,x)
```

output
```
(log(g + h*x)*(b*c*h*p*r - g*(b*d*p*r + b*d*q*r) + a*d*h*q*r))/(a*c*h^3 -
a*d*g*h^2 - b*c*g*h^2 + b*d*g^2*h) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)
*(x + g/h))/(g + h*x)^2 - (b*p*r*log(a + b*x))/(a*h^2 - b*g*h) - (d*q*r*lo
g(c + d*x))/(c*h^2 - d*g*h)
```

3.32 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx$

3.32.1 Optimal result

Integrand size = 29, antiderivative size = 202

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = {}& \frac{bpr}{2h(bg-ah)(g+hx)}+\frac{dqr}{2h(dg-ch)(g+hx)} \\ &+\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2} \\ &-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} \\ &-\frac{b^2pr\log(g+hx)}{2h(bg-ah)^2}-\frac{d^2qr\log(g+hx)}{2h(dg-ch)^2}\end{aligned}$$

output
```
1/2*b*p*r/h/(-a*h+b*g)/(h*x+g)+1/2*d*q*r/h/(-c*h+d*g)/(h*x+g)+1/2*b^2*p*r*
ln(b*x+a)/h/(-a*h+b*g)^2+1/2*d^2*q*r*ln(d*x+c)/h/(-c*h+d*g)^2-1/2*ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^2-1/2*b^2*p*r*ln(h*x+g)/h/(-a*h+b*g)^2-1
/2*d^2*q*r*ln(h*x+g)/h/(-c*h+d*g)^2
```

3.32.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$$
$$= \frac{-\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{r(g+hx)((bc-ad)(bg-ah)(dg-ch)(bdg(p+q)-h(bcp+adq))-(g+hx)(-b^2(bc-ad)(dg-ch)^2p(\log}{(bc-ad)(bg-ah)^2(dg-ch)}}{2h(g+hx)^2}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

output
```
(-Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*c - a*d)*(b*g -
a*h)*(d*g - c*h)*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*(-(b^2*(b
*c - a*d)*(d*g - c*h)^2*p*(Log[a + b*x] - Log[g + h*x])) + d^2*(-(b*c) + a
*d)*(b*g - a*h)^2*q*(Log[c + d*x] - Log[g + h*x]))))/((b*c - a*d)*(b*g - a
*h)^2*(d*g - c*h)^2))/(2*h*(g + h*x)^2)
```

3.32.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$$
$$\Big\downarrow \ 2981$$
$$\frac{bpr\int \frac{1}{(a+bx)(g+hx)^2}dx}{2h}+\frac{dqr\int \frac{1}{(c+dx)(g+hx)^2}dx}{2h}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$
$$\Big\downarrow \ 54$$
$$\frac{bpr\int \left(\frac{b^2}{(bg-ah)^2(a+bx)}-\frac{hb}{(bg-ah)^2(g+hx)}-\frac{h}{(bg-ah)(g+hx)^2}\right)dx}{2h}+$$
$$\frac{dqr\int \left(\frac{d^2}{(dg-ch)^2(c+dx)}-\frac{hd}{(dg-ch)^2(g+hx)}-\frac{h}{(dg-ch)(g+hx)^2}\right)dx}{2h}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$
$$\Big\downarrow \ 2009$$

$$-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}+\frac{bpr\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}\right)}{2h}+\frac{dqr\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}\right)}{2h}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

output
```
-1/2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)^2) + (b*p*r*(1/((b*
g - a*h)*(g + h*x)) + (b*Log[a + b*x])/(b*g - a*h)^2 - (b*Log[g + h*x])/(b
*g - a*h)^2))/(2*h) + (d*q*r*(1/((d*g - c*h)*(g + h*x)) + (d*Log[c + d*x])
/(d*g - c*h)^2 - (d*Log[g + h*x])/(d*g - c*h)^2))/(2*h)
```

3.32.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.32.4 Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(hx+g)^3}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

3.32.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**3,x)
```

output
```
Timed out
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$$
$$= \frac{\left(bfp\left(\frac{b\log(bx+a)}{b^2g^2-2\,abgh+a^2h^2} - \frac{b\log(hx+g)}{b^2g^2-2\,abgh+a^2h^2} + \frac{1}{bg^2-agh+(bgh-ah^2)x}\right) + dfq\left(\frac{d\log(dx+c)}{d^2g^2-2\,cdgh+c^2h^2} - \frac{d\log(hx+g)}{d^2g^2-2\,cdgh+c^2h^2} + \frac{}{dg\cdots}\right.}{2\,fh}$$
$$- \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{2\,(hx+g)^2h}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="maxima"
)
```

output
```
1/2*(b*f*p*(b*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) - b*log(h*x + g
)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + 1/(b*g^2 - a*g*h + (b*g*h - a*h^2)*x))
 + d*f*q*(d*log(d*x + c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) - d*log(h*x + g)/
(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + 1/(d*g^2 - c*g*h + (d*g*h - c*h^2)*x)))*
r/(f*h) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*x + g)^2*h)
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(188) = 376$.

Time = 0.45 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.99

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \frac{b^3pr\log\left(|bx+a|\right)}{2\left(b^3g^2h-2\,ab^2gh^2+a^2bh^3\right)} + \frac{d^3qr\log\left(|dx+c|\right)}{2\left(d^3g^2h-2\,cd^2gh^2+c^2dh^3\right)} - \frac{pr\log\left(bx+a\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} - \frac{qr\log\left(dx+c\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} - \frac{\left(b^2d^2g^2pr-2\,b^2cdghpr+b^2c^2h^2pr+b^2d^2g^2qr-2\,abd^2ghqr+a^2d^2h^2qr\right)\log\left(hx+g\right)}{2\left(b^2d^2g^4h-2\,b^2cdg^3h^2-2\,abd^2g^3h^2+b^2c^2g^2h^3+4\,abcdg^2h^3+a^2d^2g^2h^3-2\,abc^2gh^4-2\,a^2cdgh^4+a^2\right.} + \frac{bdghprx-bch^2prx+bdghqrx-adh^2qrx+bdg^2pr-bcghpr+bdg^2qr-adghqr-bdg^2r\log\left(f\right)+bcg}{2\left(bdg^2h^3x^2-bcgh^4x^2-adgh^4x^2+ach^5x^2+2\,bdg^3h^2x-2\,bcg^2h^3\right.}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/2*b^3*p*r*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) + 1/
2*d^3*q*r*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*
p*r*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*q*r*log(d*x + c)/(h^3
*x^2 + 2*g*h^2*x + g^2*h) - 1/2*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r + b^2
*c^2*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*log(
h*x + g)/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2*
g^2*h^3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*
d*g*h^4 + a^2*c^2*h^5) + 1/2*(b*d*g*h*p*r*x - b*c*h^2*p*r*x + b*d*g*h*q*r*
x - a*d*h^2*q*r*x + b*d*g^2*p*r - b*c*g*h*p*r + b*d*g^2*q*r - a*d*g*h*q*r
- b*d*g^2*r*log(f) + b*c*g*h*r*log(f) + a*d*g*h*r*log(f) - a*c*h^2*r*log(f
) - b*d*g^2*log(e) + b*c*g*h*log(e) + a*d*g*h*log(e) - a*c*h^2*log(e))/(b*
d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^
2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h - b*c*
g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.90

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \frac{b^2\,p\,r\,\ln\left(a+b\,x\right)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h}$$
$$-\frac{\ln\left(g+h\,x\right)\,\left(h^2\,\left(q\,r\,a^2\,d^2+p\,r\,b^2\,c^2\right)-h\left(2\,c\,g\,p\,r\,b^2\,d+2\,a\,g\,q\,r\,b\,d^2\right)+b^2\,d^2\,g^2\,p\,r+b^2\,d^2}{2\,a^2\,c^2\,h^5-4\,a^2\,c\,d\,g\,h^4+2\,a^2\,d^2\,g^2\,h^3-4\,a\,b\,c^2\,g\,h^4+8\,a\,b\,c\,d\,g^2\,h^3-4\,a\,b\,d^2\,g^3\,h^2+2\,b^2\,c^2\,g^2\,h^3-4\,b^2}$$
$$-\frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\,\left(\frac{x}{2}+\frac{g}{2\,h}\right)}{\left(g+h\,x\right)^3}$$
$$-\frac{b\,c\,h\,p\,r-b\,d\,g\,p\,r+a\,d\,h\,q\,r-b\,d\,g\,q\,r}{\left(2\,x\,h^2+2\,g\,h\right)\,\left(a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h\right)}+\frac{d^2\,q\,r\,\ln\left(c+d\,x\right)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

input

```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^3,x)
```

output

```
(b^2*p*r*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*a*b*g*h^2) - (log(g +
h*x)*(h^2*(b^2*c^2*p*r + a^2*d^2*q*r) - h*(2*a*b*d^2*g*q*r + 2*b^2*c*d*g*p
*r) + b^2*d^2*g^2*p*r + b^2*d^2*g^2*q*r))/(2*a^2*c^2*h^5 + 2*b^2*d^2*g^4*h
 + 2*a^2*d^2*g^2*h^3 + 2*b^2*c^2*g^2*h^3 - 4*a*b*c^2*g*h^4 - 4*a^2*c*d*g*h
^4 - 4*a*b*d^2*g^3*h^2 - 4*b^2*c*d*g^3*h^2 + 8*a*b*c*d*g^2*h^3) - (log(e*(
f*(a + b*x)^p*(c + d*x)^q)^r)*(x/2 + g/(2*h)))/(g + h*x)^3 - (b*c*h*p*r -
b*d*g*p*r + a*d*h*q*r - b*d*g*q*r)/((2*g*h + 2*h^2*x)*(a*c*h^2 + b*d*g^2 -
 a*d*g*h - b*c*g*h)) + (d^2*q*r*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4
*c*d*g*h^2)
```

3.33 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4}\,dx$

3.33.1 Optimal result

Integrand size = 29, antiderivative size = 260

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \frac{bpr}{6h(bg-ah)(g+hx)^2}+\frac{dqr}{6h(dg-ch)(g+hx)^2} + \frac{b^2pr}{3h(bg-ah)^2(g+hx)}+\frac{d^2qr}{3h(dg-ch)^2(g+hx)} + \frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}+\frac{d^3qr\log(c+dx)}{3h(dg-ch)^3} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} - \frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}-\frac{d^3qr\log(g+hx)}{3h(dg-ch)^3}$$

output
```
1/6*b*p*r/h/(-a*h+b*g)/(h*x+g)^2+1/6*d*q*r/h/(-c*h+d*g)/(h*x+g)^2+1/3*b^2*
p*r/h/(-a*h+b*g)^2/(h*x+g)+1/3*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*p*r*
ln(b*x+a)/h/(-a*h+b*g)^3+1/3*d^3*q*r*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^3-1/3*b^3*p*r*ln(h*x+g)/h/(-a*h+b*g)^3-1
/3*d^3*q*r*ln(h*x+g)/h/(-c*h+d*g)^3
```

3.33.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$
$$= \frac{-2\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{r(g+hx)\left((bg-ah)^2(dg-ch)^2(bdg(p+q)-h(bcp+adq))-(g+hx)\left((bg-ah)(dg-ch)\left(4abd^2ghq-2c\right.\right.\right.}{}}{6h(g+h}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

output
```
(-2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*g - a*h)^2*(d*
g - c*h)^2*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)*(d
*g - c*h)*(4*a*b*d^2*g*h*q - 2*a^2*d^2*h^2*q - 2*b^2*(-2*c*d*g*h*p + c^2*h
^2*p + d^2*g^2*(p + q))) - 2*(g + h*x)*(b^3*(d*g - c*h)^3*p*(Log[a + b*x]
- Log[g + h*x]) + d^3*(b*g - a*h)^3*q*(Log[c + d*x] - Log[g + h*x])))))/((
b*g - a*h)^3*(d*g - c*h)^3))/(6*h*(g + h*x)^3)
```

3.33.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$

$$\Big\downarrow \ 2981$$

$$\frac{bpr\int \frac{1}{(a+bx)(g+hx)^3}dx}{3h}+\frac{dqr\int \frac{1}{(c+dx)(g+hx)^3}dx}{3h}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$$\Big\downarrow \ 54$$

$$\frac{bpr\int\left(\frac{b^3}{(bg-ah)^3(a+bx)}-\frac{hb^2}{(bg-ah)^3(g+hx)}-\frac{hb}{(bg-ah)^2(g+hx)^2}-\frac{h}{(bg-ah)(g+hx)^3}\right)dx}{3h}+\frac{dqr\int\left(\frac{d^3}{(dg-ch)^3(c+dx)}-\frac{hd^2}{(dg-ch)^3(g+hx)}-\frac{hd}{(dg-ch)^2(g+hx)^2}-\frac{h}{(dg-ch)(g+hx)^3}\right)dx}{3h}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bpr\left(\frac{b^2\log(a+bx)}{(bg-ah)^3}-\frac{b^2\log(g+hx)}{(bg-ah)^3}+\frac{b}{(g+hx)(bg-ah)^2}+\frac{1}{2(g+hx)^2(bg-ah)}\right)}{3h}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+\frac{dqr\left(\frac{d^2\log(c+dx)}{(dg-ch)^3}-\frac{d^2\log(g+hx)}{(dg-ch)^3}+\frac{d}{(g+hx)(dg-ch)^2}+\frac{1}{2(g+hx)^2(dg-ch)}\right)}{3h}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

output
```
-1/3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)^3) + (b*p*r*(1/(2*(
b*g - a*h)*(g + h*x)^2) + b/((b*g - a*h)^2*(g + h*x)) + (b^2*Log[a + b*x])
/(b*g - a*h)^3 - (b^2*Log[g + h*x])/(b*g - a*h)^3))/(3*h) + (d*q*r*(1/(2*(
d*g - c*h)*(g + h*x)^2) + d/((d*g - c*h)^2*(g + h*x)) + (d^2*Log[c + d*x])
/(d*g - c*h)^3 - (d^2*Log[g + h*x])/(d*g - c*h)^3))/(3*h)
```

3.33.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeBQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.33.4 Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^{p} (dx+c)^{q})^{r}\right)}{(hx+g)^{4}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

3.33.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**4,x)
```

output
```
Timed out
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.75

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$
$$= \frac{\left(\left(\frac{2\,b^2\log(bx+a)}{b^3g^3-3\,ab^2g^2h+3\,a^2bgh^2-a^3h^3} - \frac{2\,b^2\log(hx+g)}{b^3g^3-3\,ab^2g^2h+3\,a^2bgh^2-a^3h^3} + \frac{2\,bhx+3\,bg-ah}{b^2g^4-2\,abg^3h+a^2g^2h^2+(b^2g^2h^2-2\,abgh^3+a^2h^4)x^2+2\,(b^2g^3h-2\,a}\right.\right.}{}$$
$$- \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{3\,(hx+g)^3h}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="maxima"
)
```

output
```
1/6*((2*b^2*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^
3) - 2*b^2*log(h*x + g)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3
) + (2*b*h*x + 3*b*g - a*h)/(b^2*g^4 - 2*a*b*g^3*h + a^2*g^2*h^2 + (b^2*g^
2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*x^2 + 2*(b^2*g^3*h - 2*a*b*g^2*h^2 + a^2*g*
h^3)*x))*b*f*p + (2*d^2*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*
h^2 - c^3*h^3) - 2*d^2*log(h*x + g)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h
^2 - c^3*h^3) + (2*d*h*x + 3*d*g - c*h)/(d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h
^2 + (d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*x^2 + 2*(d^2*g^3*h - 2*c*d*g^2*
h^2 + c^2*g*h^3)*x))*d*f*q)*r/(f*h) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^
r*e)/((h*x + g)^3*h)
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1783 vs. 2(242) = 484.

Time = 0.52 (sec) , antiderivative size = 1783, normalized size of antiderivative = 6.86

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="giac")
```

output
```
1/3*b^4*p*r*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h
^3 - a^3*b*h^4) + 1/3*d^4*q*r*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h
^2 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*p*r*log(b*x + a)/(h^4*x^3 + 3*g*h^
3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*q*r*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2
 + 3*g^2*h^2*x + g^3*h) - 1/3*(b^3*d^3*g^3*p*r - 3*b^3*c*d^2*g^2*h*p*r + 3
*b^3*c^2*d*g*h^2*p*r - b^3*c^3*h^3*p*r + b^3*d^3*g^3*q*r - 3*a*b^2*d^3*g^2
*h*q*r + 3*a^2*b*d^3*g*h^2*q*r - a^3*d^3*h^3*q*r)*log(h*x + g)/(b^3*d^3*g^
6*h - 3*b^3*c*d^2*g^5*h^2 - 3*a*b^2*d^3*g^5*h^2 + 3*b^3*c^2*d*g^4*h^3 + 9*
a*b^2*c*d^2*g^4*h^3 + 3*a^2*b*d^3*g^4*h^3 - b^3*c^3*g^3*h^4 - 9*a*b^2*c^2*
d*g^3*h^4 - 9*a^2*b*c*d^2*g^3*h^4 - a^3*d^3*g^3*h^4 + 3*a*b^2*c^3*g^2*h^5
+ 9*a^2*b*c^2*d*g^2*h^5 + 3*a^3*c*d^2*g^2*h^5 - 3*a^2*b*c^3*g*h^6 - 3*a^3*
c^2*d*g*h^6 + a^3*c^3*h^7) + 1/6*(2*b^2*d^2*g^2*h^2*p*r*x^2 - 4*b^2*c*d*g*
h^3*p*r*x^2 + 2*b^2*c^2*h^4*p*r*x^2 + 2*b^2*d^2*g^2*h^2*q*r*x^2 - 4*a*b*d^
2*g*h^3*q*r*x^2 + 2*a^2*d^2*h^4*q*r*x^2 + 5*b^2*d^2*g^3*h*p*r*x - 10*b^2*c
*d*g^2*h^2*p*r*x - a*b*d^2*g^2*h^2*p*r*x + 5*b^2*c^2*g*h^3*p*r*x + 2*a*b*c
*d*g*h^3*p*r*x - a*b*c^2*h^4*p*r*x + 5*b^2*d^2*g^3*h*q*r*x - b^2*c*d*g^2*h
^2*q*r*x - 10*a*b*d^2*g^2*h^2*q*r*x + 2*a*b*c*d*g*h^3*q*r*x + 5*a^2*d^2*g*
h^3*q*r*x - a^2*c*d*h^4*q*r*x + 3*b^2*d^2*g^4*p*r - 6*b^2*c*d*g^3*h*p*r -
a*b*d^2*g^3*h*p*r + 3*b^2*c^2*g^2*h^2*p*r + 2*a*b*c*d*g^2*h^2*p*r - a*b*c^
2*g*h^3*p*r + 3*b^2*d^2*g^4*q*r - b^2*c*d*g^3*h*q*r - 6*a*b*d^2*g^3*h*q...
```

3.33.9 Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.76

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$
$$= \frac{\frac{3\,b^2\,d^2\,g^3\,p\,r+3\,b^2\,d^2\,g^3\,q\,r-a\,b\,c^2\,h^3\,p\,r-a^2\,c\,d\,h^3\,q\,r+3\,b^2\,c^2\,g\,h^2\,p\,r+3\,a^2\,d^2\,g\,h^2\,q\,r-a\,b\,d^2\,g^2\,h\,p\,r-6\,a\,b\,d^2\,g^2\,h\,q\,r-6\,b^2\,c\,d\,g^2\,h\,p\,r-b^2\,c\,d\,g^2\,h\,g}{2\,(a^2\,c^2\,h^4-2\,a^2\,c\,d\,g\,h^3+a^2\,d^2\,g^2\,h^2-2\,a\,b\,c^2\,g\,h^3+4\,a\,b\,c\,d\,g^2\,h^2-2\,a\,b\,d^2\,g^3\,h+b^2\,c^2\,g^2\,h^2-2\,b^2\,c\,d\,g^3\,h+b^2\,d^2\,g}}{3\,g^2\,h+6\,g}$$
$$+ \frac{\ln(g+h\,x)\,\left(g^2\,(3\,c\,h\,p\,r\,b^3\,d^2+3\,a\,h\,q\,r\,b^2\,d^3\right)}{3\,a^3\,c^3\,h^7-9\,a^3\,c^2\,d\,g\,h^6+9\,a^3\,c\,d^2\,g^2\,h^5-3\,a^3\,d^3\,g^3\,h^4-9\,a^2\,b\,c^3\,g\,h^6+27\,a^2\,b\,c^2\,d\,g^2\,h^5-27\,a^2\,b\,c\,d^2\,g}$$
$$- \frac{\ln\left(e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(\frac{x}{3}+\frac{g}{3\,h}\right)}{(g+h\,x)^4} - \frac{b^3\,p\,r\,\ln(a+b\,x)}{3\,a^3\,h^4-9\,a^2\,b\,g\,h^3+9\,a\,b^2\,g^2\,h^2-3\,b^3\,g^3\,h}$$
$$- \frac{d^3\,q\,r\,\ln(c+d\,x)}{3\,c^3\,h^4-9\,c^2\,d\,g\,h^3+9\,c\,d^2\,g^2\,h^2-3\,d^3\,g^3\,h}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^4,x)
```

output
```
((3*b^2*d^2*g^3*p*r + 3*b^2*d^2*g^3*q*r - a*b*c^2*h^3*p*r - a^2*c*d*h^3*q*
r + 3*b^2*c^2*g*h^2*p*r + 3*a^2*d^2*g*h^2*q*r - a*b*d^2*g^2*h*p*r - 6*a*b*
d^2*g^2*h*q*r - 6*b^2*c*d*g^2*h*p*r - b^2*c*d*g^2*h*q*r + 2*a*b*c*d*g*h^2*
p*r + 2*a*b*c*d*g*h^2*q*r)/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2
 + b^2*c^2*g^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 -
 2*b^2*c*d*g^3*h + 4*a*b*c*d*g^2*h^2)) + (x*(b^2*c^2*h^3*p*r + a^2*d^2*h^3
*q*r + b^2*d^2*g^2*h*p*r + b^2*d^2*g^2*h*q*r - 2*a*b*d^2*g*h^2*q*r - 2*b^2
*c*d*g*h^2*p*r))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^
2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^
3*h + 4*a*b*c*d*g^2*h^2))/(3*g^2*h + 3*h^3*x^2 + 6*g*h^2*x) + (log(g + h*x
)*(g^2*(3*a*b^2*d^3*h*q*r + 3*b^3*c*d^2*h*p*r) - g^3*(b^3*d^3*p*r + b^3*d^
3*q*r) - g*(3*a^2*b*d^3*h^2*q*r + 3*b^3*c^2*d*h^2*p*r) + b^3*c^3*h^3*p*r +
 a^3*d^3*h^3*q*r))/(3*a^3*c^3*h^7 + 3*b^3*d^3*g^6*h - 3*a^3*d^3*g^3*h^4 -
3*b^3*c^3*g^3*h^4 - 9*a^2*b*c^3*g*h^6 - 9*a^3*c^2*d*g*h^6 + 9*a*b^2*c^3*g^
2*h^5 - 9*a*b^2*d^3*g^5*h^2 + 9*a^2*b*d^3*g^4*h^3 + 9*a^3*c*d^2*g^2*h^5 -
9*b^3*c*d^2*g^5*h^2 + 9*b^3*c^2*d*g^4*h^3 + 27*a*b^2*c*d^2*g^4*h^3 - 27*a*
b^2*c^2*d*g^3*h^4 - 27*a^2*b*c*d^2*g^3*h^4 + 27*a^2*b*c^2*d*g^2*h^5) - (lo
g(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/3 + g/(3*h)))/(g + h*x)^4 - (b^3*p*r
*log(a + b*x))/(3*a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3)
 - (d^3*q*r*log(c + d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - ...
```

3.33. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4}\,dx$

3.34 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^5}\,dx$

3.34.1 Optimal result

Integrand size = 29, antiderivative size = 318

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx &= \frac{bpr}{12h(bg-ah)(g+hx)^3}+\frac{dqr}{12h(dg-ch)(g+hx)^3}\\ &+\frac{b^2pr}{8h(bg-ah)^2(g+hx)^2}+\frac{d^2qr}{8h(dg-ch)^2(g+hx)^2}\\ &+\frac{b^3pr}{4h(bg-ah)^3(g+hx)}+\frac{d^3qr}{4h(dg-ch)^3(g+hx)}\\ &+\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}+\frac{d^4qr\log(c+dx)}{4h(dg-ch)^4}\\ &-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4}\\ &-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}-\frac{d^4qr\log(g+hx)}{4h(dg-ch)^4}\end{aligned}$$

output
```
1/12*b*p*r/h/(-a*h+b*g)/(h*x+g)^3+1/12*d*q*r/h/(-c*h+d*g)/(h*x+g)^3+1/8*b^
2*p*r/h/(-a*h+b*g)^2/(h*x+g)^2+1/8*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)^2+1/4*b^
3*p*r/h/(-a*h+b*g)^3/(h*x+g)+1/4*d^3*q*r/h/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*p*
r*ln(b*x+a)/h/(-a*h+b*g)^4+1/4*d^4*q*r*ln(d*x+c)/h/(-c*h+d*g)^4-1/4*ln(e*(
f*(b*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^4-1/4*b^4*p*r*ln(h*x+g)/h/(-a*h+b*g)^4
-1/4*d^4*q*r*ln(h*x+g)/h/(-c*h+d*g)^4
```

3.34.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.51

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx$$
$$= \frac{-6\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{r(g+hx)(2(bg-ah)^3(dg-ch)^3(bdg(p+q)-h(bcp+adq))-(g+hx)((bg-ah)^2(dg-ch)^2(6abd^2ghq}{}}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

output
```
(-6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*(2*(b*g - a*h)^3*(
d*g - c*h)^3*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)^
2*(d*g - c*h)^2*(6*a*b*d^2*g*h*q - 3*a^2*d^2*h^2*q - 3*b^2*(-2*c*d*g*h*p +
 c^2*h^2*p + d^2*g^2*(p + q))) + 6*(g + h*x)*(-((b*g - a*h)*(-(d*g) + c*h)
*(3*a*b^2*d^3*g^2*h*q - 3*a^2*b*d^3*g*h^2*q + a^3*d^3*h^3*q - b^3*(-3*c*d^
2*g^2*h*p + 3*c^2*d*g*h^2*p - c^3*h^3*p + d^3*g^3*(p + q)))) - (g + h*x)*(
b^4*(d*g - c*h)^4*p*Log[a + b*x] + d^4*(b*g - a*h)^4*q*Log[c + d*x] - (-4*
a*b^3*d^4*g^3*h*q + 6*a^2*b^2*d^4*g^2*h^2*q - 4*a^3*b*d^4*g*h^3*q + a^4*d^
4*h^4*q + b^4*(-4*c*d^3*g^3*h*p + 6*c^2*d^2*g^2*h^2*p - 4*c^3*d*g*h^3*p +
c^4*h^4*p + d^4*g^4*(p + q)))*Log[g + h*x])))))/((b*g - a*h)^4*(d*g - c*h)
^4))/(24*h*(g + h*x)^4)
```

3.34.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2981, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx$$
$$\Big\downarrow \; 2981$$
$$\frac{bpr\int \frac{1}{(a+bx)(g+hx)^4}dx}{4h}+\frac{dqr\int \frac{1}{(c+dx)(g+hx)^4}dx}{4h}-\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4}$$
$$\Big\downarrow \; 54$$

$$\frac{bpr\int\left(\frac{b^4}{(bg-ah)^4(a+bx)}-\frac{hb^3}{(bg-ah)^4(g+hx)}-\frac{hb^2}{(bg-ah)^3(g+hx)^2}-\frac{hb}{(bg-ah)^2(g+hx)^3}-\frac{h}{(bg-ah)(g+hx)^4}\right)dx}{4h}+\frac{dqr\int\left(\frac{d^4}{(dg-ch)^4(c+dx)}-\frac{hd^3}{(dg-ch)^4(g+hx)}-\frac{hd^2}{(dg-ch)^3(g+hx)^2}-\frac{hd}{(dg-ch)^2(g+hx)^3}-\frac{h}{(dg-ch)(g+hx)^4}\right)dx}{4h}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{bpr\left(\frac{b^3\log(a+bx)}{(bg-ah)^4}-\frac{b^3\log(g+hx)}{(bg-ah)^4}+\frac{b^2}{(g+hx)(bg-ah)^3}+\frac{b}{2(g+hx)^2(bg-ah)^2}+\frac{1}{3(g+hx)^3(bg-ah)}\right)}{4h}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4}+\frac{dqr\left(\frac{d^3\log(c+dx)}{(dg-ch)^4}-\frac{d^3\log(g+hx)}{(dg-ch)^4}+\frac{d^2}{(g+hx)(dg-ch)^3}+\frac{d}{2(g+hx)^2(dg-ch)^2}+\frac{1}{3(g+hx)^3(dg-ch)}\right)}{4h}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

output
```
-1/4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)^4) + (b*p*r*(1/(3*(
b*g - a*h)*(g + h*x)^3) + b/(2*(b*g - a*h)^2*(g + h*x)^2) + b^2/((b*g - a*
h)^3*(g + h*x)) + (b^3*Log[a + b*x])/(b*g - a*h)^4 - (b^3*Log[g + h*x])/(b
*g - a*h)^4))/(4*h) + (d*q*r*(1/(3*(d*g - c*h)*(g + h*x)^3) + d/(2*(d*g -
c*h)^2*(g + h*x)^2) + d^2/((d*g - c*h)^3*(g + h*x)) + (d^3*Log[c + d*x])/(
d*g - c*h)^4 - (d^3*Log[g + h*x])/(d*g - c*h)^4))/(4*h)
```

3.34.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

3.34.4 Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^p (dx+c)^q)^r\right)}{(hx+g)^5} dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

3.34.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5} dx = \text{Timed out}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5} dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**5,x)
```

output
```
Timed out
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(296) = 592$.

Time = 0.23 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.44

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx$$
$$= \frac{\left(\left(\frac{6\,b^3\log(bx+a)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4} - \frac{6\,b^3\log(hx+g)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4} + \frac{}{b^3g^6-3\,ab^2g^5h+3\,a^2bg^4h^2-a^3g^3h^3+(b^3}\right.\right.}{}$$
$$- \frac{\log\left(((bx+a)^p(dx+c)^q f)^r\,e\right)}{4\,(hx+g)^4h}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="maxima"
)
```

output
```
1/24*((6*b^3*log(b*x + a)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^2*b^2*g^2*h^2 - 4
*a^3*b*g*h^3 + a^4*h^4) - 6*b^3*log(h*x + g)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*
a^2*b^2*g^2*h^2 - 4*a^3*b*g*h^3 + a^4*h^4) + (6*b^2*h^2*x^2 + 11*b^2*g^2 -
 7*a*b*g*h + 2*a^2*h^2 + 3*(5*b^2*g*h - a*b*h^2)*x)/(b^3*g^6 - 3*a*b^2*g^5
*h + 3*a^2*b*g^4*h^2 - a^3*g^3*h^3 + (b^3*g^3*h^3 - 3*a*b^2*g^2*h^4 + 3*a^
2*b*g*h^5 - a^3*h^6)*x^3 + 3*(b^3*g^4*h^2 - 3*a*b^2*g^3*h^3 + 3*a^2*b*g^2*
h^4 - a^3*g*h^5)*x^2 + 3*(b^3*g^5*h - 3*a*b^2*g^4*h^2 + 3*a^2*b*g^3*h^3 -
a^3*g^2*h^4)*x))*b*f*p + (6*d^3*log(d*x + c)/(d^4*g^4 - 4*c*d^3*g^3*h + 6*
c^2*d^2*g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) - 6*d^3*log(h*x + g)/(d^4*g^4 -
 4*c*d^3*g^3*h + 6*c^2*d^2*g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) + (6*d^2*h^2
*x^2 + 11*d^2*g^2 - 7*c*d*g*h + 2*c^2*h^2 + 3*(5*d^2*g*h - c*d*h^2)*x)/(d^
3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3 + (d^3*g^3*h^3 - 3*c
*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*x^3 + 3*(d^3*g^4*h^2 - 3*c*d^2*g^3
*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*x^2 + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 +
 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*x))*d*f*q)*r/(f*h) - 1/4*log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)/((h*x + g)^4*h)
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3943 vs. 2(296) = 592.

Time = 0.60 (sec) , antiderivative size = 3943, normalized size of antiderivative = 12.40

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}\, dx = \text{Too large to display}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="giac")
```

output
```
1/4*b^5*p*r*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2
*h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) + 1/4*d^5*q*r*log(abs(d*x + c))/(d^5*g
^4*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5)
- 1/4*p*r*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*
x + g^4*h) - 1/4*q*r*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 +
 4*g^3*h^2*x + g^4*h) - 1/4*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b
^4*c^2*d^2*g^2*h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4
*g^4*q*r - 4*a*b^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4
*g*h^3*q*r + a^4*d^4*h^4*q*r)*log(h*x + g)/(b^4*d^4*g^8*h - 4*b^4*c*d^3*g^
7*h^2 - 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h
^3 + 6*a^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^
4 - 24*a^2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*
a*b^3*c^3*d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5
+ a^4*d^4*g^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^
3*b*c^2*d^2*g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3
*b*c^3*d*g^2*h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d
*g*h^8 + a^4*c^4*h^9) + 1/24*(6*b^3*d^3*g^3*h^3*p*r*x^3 - 18*b^3*c*d^2*g^2
*h^4*p*r*x^3 + 18*b^3*c^2*d*g*h^5*p*r*x^3 - 6*b^3*c^3*h^6*p*r*x^3 + 6*b^3*
d^3*g^3*h^3*q*r*x^3 - 18*a*b^2*d^3*g^2*h^4*q*r*x^3 + 18*a^2*b*d^3*g*h^5*q*
r*x^3 - 6*a^3*d^3*h^6*q*r*x^3 + 21*b^3*d^3*g^4*h^2*p*r*x^2 - 63*b^3*c*d...
```

3.34.9 Mupad [B] (verification not implemented)

Time = 11.66 (sec) , antiderivative size = 2215, normalized size of antiderivative = 6.97

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}\, dx = \text{Too large to display}$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^5,x)
```

output
```
((11*b^3*d^3*g^5*p*r + 11*b^3*d^3*g^5*q*r - 11*b^3*c^3*g^2*h^3*p*r - 11*a^
3*d^3*g^2*h^3*q*r - 2*a^2*b*c^3*h^5*p*r - 2*a^3*c^2*d*h^5*q*r + 7*a*b^2*c^
3*g*h^4*p*r - 7*a*b^2*d^3*g^4*h*p*r - 33*a*b^2*d^3*g^4*h*q*r + 7*a^3*c*d^2
*g*h^4*q*r - 33*b^3*c*d^2*g^4*h*p*r - 7*b^3*c*d^2*g^4*h*q*r + 2*a^2*b*d^3*
g^3*h^2*p*r + 33*a^2*b*d^3*g^3*h^2*q*r + 33*b^3*c^2*d*g^3*h^2*p*r + 2*b^3*
c^2*d*g^3*h^2*q*r + 21*a*b^2*c*d^2*g^3*h^2*p*r - 21*a*b^2*c^2*d*g^2*h^3*p*
r - 6*a^2*b*c*d^2*g^2*h^3*p*r + 21*a*b^2*c*d^2*g^3*h^2*q*r - 6*a*b^2*c^2*d
*g^2*h^3*q*r - 21*a^2*b*c*d^2*g^2*h^3*q*r + 6*a^2*b*c^2*d*g*h^4*p*r + 6*a^
2*b*c^2*d*g*h^4*q*r)/(6*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3
*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 -
 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d
^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g
^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (x^2*(b^3*c^3*h
^5*p*r + a^3*d^3*h^5*q*r - b^3*d^3*g^3*h^2*p*r - b^3*d^3*g^3*h^2*q*r - 3*a
^2*b*d^3*g*h^4*q*r - 3*b^3*c^2*d*g*h^4*p*r + 3*a*b^2*d^3*g^2*h^3*q*r + 3*b
^3*c*d^2*g^2*h^3*p*r))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*
c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 -
3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^
2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^
3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x*(a*b^2*c^3*...
```

3.34. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^5}\,dx$

3.35 $\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

3.35.1 Optimal result

Integrand size = 31, antiderivative size = 2240

$$\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \text{Too large to display}$$

output
```
1/6*(-a*h+b*g)^3*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^3/d/h+1/4*(-a*h+b*g)^2*(-c
*h+d*g)^2*p*q*r^2*ln(b*x+a)/b^2/d^2/h+1/4*(-a*h+b*g)^2*(-c*h+d*g)^2*p*q*r^
2*ln(d*x+c)/b^2/d^2/h+1/6*(-a*h+b*g)*(-c*h+d*g)^3*p*q*r^2*ln(d*x+c)/b/d^3/
h+1/6*(-a*h+b*g)*(-c*h+d*g)*p*q*r^2*(h*x+g)^2/b/d/h+1/4*(h*x+g)^4*ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r)^2/h-1/2*(-c*h+d*g)^3*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d
^3-1/4*(-c*h+d*g)^2*p*q*r^2*(h*x+g)^2*ln(b*x+a)/d^2/h-1/6*(-c*h+d*g)*p*q*r
^2*(h*x+g)^3*ln(b*x+a)/d/h-1/2*(-a*h+b*g)^3*p*q*r^2*(d*x+c)*ln(d*x+c)/b^3/
d-1/4*(-a*h+b*g)^2*p*q*r^2*(h*x+g)^2*ln(d*x+c)/b^2/h-1/6*(-a*h+b*g)*p*q*r^
2*(h*x+g)^3*ln(d*x+c)/b/h-1/2*(-a*h+b*g)^4*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c
))*ln(d*x+c)/b^4/h-1/2*(-c*h+d*g)^4*p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b
*c))/d^4/h+5/12*(-a*h+b*g)^2*(-c*h+d*g)*p*q*r^2*x/b^2/d+5/12*(-a*h+b*g)*(-
c*h+d*g)^2*p*q*r^2*x/b/d^2+1/8*p*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+1/8*q*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln
(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+2*(-a*h+b*g)^3*p^2*r^2*x/b^3+2*
(-c*h+d*g)^3*q^2*r^2*x/d^3+1/32*h^3*p^2*r^2*(b*x+a)^4/b^4+1/32*h^3*q^2*r^2
*(d*x+c)^4/d^4+1/16*p*q*r^2*(h*x+g)^4/h+1/8*(-a*h+b*g)^4*p*q*r^2*ln(b*x+a)
/b^4/h-3/2*h*(-a*h+b*g)^2*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^4-2/3*h^2*(-a*h+b*
g)*p^2*r^2*(b*x+a)^3*ln(b*x+a)/b^4+1/8*(-c*h+d*g)^4*p*q*r^2*ln(d*x+c)/d^4/
h-3/2*h*(-c*h+d*g)^2*q^2*r^2*(d*x+c)^2*ln(d*x+c)/d^4-2/3*h^2*(-c*h+d*g)*q^
2*r^2*(d*x+c)^3*ln(d*x+c)/d^4+1/4*(-a*h+b*g)^2*p*r*(h*x+g)^2*(p*r*ln(b*...
```

3.35.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 1386, normalized size of antiderivative = 0.62

$$\int (g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \frac{72ad^4(-4b^3g^3+6ab^2g^2h-4a^2bgh^2+a^3h^3)\,p^2r^2\log^2(a+bx)+12pr\log(a+bx)\left(12b^4c(-4d^3g^3+6cd^2g^2\right.}{}$$

input
```
Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(72*a*d^4*(-4*b^3*g^3 + 6*a*b^2*g^2*h - 4*a^2*b*g*h^2 + a^3*h^3)*p^2*r^2*L
og[a + b*x]^2 + 12*p*r*Log[a + b*x]*(12*b^4*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h
- 4*c^2*d*g*h^2 + c^3*h^3)*q*r*Log[c + d*x] - 12*(4*a*b^3*d^4*g^3 - 6*a^2*
b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d^3*g^3 + 6*c*
d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] +
 a*d*((12*b^3*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*q + a
^3*d^3*h^3*(25*p + 3*q) - 4*a^2*b*d^2*h^2*(22*d*g*p + 4*d*g*q - c*h*q) + 6
*a*b^2*d*h*(-4*c*d*g*h*q + c^2*h^2*q + 6*d^2*g^2*(3*p + q)))*r + 12*d^3*(4
*b^3*g^3 - 6*a*b^2*g^2*h + 4*a^2*b*g*h^2 - a^3*h^3)*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r])) + b*(72*b^3*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2
 + c^3*h^3)*q^2*r^2*Log[c + d*x]^2 + 12*q*r*Log[c + d*x]*((12*a^3*c*d^3*h^
3*p + 6*a^2*b*c*d^2*h^2*(-8*d*g + c*h)*p + 4*a*b^2*d*(12*d^3*g^3 + 18*c*d^
2*g^2*h - 6*c^2*d*g*h^2 + c^3*h^3)*p + b^3*c*(-48*d^3*g^3*(p + q) + 36*c*d
^2*g^2*h*(p + 3*q) - 8*c^2*d*g*h^2*(2*p + 11*q) + c^3*h^3*(3*p + 25*q)))*r
 - 12*b^3*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(-60*a^3*d^3*h^3*p*(5*p + 3*q)*x +
 6*a^2*b*d^2*h^2*p*x*(-20*c*h*q + 16*d*g*(11*p + 8*q) + d*h*(13*p + 9*q)*x
) + b^3*x*(-60*c^3*h^3*q*(3*p + 5*q) + 6*c^2*d*h^2*q*(16*g*(8*p + 11*q) +
h*(9*p + 13*q)*x) - 4*c*d^2*h*q*(p + q)*(324*g^2 + 60*g*h*x + 7*h^2*x^2) +
 d^3*(p + q)^2*(576*g^3 + 216*g^2*h*x + 64*g*h^2*x^2 + 9*h^3*x^3)) - 4*...
```

3.35.3 Rubi [A] (verified)

Time = 2.91 (sec) , antiderivative size = 1802, normalized size of antiderivative = 0.80, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2984, 2993, 49, 2009, 2858, 27, 2772, 2009, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$\Big\downarrow \; 2984$$

$$-\frac{bpr\int \frac{(g+hx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}dx}{2h}-\frac{dqr\int \frac{(g+hx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{2h}+\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}$$

$$\Big\downarrow \; 2993$$

$$-\frac{bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{(g+hx)^4}{a+bx}dx\right)+qr\int \frac{(g+hx)^4\log(c+dx)}{a+bx}\cdots}{2h}$$
$$\frac{dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{(g+hx)^4}{c+dx}dx\right)+pr\int \frac{(g+hx)^4\log(a+bx)}{c+dx}d\cdots}{2h}$$
$$\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}$$

$$\Big\downarrow \; 49$$

$$-\frac{bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \left(\frac{(bg-ah)^4}{b^4(a+bx)}+\frac{h(bg-ah)^3}{b^4}+\frac{h(g+hx)(bg-\cdots}{b^3}\right.\right.}{2h}$$
$$\frac{dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \left(\frac{(dg-ch)^4}{d^4(c+dx)}+\frac{h(dg-ch)^3}{d^4}+\frac{h(g+hx)(dg-\cdots}{d^3}\right.\right.}{2h}$$
$$\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}$$

$$\Big\downarrow \; 2009$$

$$-\frac{bpr\left(qr\int \frac{(g+hx)^4\log(c+dx)}{a+bx}dx+pr\int \frac{(g+hx)^4\log(a+bx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^4\log(a+bx)}{b^5}+\frac{hx(bg-ah)^3}{b^4}+\frac{(g+hx)^2(bg-ah)^2}{2b^3}+\frac{(g+\cdots}{}\right.\right.\right.}{2h}$$
$$\frac{dqr\left(pr\int \frac{(g+hx)^4\log(a+bx)}{c+dx}dx+qr\int \frac{(g+hx)^4\log(c+dx)}{c+dx}dx-\left(\left(\frac{(dg-ch)^4\log(c+dx)}{d^5}+\frac{hx(dg-ch)^3}{d^4}+\frac{(g+hx)^2(dg-ch)^2}{2d^3}+\frac{(g+h\cdots}{}\right.\right.\right.}{2h}$$
$$\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}$$

$$\Big\downarrow \; 2858$$

3.35. $\int (g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

$$-\frac{bpr\left(\frac{pr\int \frac{\left(b\left(g-\frac{ah}{b}\right)+h(a+bx)\right)^4\log(a+bx)}{b^4(a+bx)}d(a+bx)}{b}+qr\int \frac{(g+hx)^4\log(c+dx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^4\log(a+bx)}{b^5}+\frac{hx(bg-ah)^3}{b^4}+\frac{(g+hx)^2(}{2b}\right.\right.\right.}{2h}$$

$$\frac{dqr\left(pr\int \frac{(g+hx)^4\log(a+bx)}{c+dx}dx+\frac{qr\int \frac{\left(d\left(g-\frac{ch}{d}\right)+h(c+dx)\right)^4\log(c+dx)}{d^4(c+dx)}d(c+dx)}{d}-\left(\left(\frac{(dg-ch)^4\log(c+dx)}{d^5}+\frac{hx(dg-ch)^3}{d^4}+\frac{(g+hx)^2(dg}{2d^3}\right.\right.\right.}{2h}$$

$$\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}$$

$\downarrow$ 27

$$-\frac{bpr\left(\frac{pr\int \frac{(bg-ah+h(a+bx))^4\log(a+bx)}{a+bx}d(a+bx)}{b^5}+qr\int \frac{(g+hx)^4\log(c+dx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^4\log(a+bx)}{b^5}+\frac{hx(bg-ah)^3}{b^4}+\frac{(g+hx)^2(bg-}{2b^3}\right.\right.\right.}{2h}$$

$$\frac{dqr\left(pr\int \frac{(g+hx)^4\log(a+bx)}{c+dx}dx+\frac{qr\int \frac{(dg-ch+h(c+dx))^4\log(c+dx)}{c+dx}d(c+dx)}{d^5}-\left(\left(\frac{(dg-ch)^4\log(c+dx)}{d^5}+\frac{hx(dg-ch)^3}{d^4}+\frac{(g+hx)^2(dg-ch}{2d^3}\right.\right.\right.}{2h}$$

$$\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}$$

$\downarrow$ 2772

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(g+hx)^4}{4h}-$$

$$\frac{bpr\left(-\left(\left(\frac{\log(a+bx)(bg-ah)^4}{b^5}+\frac{hx(bg-ah)^3}{b^4}+\frac{(g+hx)^2(bg-ah)^2}{2b^3}+\frac{(g+hx)^3(bg-ah)}{3b^2}+\frac{(g+hx)^4}{4b}\right)(pr\log(a+bx)+qr\log(c+\right.\right.}{}$$

$$\frac{dqr\left(-\left(\left(\frac{\log(c+dx)(dg-ch)^4}{d^5}+\frac{hx(dg-ch)^3}{d^4}+\frac{(g+hx)^2(dg-ch)^2}{2d^3}+\frac{(g+hx)^3(dg-ch)}{3d^2}+\frac{(g+hx)^4}{4d}\right)(pr\log(a+bx)+qr\log(c+\right.\right.}{}$$

$\downarrow$ 2009

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(g+hx)^4}{4h}-$$

$$\frac{dqr\left(\frac{qr\left(\frac{1}{2}\log^2(c+dx)(dg-ch)^4-4h(c+dx)(dg-ch)^3+4h(c+dx)\log(c+dx)(dg-ch)^3-\frac{3}{2}h^2(c+dx)^2(dg-ch)^2+3h^2(c+dx)^2\log(c+dx)(dg-ch)^2-\right.}{d^5}\right.}{}$$

$$\frac{bpr\left(\frac{pr\left(\frac{1}{2}\log^2(a+bx)(bg-ah)^4-4h(a+bx)(bg-ah)^3+4h(a+bx)\log(a+bx)(bg-ah)^3-\frac{3}{2}h^2(a+bx)^2(bg-ah)^2+3h^2(a+bx)^2\log(a+bx)(bg-ah)^2-\right.}{b^5}\right.}{}$$

$\downarrow$ 2865

3.35. $\int (g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(g+hx)^4}{4h} -$$
$$dqr\left(\frac{qr\left(\frac{1}{2}\log^2(c+dx)(dg-ch)^4-4h(c+dx)(dg-ch)^3+4h(c+dx)\log(c+dx)(dg-ch)^3-\frac{3}{2}h^2(c+dx)^2(dg-ch)^2+3h^2(c+dx)^2\log(c+dx)(dg-ch)^2-\right.}{d^5}\right.$$
$$bpr\left(\frac{pr\left(\frac{1}{2}\log^2(a+bx)(bg-ah)^4-4h(a+bx)(bg-ah)^3+4h(a+bx)\log(a+bx)(bg-ah)^3-\frac{3}{2}h^2(a+bx)^2(bg-ah)^2+3h^2(a+bx)^2\log(a+bx)(bg-ah)^2-\right.}{b^5}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(g+hx)^4}{4h} -$$
$$dqr\left(\frac{qr\left(\frac{1}{2}\log^2(c+dx)(dg-ch)^4-4h(c+dx)(dg-ch)^3+4h(c+dx)\log(c+dx)(dg-ch)^3-\frac{3}{2}h^2(c+dx)^2(dg-ch)^2+3h^2(c+dx)^2\log(c+dx)(dg-ch)^2-\right.}{d^5}\right.$$
$$bpr\left(\frac{pr\left(\frac{1}{2}\log^2(a+bx)(bg-ah)^4-4h(a+bx)(bg-ah)^3+4h(a+bx)\log(a+bx)(bg-ah)^3-\frac{3}{2}h^2(a+bx)^2(bg-ah)^2+3h^2(a+bx)^2\log(a+bx)(bg-ah)^2-\right.}{b^5}\right.$$

input
```
Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(4*h) - (d*q*r*((q*r*
(-4*h*(d*g - c*h)^3*(c + d*x) - (3*h^2*(d*g - c*h)^2*(c + d*x)^2)/2 - (4*h
^3*(d*g - c*h)*(c + d*x)^3)/9 - (h^4*(c + d*x)^4)/16 + 4*h*(d*g - c*h)^3*(
c + d*x)*Log[c + d*x] + 3*h^2*(d*g - c*h)^2*(c + d*x)^2*Log[c + d*x] + (4*
h^3*(d*g - c*h)*(c + d*x)^3*Log[c + d*x])/3 + (h^4*(c + d*x)^4*Log[c + d*x
])/4 + ((d*g - c*h)^4*Log[c + d*x]^2)/2))/d^5 - ((h*(d*g - c*h)^3*x)/d^4 +
 ((d*g - c*h)^2*(g + h*x)^2)/(2*d^3) + ((d*g - c*h)*(g + h*x)^3)/(3*d^2) +
 (g + h*x)^4/(4*d) + ((d*g - c*h)^4*Log[c + d*x])/d^5)*(p*r*Log[a + b*x] +
 q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + p*r*(-1/4*(h*(
b*g - a*h)^3*x)/(b^3*d) - (h*(b*g - a*h)^2*(d*g - c*h)*x)/(3*b^2*d^2) - (h
*(b*g - a*h)*(d*g - c*h)^2*x)/(2*b*d^3) - (h*(d*g - c*h)^3*x)/d^4 - ((b*g
- a*h)^2*(g + h*x)^2)/(8*b^2*d) - ((b*g - a*h)*(d*g - c*h)*(g + h*x)^2)/(6
*b*d^2) - ((d*g - c*h)^2*(g + h*x)^2)/(4*d^3) - ((b*g - a*h)*(g + h*x)^3)/
(12*b*d) - ((d*g - c*h)*(g + h*x)^3)/(9*d^2) - (g + h*x)^4/(16*d) - ((b*g
- a*h)^4*Log[a + b*x])/(4*b^4*d) - ((b*g - a*h)^3*(d*g - c*h)*Log[a + b*x]
)/(3*b^3*d^2) - ((b*g - a*h)^2*(d*g - c*h)^2*Log[a + b*x])/(2*b^2*d^3) + (
h*(d*g - c*h)^3*(a + b*x)*Log[a + b*x])/(b*d^4) + ((d*g - c*h)^2*(g + h*x)
^2*Log[a + b*x])/(2*d^3) + ((d*g - c*h)*(g + h*x)^3*Log[a + b*x])/(3*d^2)
+ ((g + h*x)^4*Log[a + b*x])/(4*d) + ((d*g - c*h)^4*Log[a + b*x]*Log[(b*(c
 + d*x))/(b*c - a*d)])/d^5 + ((d*g - c*h)^4*PolyLog[2, -((d*(a + b*x))/...
```

3.35. $\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.35.4 Maple [F]

$$\int (hx+g)^3 \ln\left(e\left(f(bx+a)^p (dx+c)^q\right)^r\right)^2 dx$$

input
```
int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.35.5 Fricas [F]

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (hx+g)^3 \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

input
```
integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="frica
s")
```

output
```
integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2, x)
```

3.35.6 Sympy [F]

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (g+hx)^3 \log\left(e\left(f(a+bx)^p (c+dx)^q\right)^r\right)^2 dx$$

input
```
integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral((g + h*x)**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1799, normalized size of antiderivative = 0.80

$$\int (g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxim
a")
```

output
```
1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x
+ c)^q*f)^r*e)^2 + 1/24*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a
^3*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c
^2*d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*
b^3*d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q)
- c*d^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p -
 (6*d^3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12
*(6*a*b^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3
*f*g^3*(p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)
*x)/(b^3*d^3))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*(12*
a^3*c*d^3*f^2*h^3*p*q - 6*(8*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q)*a^
2*b + 4*(18*c*d^3*f^2*g^2*h*p*q - 6*c^2*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*
p*q)*a*b^2 - (48*(p*q + q^2)*c*d^3*f^2*g^3 - 36*(p*q + 3*q^2)*c^2*d^2*f^2*
g^2*h + 8*(2*p*q + 11*q^2)*c^3*d*f^2*g*h^2 - (3*p*q + 25*q^2)*c^4*f^2*h^3)
*b^3)*log(d*x + c)/(b^3*d^4) - 144*(4*a*b^3*d^4*f^2*g^3*p*q - 6*a^2*b^2*d^
4*f^2*g^2*h*p*q + 4*a^3*b*d^4*f^2*g*h^2*p*q - a^4*d^4*f^2*h^3*p*q - (4*c*d
^3*f^2*g^3*p*q - 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2
*h^3*p*q)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(
b*d*x + a*d)/(b*c - a*d)))/(b^4*d^4) + (9*(p^2 + 2*p*q + q^2)*b^4*d^4*f^2*
h^3*x^4 - 144*(4*c*d^3*f^2*g^3*p*q - 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*...
```

3.35.8 Giac [F]

$$\int (g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx=\int (hx+g)^3\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx\right.$$

input
```
integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac"
)
```

output
```
sage0*x
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2\left(g+h\,x\right)^3\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3, x)
```

3.36 $\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

3.36.1 Optimal result

Integrand size = 31, antiderivative size = 1645

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Too large to display}$$

output
```
1/3*(-a*h+b*g)^2*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^2/d/h+1/3*(-a*h+b*g)*(-c*h
+d*g)^2*p*q*r^2*ln(d*x+c)/b/d^2/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)^2/h-2/3*(-c*h+d*g)^2*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d^2-1/3*(-c*h+d*g)*
p*q*r^2*(h*x+g)^2*ln(b*x+a)/d/h-2/3*(-a*h+b*g)^2*p*q*r^2*(d*x+c)*ln(d*x+c)
/b^2/d-1/3*(-a*h+b*g)*p*q*r^2*(h*x+g)^2*ln(d*x+c)/b/h-2/3*(-a*h+b*g)^3*p*q
*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^3/h-2/3*(-c*h+d*g)^3*p*q*r^2*ln
(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/d^3/h+2/3*(-a*h+b*g)*(-c*h+d*g)*p*q*r^2*x
/b/d+2/9*p*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r))/h+2/9*q*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a
)^p*(d*x+c)^q)^r))/h+2*(-a*h+b*g)^2*p^2*r^2*x/b^2+2*(-c*h+d*g)^2*q^2*r^2*x
/d^2+2/27*h^2*p^2*r^2*(b*x+a)^3/b^3+2/27*h^2*q^2*r^2*(d*x+c)^3/d^3+4/27*p*
q*r^2*(h*x+g)^3/h+2/9*(-a*h+b*g)^3*p*q*r^2*ln(b*x+a)/b^3/h+2/9*(-c*h+d*g)^
3*p*q*r^2*ln(d*x+c)/d^3/h-2*(-a*h+b*g)^2*p^2*r^2*(b*x+a)*ln(b*x+a)/b^3-2/9
*h^2*p^2*r^2*(b*x+a)^3*ln(b*x+a)/b^3-2/9*p*q*r^2*(h*x+g)^3*ln(b*x+a)/h-1/3
*(-a*h+b*g)^3*p^2*r^2*ln(b*x+a)^2/b^3/h-2*(-c*h+d*g)^2*q^2*r^2*(d*x+c)*ln(
d*x+c)/d^3-2/9*h^2*q^2*r^2*(d*x+c)^3*ln(d*x+c)/d^3-2/9*p*q*r^2*(h*x+g)^3*l
n(d*x+c)/h-1/3*(-c*h+d*g)^3*q^2*r^2*ln(d*x+c)^2/d^3/h+2/3*(-a*h+b*g)^2*p*r
*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2+2/3*(
-c*h+d*g)^2*q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r))/d^2+8/9*(-a*h+b*g)^2*p*q*r^2*x/b^2+8/9*(-c*h+d*g)^2*p*q*r^2*x/d^2...
```

3.36.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 899, normalized size of antiderivative = 0.55

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{-18ad^3(3b^2g^2-3abgh+a^2h^2)\,p^2r^2\log^2(a+bx)-6pr\log(a+bx)\left(6b^3c(3d^2g^2-3cdgh+c^2h^2)\,qr\log(c\right.}{}$$

input
```
Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(-18*a*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*p^2*r^2*Log[a + b*x]^2 - 6*p*
r*Log[a + b*x]*(6*b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*q*r*Log[c + d*x]
 - 6*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 -
3*c*d*g*h + c^2*h^2))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*((6*b^2*(3*
d^2*g^2 - 3*c*d*g*h + c^2*h^2)*q + a^2*d^2*h^2*(11*p + 2*q) - 3*a*b*d*h*(-
(c*h*q) + 3*d*g*(3*p + q)))*r - 6*d^2*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) + b*(-18*b^2*c*(3*d^2*g^2 - 3*c*d*g*h
 + c^2*h^2)*q^2*r^2*Log[c + d*x]^2 - 6*q*r*Log[c + d*x]*((6*a^2*c*d^2*h^2*
p - 3*a*b*d*(6*d^2*g^2 + 6*c*d*g*h - c^2*h^2)*p + b^2*c*(18*d^2*g^2*(p + q
) - 9*c*d*g*h*(p + 3*q) + c^2*h^2*(2*p + 11*q)))*r - 6*b^2*c*(3*d^2*g^2 -
3*c*d*g*h + c^2*h^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(6*a^2
*d^2*h^2*p*(11*p + 8*q)*x + b^2*x*(6*c^2*h^2*q*(8*p + 11*q) - 3*c*d*h*q*(p
 + q)*(54*g + 5*h*x) + d^2*(p + q)^2*(108*g^2 + 27*g*h*x + 4*h^2*x^2)) - 3
*a*b*p*(-12*c^2*h^2*q - 12*c*d*h*q*(-3*g + h*x) + d^2*(-36*g^2*q + 54*g*h*
(p + q)*x + 5*h^2*(p + q)*x^2))) - 6*r*(6*a^2*d^2*h^2*p*x + 3*a*b*d^2*p*(6
*g^2 - 6*g*h*x - h^2*x^2) + b^2*x*(6*c^2*h^2*q - 3*c*d*h*q*(6*g + h*x) + d
^2*(p + q)*(18*g^2 + 9*g*h*x + 2*h^2*x^2)))*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r] + 18*b^2*d^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]^2)) + 36*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) +
b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*p*q*r^2*PolyLog[2, (d*(a + b*x))...
```

3.36.3 Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 1286, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2984, 2993, 49, 2009, 2858, 27, 2772, 2009, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$\Big\downarrow \; 2984$$

$$-\frac{2bpr \int \frac{(g+hx)^3 \log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}dx}{3h} - \frac{2dqr \int \frac{(g+hx)^3 \log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3h} + \frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}$$

$$\Big\downarrow \; 2993$$

$$-\frac{2bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{(g+hx)^3}{a+bx}dx\right)+qr\int \frac{(g+hx)^3\log(c+dx}{a+bx}}{3h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{(g+hx)^3}{c+dx}dx\right)+pr\int \frac{(g+hx)^3\log(a+bx)}{c+dx}}{3h}$$
$$\frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}$$

$$\Big\downarrow \; 49$$

$$-\frac{2bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \left(\frac{(bg-ah)^3}{b^3(a+bx)}+\frac{h(bg-ah)^2}{b^3}+\frac{h(g+hx)(b}{b^2}\right.\right.\right.}{3h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \left(\frac{(dg-ch)^3}{d^3(c+dx)}+\frac{h(dg-ch)^2}{d^3}+\frac{h(g+hx)(dg-}{d^2}\right.\right.\right.}{3h}$$
$$\frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2bpr\left(qr\int \frac{(g+hx)^3\log(c+dx)}{a+bx}dx+pr\int \frac{(g+hx)^3\log(a+bx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^3\log(a+bx)}{b^4}+\frac{hx(bg-ah)^2}{b^3}+\frac{(g+hx)^2(bg-ah)}{2b^2}+\frac{(g+}{}\right.\right.\right.}{3h}$$
$$\frac{2dqr\left(pr\int \frac{(g+hx)^3\log(a+bx)}{c+dx}dx+qr\int \frac{(g+hx)^3\log(c+dx)}{c+dx}dx-\left(\left(\frac{(dg-ch)^3\log(c+dx)}{d^4}+\frac{hx(dg-ch)^2}{d^3}+\frac{(g+hx)^2(dg-ch)}{2d^2}+\frac{(g+h}{3d}\right.\right.\right.}{3h}$$
$$\frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}$$

$$\Big\downarrow \; 2858$$

3.36. $\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

$$-\frac{2bpr\left(\frac{pr\int\frac{\left(b\left(g-\frac{ah}{b}\right)+h(a+bx)\right)^3\log(a+bx)}{b^3(a+bx)}d(a+bx)}{b}+qr\int\frac{(g+hx)^3\log(c+dx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^3\log(a+bx)}{b^4}+\frac{hx(bg-ah)^2}{b^3}+\frac{(g+hx)^2 \cdots}{2 \cdots}\right.\right.}{3h}$$

$$\frac{2dqr\left(pr\int\frac{(g+hx)^3\log(a+bx)}{c+dx}dx+\frac{qr\int\frac{\left(d\left(g-\frac{ch}{d}\right)+h(c+dx)\right)^3\log(c+dx)}{d^3(c+dx)}d(c+dx)}{d}-\left(\left(\frac{(dg-ch)^3\log(c+dx)}{d^4}+\frac{hx(dg-ch)^2}{d^3}+\frac{(g+hx)^2 \cdots}{2d^2}\right.\right.}{3h}$$

$$\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

$\downarrow$ 27

$$-\frac{2bpr\left(\frac{pr\int\frac{(bg-ah+h(a+bx))^3\log(a+bx)}{a+bx}d(a+bx)}{b^4}+qr\int\frac{(g+hx)^3\log(c+dx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^3\log(a+bx)}{b^4}+\frac{hx(bg-ah)^2}{b^3}+\frac{(g+hx)^2(bg \cdots}{2b^2}\right.\right.}{3h}$$

$$\frac{2dqr\left(pr\int\frac{(g+hx)^3\log(a+bx)}{c+dx}dx+\frac{qr\int\frac{(dg-ch+h(c+dx))^3\log(c+dx)}{c+dx}d(c+dx)}{d^4}-\left(\left(\frac{(dg-ch)^3\log(c+dx)}{d^4}+\frac{hx(dg-ch)^2}{d^3}+\frac{(g+hx)^2(dg- \cdots}{2d^2}\right.\right.}{3h}$$

$$\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

$\downarrow$ 2772

$$-\frac{2bpr\left(\frac{pr\left(-\int\left(\frac{1}{3}(a+bx)^2h^3+\frac{3}{2}(bg-ah)(a+bx)h^2+3(bg-ah)^2h+\frac{(bg-ah)^3\log(a+bx)}{a+bx}\right)d(a+bx)+\frac{3}{2}h^2(a+bx)^2(bg-ah)\log(a+bx)+(bg-ah)^3\log \cdots}{b^4}\right.}{}$$

$$\frac{2dqr\left(pr\int\frac{(g+hx)^3\log(a+bx)}{c+dx}dx+\frac{qr\left(-\int\left(\frac{1}{3}(c+dx)^2h^3+\frac{3}{2}(dg-ch)(c+dx)h^2+3(dg-ch)^2h+\frac{(dg-ch)^3\log(c+dx)}{c+dx}\right)d(c+dx)+\frac{3}{2}h^2(c+dx)^2(\cdots}{d^4}\right.}{}$$

$$\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

$\downarrow$ 2009

$$-\frac{2bpr\left(qr\int\frac{(g+hx)^3\log(c+dx)}{a+bx}dx+\frac{pr\left(-\frac{3}{4}h^2(a+bx)^2(bg-ah)+\frac{3}{2}h^2(a+bx)^2(bg-ah)\log(a+bx)-3h(a+bx)(bg-ah)^2+\frac{1}{2}(bg-ah)^3\log^2(a+ \cdots\right.}{b^4}\right.}{}$$

$$\frac{2dqr\left(pr\int\frac{(g+hx)^3\log(a+bx)}{c+dx}dx-\left(\frac{(dg-ch)^3\log(c+dx)}{d^4}+\frac{hx(dg-ch)^2}{d^3}+\frac{(g+hx)^2(dg-ch)}{2d^2}+\frac{(g+hx)^3}{3d}\right)\left(-\log\left(e\left(f(a+bx)^p(c \cdots\right.\right.\right.}{}$$

$$\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

$\downarrow$ 2865

3.36. $\int(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(g+hx)^3}{3h}-$$

$$2dqr\left(\frac{qr\left(\frac{1}{2}\log^2(c+dx)(dg-ch)^3-3h(c+dx)(dg-ch)^2+3h(c+dx)\log(c+dx)(dg-ch)^2-\frac{3}{4}h^2(c+dx)^2(dg-ch)+\frac{3}{2}h^2(c+dx)^2\log(c+dx)(dg-ch)-\right.}{d^4}\right.$$

$$2bpr\left(\frac{pr\left(\frac{1}{2}\log^2(a+bx)(bg-ah)^3-3h(a+bx)(bg-ah)^2+3h(a+bx)\log(a+bx)(bg-ah)^2-\frac{3}{4}h^2(a+bx)^2(bg-ah)+\frac{3}{2}h^2(a+bx)^2\log(a+bx)(bg-ah)-\right.}{b^4}\right.$$

$\downarrow$ 2009

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)(g+hx)^3}{3h}-$$

$$2dqr\left(\frac{qr\left(\frac{1}{2}\log^2(c+dx)(dg-ch)^3-3h(c+dx)(dg-ch)^2+3h(c+dx)\log(c+dx)(dg-ch)^2-\frac{3}{4}h^2(c+dx)^2(dg-ch)+\frac{3}{2}h^2(c+dx)^2\log(c+dx)(dg-ch)-\right.}{d^4}\right.$$

$$2bpr\left(\frac{pr\left(\frac{1}{2}\log^2(a+bx)(bg-ah)^3-3h(a+bx)(bg-ah)^2+3h(a+bx)\log(a+bx)(bg-ah)^2-\frac{3}{4}h^2(a+bx)^2(bg-ah)+\frac{3}{2}h^2(a+bx)^2\log(a+bx)(bg-ah)-\right.}{b^4}\right.$$

input
```
Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
((g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) - (2*d*q*r*((q*
r*(-3*h*(d*g - c*h)^2*(c + d*x) - (3*h^2*(d*g - c*h)*(c + d*x)^2)/4 - (h^3
*(c + d*x)^3)/9 + 3*h*(d*g - c*h)^2*(c + d*x)*Log[c + d*x] + (3*h^2*(d*g -
 c*h)*(c + d*x)^2*Log[c + d*x])/2 + (h^3*(c + d*x)^3*Log[c + d*x])/3 + ((d
*g - c*h)^3*Log[c + d*x]^2)/2))/d^4 - ((h*(d*g - c*h)^2*x)/d^3 + ((d*g - c
*h)*(g + h*x)^2)/(2*d^2) + (g + h*x)^3/(3*d) + ((d*g - c*h)^3*Log[c + d*x]
)/d^4)*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]) + p*r*(-1/3*(h*(b*g - a*h)^2*x)/(b^2*d) - (h*(b*g - a*h)*(d*g -
c*h)*x)/(2*b*d^2) - (h*(d*g - c*h)^2*x)/d^3 - ((b*g - a*h)*(g + h*x)^2)/(6
*b*d) - ((d*g - c*h)*(g + h*x)^2)/(4*d^2) - (g + h*x)^3/(9*d) - ((b*g - a*
h)^3*Log[a + b*x])/(3*b^3*d) - ((b*g - a*h)^2*(d*g - c*h)*Log[a + b*x])/(2
*b^2*d^2) + (h*(d*g - c*h)^2*(a + b*x)*Log[a + b*x])/(b*d^3) + ((d*g - c*h
)*(g + h*x)^2*Log[a + b*x])/(2*d^2) + ((g + h*x)^3*Log[a + b*x])/(3*d) + (
(d*g - c*h)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d^4 + ((d*g - c
*h)^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/d^4)))/(3*h) - (2*b*p*r*((
p*r*(-3*h*(b*g - a*h)^2*(a + b*x) - (3*h^2*(b*g - a*h)*(a + b*x)^2)/4 - (h
^3*(a + b*x)^3)/9 + 3*h*(b*g - a*h)^2*(a + b*x)*Log[a + b*x] + (3*h^2*(b*g
 - a*h)*(a + b*x)^2*Log[a + b*x])/2 + (h^3*(a + b*x)^3*Log[a + b*x])/3 + (
(b*g - a*h)^3*Log[a + b*x]^2)/2))/b^4 - ((h*(b*g - a*h)^2*x)/b^3 + ((b*g -
 a*h)*(g + h*x)^2)/(2*b^2) + (g + h*x)^3/(3*b) + ((b*g - a*h)^3*Log[a +...
```

3.36. $\int(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.36.4 Maple [F]

$$\int (hx+g)^2 \ln\left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2dx$$

input
```
int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.36.5 Fricas [F]

$$\int (g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (hx+g)^2\log\left(\left(((bx+a)^p(dx+c)^qf)^r\,e\right)^2\,dx\right.$$

input
```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="frica
s")
```

output
```
integral((h^2*x^2 + 2*g*h*x + g^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
 x)
```

3.36.6 Sympy [F]

$$\int (g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (g+hx)^2\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2dx$$

input
```
integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral((g + h*x)**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1123, normalized size of antiderivative = 0.68

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxim
a")
```

output
```
1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
 + 1/9*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)
/b^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^
3 - (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p +
q) - c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d
^2*f*g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))*log(((b
*x + a)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*(6*a^2*c*d^2*f^2*h^2*p*q - 3
*(6*c*d^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q)*a*b + (18*(p*q + q^2)*c*d^2*f^2
*g^2 - 9*(p*q + 3*q^2)*c^2*d*f^2*g*h + (2*p*q + 11*q^2)*c^3*f^2*h^2)*b^2)*
log(d*x + c)/(b^2*d^3) + 36*(3*a*b^2*d^3*f^2*g^2*p*q - 3*a^2*b*d^3*f^2*g*h
*p*q + a^3*d^3*f^2*h^2*p*q - (3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q +
c^3*f^2*h^2*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + d
ilog(-(b*d*x + a*d)/(b*c - a*d)))/(b^3*d^3) - (4*(p^2 + 2*p*q + q^2)*b^3*d
^3*f^2*h^2*x^3 - 36*(3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h
^2*p*q)*b^3*log(b*x + a)*log(d*x + c) - 18*(3*c*d^2*f^2*g^2*q^2 - 3*c^2*d*
f^2*g*h*q^2 + c^3*f^2*h^2*q^2)*b^3*log(d*x + c)^2 - 3*(5*(p^2 + p*q)*a*b^2
*d^3*f^2*h^2 - (9*(p^2 + 2*p*q + q^2)*d^3*f^2*g*h - 5*(p*q + q^2)*c*d^2*f^
2*h^2)*b^3)*x^2 - 18*(3*a*b^2*d^3*f^2*g^2*p^2 - 3*a^2*b*d^3*f^2*g*h*p^2 +
a^3*d^3*f^2*h^2*p^2)*log(b*x + a)^2 + 6*((11*p^2 + 8*p*q)*a^2*b*d^3*f^2*h^
2 + 3*(2*c*d^2*f^2*h^2*p*q - 9*(p^2 + p*q)*d^3*f^2*g*h)*a*b^2 + (18*(p^...
```

3.36.8 Giac [F(-1)]

Timed out.

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac"
)
```

output
```
Timed out
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \,dx = \int \ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2 \left(g+h\,x\right)^2 \,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2, x)
```

3.37 $\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

3.37.1 Optimal result

Integrand size = 29, antiderivative size = 1063

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{3(bg-ah)pqr^2x}{2b}+\frac{3(dg-ch)pqr^2x}{2d}+\frac{pqr^2(g+hx)^2}{2h}+\frac{p^2r^2(4bg-3ah+bhx)^2}{4b^2h}$$
$$+\frac{q^2r^2(4dg-3ch+dhx)^2}{4d^2h}+\frac{(bg-ah)^2pqr^2\log(a+bx)}{2b^2h}$$
$$-\frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2}-\frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd}$$
$$-\frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2}-\frac{pqr^2(g+hx)^2\log(a+bx)}{2h}-\frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{2b^2h}$$
$$+\frac{(dg-ch)^2pqr^2\log(c+dx)}{2d^2h}-\frac{(bg-ah)pqr^2(c+dx)\log(c+dx)}{bd}$$
$$-\frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2}-\frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2}$$
$$-\frac{pqr^2(g+hx)^2\log(c+dx)}{2h}-\frac{(bg-ah)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2h}$$
$$-\frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{2d^2h}-\frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h}$$
$$+\frac{(bg-ah)prx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{b}$$
$$+\frac{(dg-ch)qrx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{d}$$
$$+\frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h}$$
$$+\frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h}$$
$$+\frac{(bg-ah)^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{b^2h}$$
$$+\frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{d^2h}$$
$$+\frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h}$$
$$-\frac{(dg-ch)^2pqr^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{d^2h}-\frac{(bg-ah)^2pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2h}$$

output
```
(-a*h+b*g)*p*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r))/b+(-c*h+d*g)*q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r))/d-(-a*h+b*g)^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b^2/h+(
-a*h+b*g)^2*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(
d*x+c)^q)^r))/b^2/h+(-c*h+d*g)^2*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c
)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2/h-(-c*h+d*g)^2*p*q*r^2*polylog(2,-d
*(b*x+a)/(-a*d+b*c))/d^2/h+1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2
/h+1/2*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r))/h+1/2*q*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r))/h+1/2*p*q*r^2*(h*x+g)^2/h+1/4*p^2*r^2*(b*h*x-3*a*h+4*b*g)
^2/b^2/h+1/4*q^2*r^2*(d*h*x-3*c*h+4*d*g)^2/d^2/h-(-c*h+d*g)*p*q*r^2*(b*x+a
)*ln(b*x+a)/b/d-(-a*h+b*g)*p*q*r^2*(d*x+c)*ln(d*x+c)/b/d-(-a*h+b*g)^2*p*q*
r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2/h-(-c*h+d*g)^2*p*q*r^2*ln(b*x+
a)*ln(b*(d*x+c)/(-a*d+b*c))/d^2/h-1/2*(-a*h+b*g)^2*p^2*r^2*ln(b*x+a)^2/b^2
/h-2*(-c*h+d*g)*q^2*r^2*(d*x+c)*ln(d*x+c)/d^2-1/2*h*q^2*r^2*(d*x+c)^2*ln(d
*x+c)/d^2-1/2*p*q*r^2*(h*x+g)^2*ln(d*x+c)/h-1/2*(-c*h+d*g)^2*q^2*r^2*ln(d*
x+c)^2/d^2/h+3/2*(-a*h+b*g)*p*q*r^2*x/b+3/2*(-c*h+d*g)*p*q*r^2*x/d-2*(-a*h
+b*g)*p^2*r^2*(b*x+a)*ln(b*x+a)/b^2-1/2*h*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^2-
1/2*p*q*r^2*(h*x+g)^2*ln(b*x+a)/h+1/2*(-a*h+b*g)^2*p*q*r^2*ln(b*x+a)/b^2/h
+1/2*(-c*h+d*g)^2*p*q*r^2*ln(d*x+c)/d^2/h
```

3.37.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.45

$$\int (g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$

$$= \frac{2ad^2(-2bg+ah)p^2r^2\log^2(a+bx)+2pr\log(a+bx)\left(2b^2c(-2dg+ch)qr\log(c+dx)-2(bc-ad)(-2bd\right.}{}$$

input
```
Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(2*a*d^2*(-2*b*g + a*h)*p^2*r^2*Log[a + b*x]^2 + 2*p*r*Log[a + b*x]*(2*b^2
*c*(-2*d*g + c*h)*q*r*Log[c + d*x] - 2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d
*h)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b*(-2*d*g + c*h)*q*r + a*d
*h*(3*p + q)*r + (4*b*d*g - 2*a*d*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
) + b*(2*b*c*(-2*d*g + c*h)*q^2*r^2*Log[c + d*x]^2 + 2*q*r*Log[c + d*x]*(2
*a*d*(2*d*g + c*h)*p*r + b*c*(-4*d*g*(p + q) + c*h*(p + 3*q))*r - 2*b*c*(-
2*d*g + c*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(-2*a*p*(-4*d*
g*q + 2*c*h*q + 3*d*h*(p + q)*x) + b*(p + q)*x*(-6*c*h*q + d*(p + q)*(8*g
+ h*x))) - 2*r*(2*a*d*p*(2*g - h*x) + b*x*(-2*c*h*q + d*(p + q)*(4*g + h*x
)))*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*x*(2*g + h*x)*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^2)) - 4*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*p
*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b^2*d^2)
```

3.37.3 Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 828, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2984, 2993, 49, 2009, 2858, 27, 2772, 2009, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$$

$$\Big\downarrow \text{2984}$$

$$-\frac{bpr\int \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}dx}{h}-\frac{dqr\int \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}dx}{h}+\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$$\Big\downarrow \text{2993}$$

$$-\frac{bpr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{(g+hx)^2}{a+bx}dx\right)+qr\int \frac{(g+hx)^2\log(c+dx)}{a+bx}\right.}{h}$$
$$\frac{dqr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{(g+hx)^2}{c+dx}dx\right)+pr\int \frac{(g+hx)^2\log(a+bx)}{c+dx}d\right.}{h}$$
$$\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$$\Big\downarrow \text{49}$$

$$-\frac{bpr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int\left(\frac{(bg-ah)^2}{b^2(a+bx)}+\frac{h(bg-ah)}{b^2}+\frac{h(g+hx)}{b}\right)dx\right.}{h}$$
$$\frac{dqr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int\left(\frac{(dg-ch)^2}{d^2(c+dx)}+\frac{h(dg-ch)}{d^2}+\frac{h(g+hx)}{d}\right)dx\right)}{h}$$
$$\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$\downarrow$ 2009

$$-\frac{bpr\left(qr\int\frac{(g+hx)^2\log(c+dx)}{a+bx}dx+pr\int\frac{(g+hx)^2\log(a+bx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^2\log(a+bx)}{b^3}+\frac{hx(bg-ah)}{b^2}+\frac{(g+hx)^2}{2b}\right)\left(-\log\left(e\left(f(a\right.\right.\right.\right.}{h}$$
$$\frac{dqr\left(pr\int\frac{(g+hx)^2\log(a+bx)}{c+dx}dx+qr\int\frac{(g+hx)^2\log(c+dx)}{c+dx}dx-\left(\left(\frac{(dg-ch)^2\log(c+dx)}{d^3}+\frac{hx(dg-ch)}{d^2}+\frac{(g+hx)^2}{2d}\right)\left(-\log\left(e\left(f(a\right.\right.\right.\right.}{h}$$
$$\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$\downarrow$ 2858

$$-\frac{bpr\left(\frac{pr\int\frac{\left(b\left(g-\frac{ah}{b}\right)+h(a+bx)\right)^2\log(a+bx)}{b^2(a+bx)}d(a+bx)}{b}+qr\int\frac{(g+hx)^2\log(c+dx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^2\log(a+bx)}{b^3}+\frac{hx(bg-ah)}{b^2}+\frac{(g+hx)^2}{2b}\right)\right.\right.}{h}$$
$$\frac{dqr\left(pr\int\frac{(g+hx)^2\log(a+bx)}{c+dx}dx+\frac{qr\int\frac{\left(d\left(g-\frac{ch}{d}\right)+h(c+dx)\right)^2\log(c+dx)}{d^2(c+dx)}d(c+dx)}{d}-\left(\left(\frac{(dg-ch)^2\log(c+dx)}{d^3}+\frac{hx(dg-ch)}{d^2}+\frac{(g+hx)^2}{2d}\right)\right.\right.}{h}$$
$$\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$\downarrow$ 27

$$-\frac{bpr\left(\frac{pr\int\frac{(bg-ah+h(a+bx))^2\log(a+bx)}{a+bx}d(a+bx)}{b^3}+qr\int\frac{(g+hx)^2\log(c+dx)}{a+bx}dx-\left(\left(\frac{(bg-ah)^2\log(a+bx)}{b^3}+\frac{hx(bg-ah)}{b^2}+\frac{(g+hx)^2}{2b}\right)\left(-\right.\right.\right.}{h}$$
$$\frac{dqr\left(pr\int\frac{(g+hx)^2\log(a+bx)}{c+dx}dx+\frac{qr\int\frac{(dg-ch+h(c+dx))^2\log(c+dx)}{c+dx}d(c+dx)}{d^3}-\left(\left(\frac{(dg-ch)^2\log(c+dx)}{d^3}+\frac{hx(dg-ch)}{d^2}+\frac{(g+hx)^2}{2d}\right)\left(-\log\right.\right.\right.}{h}$$
$$\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

$\downarrow$ 2772

3.37. $\int(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

$$-\frac{bpr\left(\frac{pr\left(-\int\left(\frac{\log(a+bx)(bg-ah)^2}{a+bx}+\frac{1}{2}h(4(bg-ah)+h(a+bx))\right)d(a+bx)+(bg-ah)^2\log^2(a+bx)+2h(a+bx)(bg-ah)\log(a+bx)+\frac{1}{2}h^2(a+bx)^2\log}{b^3}\right.}{dqr\left(pr\int\frac{(g+hx)^2\log(a+bx)}{c+dx}dx+\frac{qr\left(-\int\left(\frac{\log(c+dx)(dg-ch)^2}{c+dx}+\frac{1}{2}h(4(dg-ch)+h(c+dx))\right)d(c+dx)+(dg-ch)^2\log^2(c+dx)+2h(c+dx)(dg-c}{d^3}\right.}$$

$$\frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h}$$

$\Big\downarrow$ 2009

$$-\frac{bpr\left(qr\int\frac{(g+hx)^2\log(c+dx)}{a+bx}dx+\frac{pr\left(-\frac{1}{4}(4(bg-ah)+h(a+bx))^2+\frac{1}{2}(bg-ah)^2\log^2(a+bx)+2h(a+bx)(bg-ah)\log(a+bx)+\frac{1}{2}h^2(a+bx)^2\log(\right.}{b^3}\right.}{dqr\left(pr\int\frac{(g+hx)^2\log(a+bx)}{c+dx}dx-\left(\frac{(dg-ch)^2\log(c+dx)}{d^3}+\frac{hx(dg-ch)}{d^2}+\frac{(g+hx)^2}{2d}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(\right.\right.}$$

$$\frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h}$$

$\Big\downarrow$ 2865

$$-\frac{bpr\left(qr\int\left(\frac{\log(c+dx)(bg-ah)^2}{b^2(a+bx)}+\frac{h\log(c+dx)(bg-ah)}{b^2}+\frac{h(g+hx)\log(c+dx)}{b}\right)dx+\frac{pr\left(-\frac{1}{4}(4(bg-ah)+h(a+bx))^2+\frac{1}{2}(bg-ah)^2\log^2(a+\right.}{}\right.}{dqr\left(pr\int\left(\frac{\log(a+bx)(dg-ch)^2}{d^2(c+dx)}+\frac{h\log(a+bx)(dg-ch)}{d^2}+\frac{h(g+hx)\log(a+bx)}{d}\right)dx-\left(\frac{(dg-ch)^2\log(c+dx)}{d^3}+\frac{hx(dg-ch)}{d^2}+\frac{(g+hx)^2}{2d}\right)\right.}$$

$$\frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h}$$

$\Big\downarrow$ 2009

$$\frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h}-\frac{dqr\left(\frac{qr\left(\frac{1}{2}h^2\log(c+dx)(c+dx)^2+2h(dg-ch)\log(c+dx)(c+dx)-\frac{1}{4}(4(dg-ch)+h(c+dx))^2+\frac{1}{2}(dg-ch)^2\log^2(c+dx)\right)}{d^3}-\left(\frac{\log(c+dx)(dg-ch)^2}{d^3}+\right.\right.}{bpr\left(\frac{pr\left(\frac{1}{2}h^2\log(a+bx)(a+bx)^2+2h(bg-ah)\log(a+bx)(a+bx)-\frac{1}{4}(4(bg-ah)+h(a+bx))^2+\frac{1}{2}(bg-ah)^2\log^2(a+bx)\right)}{b^3}-\left(\frac{\log(a+bx)(bg-ah)^2}{b^3}+\right.\right.}$$

input
```
Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

3.37. $\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

output
```
((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*h) - (d*q*r*((q*r*
(-1/4*(4*(d*g - c*h) + h*(c + d*x))^2 + 2*h*(d*g - c*h)*(c + d*x)*Log[c +
d*x] + (h^2*(c + d*x)^2*Log[c + d*x])/2 + ((d*g - c*h)^2*Log[c + d*x]^2)/2
))/d^3 - ((h*(d*g - c*h)*x)/d^2 + (g + h*x)^2/(2*d) + ((d*g - c*h)^2*Log[c
 + d*x])/d^3)*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]) + p*r*(-1/2*(h*(b*g - a*h)*x)/(b*d) - (h*(d*g - c*h)*x)/d
^2 - (g + h*x)^2/(4*d) - ((b*g - a*h)^2*Log[a + b*x])/(2*b^2*d) + (h*(d*g
- c*h)*(a + b*x)*Log[a + b*x])/(b*d^2) + ((g + h*x)^2*Log[a + b*x])/(2*d)
+ ((d*g - c*h)^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/d^3 + ((d*g
- c*h)^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/d^3)))/h - (b*p*r*((p*r
*(-1/4*(4*(b*g - a*h) + h*(a + b*x))^2 + 2*h*(b*g - a*h)*(a + b*x)*Log[a +
 b*x] + (h^2*(a + b*x)^2*Log[a + b*x])/2 + ((b*g - a*h)^2*Log[a + b*x]^2)/
2))/b^3 - ((h*(b*g - a*h)*x)/b^2 + (g + h*x)^2/(2*b) + ((b*g - a*h)^2*Log[
a + b*x])/b^3)*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]) + q*r*(-((h*(b*g - a*h)*x)/b^2) - (h*(d*g - c*h)*x)/(2*b
*d) - (g + h*x)^2/(4*b) - ((d*g - c*h)^2*Log[c + d*x])/(2*b*d^2) + (h*(b*g
 - a*h)*(c + d*x)*Log[c + d*x])/(b^2*d) + ((g + h*x)^2*Log[c + d*x])/(2*b)
 + ((b*g - a*h)^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/b^3 + ((
b*g - a*h)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^3)))/h
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.37.4 Maple [F]

$$\int (hx+g)\ln \left(e\left(f(bx+a)^{p} (dx+c)^{q}\right)^{r}\right)^{2}dx$$

input
```
int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.37.5 Fricas [F]

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int (hx+g)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx$$

input
```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas"
)
```

output
```
integral((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.37.6 Sympy [F]

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)^2 dx$$

input
```
integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral((g + h*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.59

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\frac{1}{2}\left(hx^2+2\,gx\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2$$
$$+\frac{r\left(\frac{2\,(2\,abfgp-a^2fhp)\log(bx+a)}{b^2}+\frac{2\,(2\,cdfgq-c^2fhq)\log(dx+c)}{d^2}-\frac{bdfh(p+q)x^2-2\,(adfhp-(2\,dfg(p+q)-cfhq)b)x}{bd}\right)\log\left(\left((bx+a\right.\right.}{2\,f}$$
$$+\frac{r^2\left(\frac{2\,(2\,acdf^2hpq-(4\,(pq+q^2)cdf^2g-(pq+3\,q^2)c^2f^2h)b)\log(dx+c)}{bd^2}-\frac{4\,(2\,abd^2f^2gpq-a^2d^2f^2hpq-(2\,cdf^2gpq-c^2f^2hpq)b^2)\left(\log(bx+a)\right.}{b^2d^2}\right.}{}$$

input
```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima"
)
```

output
```
1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*r*(2*(2*a
*b*f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d
*x + c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f
*h*q)*b)*x)/(b*d))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/4*r^2*(2*(2*
a*c*d*f^2*h*p*q - (4*(p*q + q^2)*c*d*f^2*g - (p*q + 3*q^2)*c^2*f^2*h)*b)*l
og(d*x + c)/(b*d^2) - 4*(2*a*b*d^2*f^2*g*p*q - a^2*d^2*f^2*h*p*q - (2*c*d*
f^2*g*p*q - c^2*f^2*h*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b^2*d^2) + ((p^2 + 2*p*q + q^
2)*b^2*d^2*f^2*h*x^2 - 4*(2*c*d*f^2*g*p*q - c^2*f^2*h*p*q)*b^2*log(b*x + a
)*log(d*x + c) - 2*(2*c*d*f^2*g*q^2 - c^2*f^2*h*q^2)*b^2*log(d*x + c)^2 -
2*(2*a*b*d^2*f^2*g*p^2 - a^2*d^2*f^2*h*p^2)*log(b*x + a)^2 - 2*(3*(p^2 + p
*q)*a*b*d^2*f^2*h - (4*(p^2 + 2*p*q + q^2)*d^2*f^2*g - 3*(p*q + q^2)*c*d*f
^2*h)*b^2)*x + 2*((3*p^2 + p*q)*a^2*d^2*f^2*h + 2*(c*d*f^2*h*p*q - 2*(p^2
+ p*q)*d^2*f^2*g)*a*b)*log(b*x + a))/(b^2*d^2))/f^2
```

3.37.8 Giac [F]

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int (hx+g)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx$$

input
```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

output
```
integrate((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2(g+h\,x)\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x), x)
```

3.38 $\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

3.38.1 Optimal result

Integrand size = 23, antiderivative size = 269

$$\begin{aligned}\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx &= 2(p+q)^2r^2x-\frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}\\ &\quad-\frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd}-\frac{(bc-ad)q^2r^2\log^2(c+dx)}{bd}\\ &\quad-\frac{2(p+q)r(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}\\ &\quad+\frac{2(bc-ad)qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd}\\ &\quad+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{2(bc-ad)pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd}\end{aligned}$$

output
```
2*(p+q)^2*r^2*x-2*(-a*d+b*c)*q*(p+q)*r^2*ln(d*x+c)/b/d-2*(-a*d+b*c)*p*q*r^
2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d-(-a*d+b*c)*q^2*r^2*ln(d*x+c)^2/b
/d-2*(p+q)*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2*(-a*d+b*c)*q*r*ln
(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r)^2/b-2*(-a*d+b*c)*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.45

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \frac{2adpqr^2 + 2bdp^2r^2x + 4bdpqr^2x + 2bdq^2r^2x - adp^2r^2\log^2(a+bx) - 2bcpqr^2\log(c+dx) + 2adpqr^2\log(c}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

output
```
(2*a*d*p*q*r^2 + 2*b*d*p^2*r^2*x + 4*b*d*p*q*r^2*x + 2*b*d*q^2*r^2*x - a*d
*p^2*r^2*Log[a + b*x]^2 - 2*b*c*p*q*r^2*Log[c + d*x] + 2*a*d*p*q*r^2*Log[c
 + d*x] - 2*b*c*q^2*r^2*Log[c + d*x] - b*c*q^2*r^2*Log[c + d*x]^2 - 2*p*r*
Log[a + b*x]*(b*c*q*r*Log[c + d*x] + (-(b*c) + a*d)*q*r*Log[(b*(c + d*x))/
(b*c - a*d)] + a*d*(q*r - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 2*a*d*p
*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*p*r*x*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] - 2*b*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*
c*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + b*d*x*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r]^2 + 2*(b*c - a*d)*p*q*r^2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)])/(b*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2979, 2979, 16, 24, 2980, 2837, 2738, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$\Big\downarrow \text{2979}$$

$$-2r(p+q)\int \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)dx + \frac{2qr(bc-ad)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b} + \frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

$$\Big\downarrow \text{2979}$$

$$-2r(p+q)\left(\frac{qr(bc-ad)\int \frac{1}{c+dx}dx}{b}-r(p+q)\int 1dx+\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}\right)+\frac{2qr(bc-ad)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

$$\Big\downarrow \; 16$$

$$-2r(p+q)\left(-r(p+q)\int 1dx+\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}\right)+\frac{2qr(bc-ad)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{2qr(bc-ad)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

$$\Big\downarrow \; 2980$$

$$\frac{2qr(bc-ad)\left(-\frac{bpr\int \frac{\log(c+dx)}{a+bx}dx}{d}-qr\int \frac{\log(c+dx)}{c+dx}dx+\frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}\right)}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

$$\Big\downarrow \; 2837$$

$$\frac{2qr(bc-ad)\left(-\frac{bpr\int \frac{\log(c+dx)}{a+bx}dx}{d}-\frac{qr\int \frac{\log(c+dx)}{c+dx}d(c+dx)}{d}+\frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}\right)}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

$$\Big\downarrow \; 2738$$

$$\frac{2qr(bc-ad)\left(-\frac{bpr\int \frac{\log(c+dx)}{a+bx}dx}{d}+\frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{qr\log^2(c+dx)}{2d}\right)}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

$$\Big\downarrow \; 2841$$

3.38. $\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

$$\frac{2qr(bc-ad)\left(-\frac{bpr\left(\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{d\int\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{c+dx}dx}{b}\right)}{d}+\frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{qr\log^2(c+dx)}{2d}\right)}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

$\downarrow$ 2840

$$\frac{2qr(bc-ad)\left(-\frac{bpr\left(\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(c+dx)}{bc-ad}\right)}{c+dx}d(c+dx)}{b}\right)}{d}+\frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{qr\log^2(c+dx)}{2d}\right)}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

$\downarrow$ 2838

$$\frac{2qr(bc-ad)\left(\frac{\log(c+dx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}-\frac{bpr\left(\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b}\right)}{d}-\frac{qr\log^2(c+dx)}{2d}\right)}{b}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-2r(p+q)\left(\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))\right)$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

3.38. $\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

output
```
((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/b - 2*(p + q)*r*(-((p +
 q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/b) + (2*(b*c - a*d)*q*r*(-1/2*(q*r*Log[c + d*x]^2
)/d + (Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (b*p*r*((Log
[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/b + PolyLog[2, (b*(c + d*x))/
(b*c - a*d)]/b))/d))/b
```

3.38.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2979
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Simp[q*r*s*((b*c - a*d)/b)   Int[Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Simp[r*s*(p + q)   Int[Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q
, r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4
]
```

rule 2980
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Simp[b*p*(r/h)   Int[Log[g + h*x]/(a + b
*x), x], x] - Simp[d*q*(r/h)   Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

3.38.4 Maple [F]

$$\int \ln \left(e{\left(f{\left(bx+a\right)}^{p} {\left(dx+c\right)}^{q}\right)}^{r}\right)^{2}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

3.38.5 Fricas [F]

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.38.6 Sympy [F]

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \log\left(e\left(f(a+bx)^p\,(c+dx)^q\right)^r\right)^2\,dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.11

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = x\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 - \frac{2\left(f(p+q)x - \frac{afp\log(bx+a)}{b} - \frac{cfq\log(dx+c)}{d}\right)r\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{f} - \frac{\left(\frac{2(pq+q^2)cf^2\log(dx+c)}{d} - \frac{2(bcf^2pq-adf^2pq)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bd} + \frac{adf^2p^2\log(bx+a)^2+2\,bcf^2pq\log(bx+a\cdots}{\cdots}\right.}{f^2}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

output
```
x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 - 2*(f*(p + q)*x - a*f*p*log(b*x
+ a)/b - c*f*q*log(d*x + c)/d)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f -
(2*(p*q + q^2)*c*f^2*log(d*x + c)/d - 2*(b*c*f^2*p*q - a*d*f^2*p*q)*(log(b
*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a
*d)))/(b*d) + (a*d*f^2*p^2*log(b*x + a)^2 + 2*b*c*f^2*p*q*log(b*x + a)*log
(d*x + c) + b*c*f^2*q^2*log(d*x + c)^2 - 2*(p^2 + 2*p*q + q^2)*b*d*f^2*x +
 2*(p^2 + p*q)*a*d*f^2*log(b*x + a))/(b*d))*r^2/f^2
```

3.38.8 Giac [F]

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2, x)
```

3.39 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx} dx$

3.39.1 Optimal result

Integrand size = 31, antiderivative size = 1471

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx} dx = \text{Too large to display}$$

output
```
p*q*r^2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^
2/h+p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(b*(h*x+g)/(-a*h+b*g))/h+p*q*r^2
*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(b*(h*x+g)/(-a*h+b*g))/h-p*
q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(d*(h*x+g)/(-c*h+d*g))/h-p*q*r^2*ln((-
a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(
b*x+a))/h+p^2*r^2*ln(b*x+a)^2*ln(h*x+g)/h+q^2*r^2*ln(d*x+c)^2*ln(h*x+g)/h-
p^2*r^2*ln(b*x+a)^2*ln(b*(h*x+g)/(-a*h+b*g))/h-q^2*r^2*ln(d*x+c)^2*ln(d*(h
*x+g)/(-c*h+d*g))/h-2*p^2*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-2*q^2*r^2
*polylog(3,-h*(d*x+c)/(-c*h+d*g))/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2*ln(h
*x+g)/h-2*p*r*(q*r*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))-ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r))*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h+2*q*r*(p*r*ln((-a*h
+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*polylog
(2,-h*(d*x+c)/(-c*h+d*g))/h-2*p*q*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-2
*p*q*r^2*polylog(3,-h*(d*x+c)/(-c*h+d*g))/h-2*p*q*r^2*polylog(3,b*(d*x+c)/
d/(b*x+a))/h+2*p*q*r^2*polylog(3,(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))/h-
2*p*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-2*q*r*ln(d*x+c
)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h+2*p*r*ln(b*x+a)*ln(e*(f*(b*x
+a)^p*(d*x+c)^q)^r)*ln(b*(h*x+g)/(-a*h+b*g))/h+2*q*r*ln(d*x+c)*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln((-a*h+b*g)*(d
*x+c)/(-c*h+d*g)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/h-2*p*q*r^2*ln...
```

3.39.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 1370, normalized size of antiderivative = 0.93

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

output
```
(p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d
*g - c*h)*(a + b*x))]^2 + p^2*r^2*Log[a + b*x]^2*Log[g + h*x] + 2*p*q*r^2*
Log[a + b*x]*Log[c + d*x]*Log[g + h*x] + q^2*r^2*Log[c + d*x]^2*Log[g + h*
x] - 2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]
- 2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x] - p^2*r^2*Log[a + b*x]^
2*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[(h*(c + d*x
))/(-(d*g) + c*h)]^2*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[(h*(c
+ d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*Log[(b*(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))]^2*Log[(b*(g + h*x))/(b*g - a*h)] + 2*p*r*Log[a + b*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] - 2
*p*q*r^2*Log[a + b*x]*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] - q^2*r^
2*Log[c + d*x]^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[a + b*x]*L
og[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*
Log[(h*(c + d*x))/(-(d*g) + c*h)]^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q
*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*q*r*Log[c + d*x]*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r...
```

3.39.3 Rubi [A] (verified)

Time = 5.27 (sec) , antiderivative size = 2728, normalized size of antiderivative = 1.85, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {2983, 2986, 2841, 2840, 2838, 2881, 2822, 27, 2754, 2821, 2890, 2887, 2841, 27, 2752, 2885, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$$

$$\Big\downarrow \text{ 2983}$$

$$-\frac{2bpr\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{a+bx}dx}{h}-\frac{2dqr\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{c+dx}dx}{h}+\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$$\Big\downarrow \text{ 2986}$$

$$-\frac{2bpr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\int \frac{\log(g+hx)}{a+bx}dx\right)+\int \frac{\log((c+dx)^{qr})\,\mathrm{lo}}{a+bx}}{h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\int \frac{\log(g+hx)}{c+dx}dx\right)+\int \frac{\log((a+bx)^{pr})\log(}{c+dx}}{h}$$
$$\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$$\Big\downarrow \text{ 2841}$$

$$-\frac{2bpr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{h\int -\frac{\mathrm{lo}}{}}{}\right.\right.\right.}{h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{h\int \frac{\log(}{}}{}\right.\right.\right.}{h}$$
$$\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$$\Big\downarrow \text{ 2840}$$

$$-\frac{2bpr\left(-\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\int \frac{\log\left(1-\right.}{}\right.\right.}{h}$$
$$\frac{2dqr\left(-\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\int \frac{\log\left(1-\frac{d}{}\right.}{g+}\right.\right.}{h}$$
$$\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$$\Big\downarrow \text{ 2838}$$

3.39. $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

$$-\frac{2bpr\left(\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}dx+\int \frac{\log((a+bx)^{pr})\log(g+hx)}{a+bx}dx-\left(\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}\right)(-\right.\right.}{h}$$

$$\frac{2dqr\left(\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}dx+\int \frac{\log((c+dx)^{qr})\log(g+hx)}{c+dx}dx-\left(\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}+\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}\right)(-\log\right.\right.}{h}$$

$$\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$\Big\downarrow$ 2881

$$-\frac{2bpr\left(\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}dx+\frac{\int \frac{\log((a+bx)^{pr})\log\left(g-\frac{ah}{b}+\frac{h(a+bx)}{b}\right)}{a+bx}d(a+bx)}{b}-\left(\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\frac{\log(g+hx)\log\left(-\frac{h(}{b}\right.}{b}\right.\right.\right.}{h}$$

$$\frac{2dqr\left(\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}dx+\frac{\int \frac{\log((c+dx)^{qr})\log\left(g-\frac{ch}{d}+\frac{h(c+dx)}{d}\right)}{c+dx}d(c+dx)}{d}-\left(\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}+\frac{\log(g+hx)\log\left(-\frac{h(c+}{dg-}\right.}{d}\right.\right.\right.}{h}$$

$$\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$\Big\downarrow$ 2822

$$-\frac{2dqr\left(\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}dx+\frac{\frac{\log\left(\frac{h(c+dx)}{d}-\frac{ch}{d}+g\right)\log^2((c+dx)^{qr})}{2qr}-\frac{h\int \frac{d\log^2((c+dx)^{qr})}{d\left(g-\frac{ch}{d}\right)+h(c+dx)}d(c+dx)}{2dqr}}{d}-\left(\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}\right.\right.\right.}{h}$$

$$\frac{2bpr\left(\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}dx+\frac{\frac{\log\left(\frac{h(a+bx)}{b}-\frac{ah}{b}+g\right)\log^2((a+bx)^{pr})}{2pr}-\frac{h\int \frac{b\log^2((a+bx)^{pr})}{b\left(g-\frac{ah}{b}\right)+h(a+bx)}d(a+bx)}{2bpr}}{b}-\left(\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\right.\right.\right.}{h}$$

$$\frac{\log(g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}$$

$\Big\downarrow$ 27

3.39. $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

$$-\frac{2dqr\left(\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}dx+\frac{\frac{\log\left(\frac{h(c+dx)}{d}-\frac{ch}{d}+g\right)\log^2((c+dx)^{qr})}{2qr}-\frac{h\int \frac{\log^2((c+dx)^{qr})}{dg-ch+h(c+dx)}d(c+dx)}{2qr}}{d}-\left(\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}+\right.\right.\right.}{h}$$

$$\frac{2bpr\left(\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}dx+\frac{\frac{\log\left(\frac{h(a+bx)}{b}-\frac{ah}{b}+g\right)\log^2((a+bx)^{pr})}{2pr}-\frac{h\int \frac{\log^2((a+bx)^{pr})}{bg-ah+h(a+bx)}d(a+bx)}{2pr}}{b}-\left(\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\frac{\log}{}\right.\right.\right.}{h}$$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$

$$\Big\downarrow \ 2754$$

$$-\frac{2bpr\left(\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}dx+\frac{\frac{\log\left(\frac{h(a+bx)}{b}-\frac{ah}{b}+g\right)\log^2((a+bx)^{pr})}{2pr}-\frac{h\left(\frac{\log\left(\frac{h(a+bx)}{bg-ah}+1\right)\log^2((a+bx)^{pr})}{h}-\frac{2pr\int \frac{\log((a+bx)^{pr})\log\left(\frac{h(}{bg}\right.}{a+bx}}{h}\right.}{2pr}}{b}\right.}{}$$

$$\frac{2dqr\left(\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}dx+\frac{\frac{\log\left(\frac{h(c+dx)}{d}-\frac{ch}{d}+g\right)\log^2((c+dx)^{qr})}{2qr}-\frac{h\left(\frac{\log\left(\frac{h(c+dx)}{dg-ch}+1\right)\log^2((c+dx)^{qr})}{h}-\frac{2qr\int \frac{\log((c+dx)^{qr})\log\left(\frac{h(c+}{dg-}\right.}{c+dx}}{h}\right.}{2qr}}{d}\right.}{}$$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$

$$\Big\downarrow \ 2821$$

3.39. $\displaystyle\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

$$-\frac{2bpr\left(\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}dx+\frac{\frac{\log\left(\frac{h(a+bx)}{b}-\frac{ah}{b}+g\right)\log^2((a+bx)^{pr})}{2pr}-\frac{h\left(\frac{\log\left(\frac{h(a+bx)}{bg-ah}+1\right)\log^2((a+bx)^{pr})}{h}-\frac{2pr\left(pr\int\frac{\text{PolyLog}\left(2,-\frac{h(a-}{bg-}\right.}{a+bx}\right.}{}\right.}{2pr}}{b}\right.}{}$$

$$\frac{2dqr\left(\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}dx+\frac{\frac{\log\left(\frac{h(c+dx)}{d}-\frac{ch}{d}+g\right)\log^2((c+dx)^{qr})}{2qr}-\frac{h\left(\frac{\log\left(\frac{h(c+dx)}{dg-ch}+1\right)\log^2((c+dx)^{qr})}{h}-\frac{2qr\left(qr\int\frac{\text{PolyLog}\left(2,-\frac{h(c+dx}{dg-ch}\right.}{c+dx}\right.}{}\right.}{2qr}}{d}\right.}{}$$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$

$\downarrow$ 2890

3.39. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

$$-\frac{2bpr\left(\frac{\int \frac{\log\left(\left(c-\frac{ad}{b}+\frac{d(a+bx)}{b}\right)^{qr}\right)\log\left(g-\frac{ah}{b}+\frac{h(a+bx)}{b}\right)}{a+bx}d(a+bx)}{b}+\frac{\log\left(\frac{h(a+bx)}{b}-\frac{ah}{b}+g\right)\log^2\left((a+bx)^{pr}\right)}{2pr}-\frac{h\left(\frac{\log\left(\frac{h(a+bx)}{bg-ah}+1\right)\log^2\left((a+bx)^{pr}\right)}{h}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2dqr\left(\frac{\int \frac{\log\left(\left(a+\frac{b(c+dx)}{d}-\frac{bc}{d}\right)^{pr}\right)\log\left(g-\frac{ch}{d}+\frac{h(c+dx)}{d}\right)}{c+dx}d(c+dx)}{d}+\frac{\log\left(\frac{h(c+dx)}{d}-\frac{ch}{d}+g\right)\log^2\left((c+dx)^{qr}\right)}{2qr}-\frac{h\left(\frac{\log\left(\frac{h(c+dx)}{dg-ch}+1\right)\log^2\left((c+dx)^{qr}\right)}{h}-\frac{2q\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}$$

$\downarrow$ 2887

3.39. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h} -$$

$$2bpr\left(-\left((\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{b}+\frac{\text{PolyLog}\left(2\right.}{b}\right.\right.\right.$$

$$2dqr\left(-\left((\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{d}+\frac{\text{PolyLog}\left(2\right.}{d}\right.\right.\right.$$

$\downarrow$ 2841

3.39. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}-$$

$$2bpr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{b}+\frac{\text{PolyLog}\left(2\right.}{b}\right.\right.\right.$$

$$2dqr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{d}+\frac{\text{PolyLog}\left(2\right.}{d}\right.\right.\right.$$

$\downarrow$ 27

3.39. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}-$$

$$2bpr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{b}+\frac{\text{PolyLog}\left(2\right.}{b}\right.\right.\right.$$

$$2dqr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{d}+\frac{\text{PolyLog}\left(2\right.}{d}\right.\right.\right.$$

$\downarrow$ 2752

3.39. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h} -$$

$$2bpr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{b}+\frac{\text{PolyLog}\left(2\right.}{b}\right.\right.\right.$$

$$2dqr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{d}+\frac{\text{PolyLog}\left(2\right.}{d}\right.\right.\right.$$

$\downarrow$ 2885

3.39. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}-$$

$$2bpr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{b}+\frac{\text{PolyLog}\left(2\right.}{b}\right.\right.\right.$$

$$2dqr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{d}+\frac{\text{PolyLog}\left(2\right.}{d}\right.\right.\right.$$

$$\Big\downarrow 7143$$

$$\frac{\log(g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}-$$

$$2bpr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{b}+\frac{\text{PolyLog}\left(2\right.}{b}\right.\right.\right.$$

$$2dqr\left(-\left(\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{d}+\frac{\text{PolyLog}\left(2\right.}{d}\right.\right.\right.$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

3.39. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$

output
```
(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x])/h - (2*b*p*r*(-((Log
[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])*((Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/b + PolyLog[2, (b
*(g + h*x))/(b*g - a*h)]/b)) + ((Log[(a + b*x)^(p*r)]^2*Log[g - (a*h)/b +
(h*(a + b*x))/b])/(2*p*r) - (h*((Log[(a + b*x)^(p*r)]^2*Log[1 + (h*(a + b*
x))/(b*g - a*h)])/h - (2*p*r*(-(Log[(a + b*x)^(p*r)]*PolyLog[2, -((h*(a +
b*x))/(b*g - a*h))]) + p*r*PolyLog[3, -((h*(a + b*x))/(b*g - a*h))]))/h))/
(2*p*r))/b + (-((q*r*Log[c - (a*d)/b + (d*(a + b*x))/b] - Log[(c - (a*d)/b
 + (d*(a + b*x))/b)^(q*r)])*(Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g - (a*
h)/b + (h*(a + b*x))/b] + PolyLog[2, 1 + (h*(a + b*x))/(b*g - a*h)])) + q*
r*(((Log[-((d*(a + b*x))/(b*c - a*d))] + Log[(b*(d*g - c*h))/(d*(b*(g - (a
*h)/b) + h*(a + b*x)))] - Log[-((b*(d*g - c*h)*(a + b*x))/((b*c - a*d)*(b*
(g - (a*h)/b) + h*(a + b*x))))])*Log[((b*c - a*d)*(b*(g - (a*h)/b) + h*(a
+ b*x)))/((b*g - a*h)*(b*(c - (a*d)/b) + d*(a + b*x)))]^2)/2 - ((Log[-((d*
(a + b*x))/(b*c - a*d))] - Log[-((h*(a + b*x))/(b*g - a*h))])*(Log[c - (a*
d)/b + (d*(a + b*x))/b] + Log[((b*c - a*d)*(b*(g - (a*h)/b) + h*(a + b*x))
)/((b*g - a*h)*(b*(c - (a*d)/b) + d*(a + b*x)))])^2)/2 + Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[c - (a*d)/b + (d*(a + b*x))/b]*Log[g - (a*h)/b + (h*(
a + b*x))/b] - (Log[((b*c - a*d)*(b*(g - (a*h)/b) + h*(a + b*x)))/((b*g -
a*h)*(b*(c - (a*d)/b) + d*(a + b*x)))] - Log[g - (a*h)/b + (h*(a + b*x)...
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

3.39. $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

rule 2885
```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

rule 2887
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Simp[m   Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x),
x], x] - Simp[(m*Log[i + j*x] - Log[h*(i + j*x)^m])   Int[Log[c*(d + e*x)^n
]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && N
eQ[i + j*x, h*(i + j*x)^m]
```

rule 2890
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Simp[1/l   Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f +
g*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
 c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

rule 2983
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^2/h), x] + (-Simp[2*b*p*(r/h)   Int[Log[g + h*x]*
(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)), x], x] - Simp[2*d*q*(r/h)
   Int[Log[g + h*x]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(c + d*x)), x], x
]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

3.39. $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

rule 2986
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Simp[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
 + b*x)^(p*r)] - Log[(c + d*x)^(q*r)])   Int[(s + t*Log[i*(g + h*x)^n])/(j
+ k*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j
+ k*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x
), x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] &&
NeQ[b*c - a*d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.39.4 Maple [F]

$$\int \frac{\ln \left(e{\left(f{\left(bx+a\right)}^{p}\left(dx+c\right)^{q}\right)}^{r}\right)^{2}}{hx+g}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

3.39.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="fricas"
)
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g),x)
```

output
```
Timed out
```

3.39.7 Maxima [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\log\left(\left(((bx+a)^p(dx+c)^q f)^r e\right)^2\right)}{hx+g}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="maxima"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

3.39.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\log\left(\left(((bx+a)^p(dx+c)^q f)^r e\right)^2\right)}{hx+g}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2}{g+h\,x}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x), x)
```

3.40 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2} \, dx$

3.40.1 Optimal result

Integrand size = 31, antiderivative size = 832

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx$$
$$= \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)}$$
$$- \frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)}$$
$$- \frac{2dqr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)}$$
$$- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$
$$+ \frac{2bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(bg-ah)}$$
$$+ \frac{2dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)}$$
$$- \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)} - \frac{2bpqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)}$$
$$- \frac{2bp^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} - \frac{2dq^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$+ \frac{2bp^2r^2\,\mathrm{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} + \frac{2dpqr^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)}$$
$$- \frac{2dpqr^2\,\mathrm{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)} + \frac{2dq^2r^2\,\mathrm{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$+ \frac{2bpqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{h(bg-ah)} - \frac{2bpqr^2\,\mathrm{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)}{h(bg-ah)}$$

output
```
2*b*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)+2*d*p*q*r^2*l
n(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)-2*b*p*r*ln(b*x+a)*(p*r*ln(b
*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)-2*d*q*r*
ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/
(-c*h+d*g)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)+2*b*p*r*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)+
2*d*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h
*x+g)/h/(-c*h+d*g)-2*d*p*q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+
d*g)-2*b*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*h+b*g)-2*b*p^2*r
^2*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)-2*d*q^2*r^2*ln(d*x+c)
*ln(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p^2*r^2*polylog(2,(a*h-b*g)/h
/(b*x+a))/h/(-a*h+b*g)+2*d*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*
h+d*g)-2*d*p*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)+2*d*q^2*r
^2*polylog(2,(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p*q*r^2*polylog(2,b*(d*
x+c)/(-a*d+b*c))/h/(-a*h+b*g)-2*b*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))
/h/(-a*h+b*g)
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2930 vs. 2(832) = 1664.

Time = 0.53 (sec) , antiderivative size = 2930, normalized size of antiderivative = 3.52

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

output
```
(-(b*d*g^2*p^2*r^2*Log[a + b*x]^2) + b*c*g*h*p^2*r^2*Log[a + b*x]^2 - b*d*
g*h*p^2*r^2*x*Log[a + b*x]^2 + b*c*h^2*p^2*r^2*x*Log[a + b*x]^2 - 2*b*d*g^
2*p*q*r^2*Log[a + b*x]*Log[c + d*x] + 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[c
 + d*x] - 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[c + d*x] + 2*a*d*h^2*p*q*r^
2*x*Log[a + b*x]*Log[c + d*x] - b*d*g^2*q^2*r^2*Log[c + d*x]^2 + a*d*g*h*q
^2*r^2*Log[c + d*x]^2 - b*d*g*h*q^2*r^2*x*Log[c + d*x]^2 + a*d*h^2*q^2*r^2
*x*Log[c + d*x]^2 + 2*b*c*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*
g) + c*h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*
h)] + 2*b*c*h^2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] -
 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - b*c*
g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + a*d*g*h*p*q*r^2*Log[(h*(
c + d*x))/(-(d*g) + c*h)]^2 - b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g)
+ c*h)]^2 + a*d*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + 2*b*c*
g*h*p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/
((d*g - c*h)*(a + b*x))] - 2*a*d*g*h*p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*
x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*
r^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g
- c*h)*(a + b*x))] - 2*a*d*h^2*p*q*r^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]
*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*g*h*p*q*r^2*
Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c...
```

3.40.3 Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.77, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2984, 2993, 47, 16, 2858, 27, 2779, 2838, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx$$

$$\Big\downarrow \text{2984}$$

$$\frac{2bpr\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)}dx}{h}+\frac{2dqr\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(c+dx)(g+hx)}dx}{h}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}$$

$$\Big\downarrow \text{2993}$$

$$\frac{2bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int\frac{1}{(a+bx)(g+hx)}dx\right)+qr\int\frac{\log(c+dx)}{(a+bx)(g+hx)}}{h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int\frac{1}{(c+dx)(g+hx)}dx\right)+pr\int\frac{\log(a+bx)}{(c+dx)(g+hx)}}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\big\downarrow$ 47

$$\frac{2bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\left(\frac{b\int\frac{1}{a+bx}dx}{bg-ah}-\frac{h\int\frac{1}{g+hx}dx}{bg-ah}\right)\right)+qr\int\frac{}{(a}}{h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\left(\frac{d\int\frac{1}{c+dx}dx}{dg-ch}-\frac{h\int\frac{1}{g+hx}dx}{dg-ch}\right)\right)+pr\int\frac{}{(c}}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\big\downarrow$ 16

$$\frac{2bpr\left(qr\int\frac{\log(c+dx)}{(a+bx)(g+hx)}dx+pr\int\frac{\log(a+bx)}{(a+bx)(g+hx)}dx-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\,\mathrm{l}\right.\right.}{h}$$
$$\frac{2dqr\left(pr\int\frac{\log(a+bx)}{(c+dx)(g+hx)}dx+qr\int\frac{\log(c+dx)}{(c+dx)(g+hx)}dx-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\,\mathrm{l}\right.\right.}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\big\downarrow$ 2858

$$\frac{2bpr\left(qr\int\frac{\log(c+dx)}{(a+bx)(g+hx)}dx+\frac{pr\int\frac{b\log(a+bx)}{(a+bx)\left(b\left(g-\frac{ah}{b}\right)+h(a+bx)\right)}d(a+bx)}{b}-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\log\left(e(f(a+bx)^p(c+\right.\right.\right.\right.}{h}$$
$$\frac{2dqr\left(pr\int\frac{\log(a+bx)}{(c+dx)(g+hx)}dx+\frac{qr\int\frac{d\log(c+dx)}{(c+dx)\left(d\left(g-\frac{ch}{d}\right)+h(c+dx)\right)}d(c+dx)}{d}-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\log\left(e(f(a+bx)^p(c+\right.\right.\right.\right.}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\big\downarrow$ 27

3.40. $\int\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx$

$$\frac{2bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)}dx+pr\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\log\left(e(f(a+bx)^p(c\right.\right.\right.}{h}$$

$$\frac{2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)}dx+qr\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\log\left(e(f(a+bx)^p(c\right.\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\downarrow$ 2779

$$\frac{2bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)}dx+pr\left(\frac{\int \frac{\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{a+bx}d(a+bx)}{bg-ah}-\frac{\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{bg-ah}\right)-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\text{lo}\right.\right.\right.}{h}$$

$$\frac{2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)}dx+qr\left(\frac{\int \frac{\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{c+dx}d(c+dx)}{dg-ch}-\frac{\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{dg-ch}\right)-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\text{lo}\right.\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\downarrow$ 2838

$$\frac{2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)}dx-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c\right.\right.\right.}{h}$$

$$\frac{2bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)}dx-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c\right.\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\downarrow$ 2865

$$\frac{2dqr\left(pr\int \left(\frac{d\log(a+bx)}{(dg-ch)(c+dx)}-\frac{h\log(a+bx)}{(dg-ch)(g+hx)}\right)dx-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\text{ log}\right.\right.\right.}{h}$$

$$\frac{2bpr\left(qr\int \left(\frac{b\log(c+dx)}{(bg-ah)(a+bx)}-\frac{h\log(c+dx)}{(bg-ah)(g+hx)}\right)dx-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\text{ log}\right.\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

$\downarrow$ 2009

3.40. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2}\,dx$

$$2dqr\left(-\left(\left(\frac{\log(c+dx)}{dg-ch}-\frac{\log(g+hx)}{dg-ch}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\right)+pr\left(\frac{\text{PolyL}}{}\right.$$

$$2bpr\left(-\left(\left(\frac{\log(a+bx)}{bg-ah}-\frac{\log(g+hx)}{bg-ah}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\right)+qr\left(\frac{\text{PolyL}}{}\right.$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

output
```
-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g + h*x))) + (2*d*q*r*(-((p*r
*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*(
Log[c + d*x]/(d*g - c*h) - Log[g + h*x]/(d*g - c*h))) + p*r*((Log[a + b*x]
*Log[(b*(c + d*x))/(b*c - a*d)])/(d*g - c*h) - (Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)])/(d*g - c*h) + PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]/
(d*g - c*h) - PolyLog[2, -((h*(a + b*x))/(b*g - a*h))]/(d*g - c*h)) + q*r*
(-((Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])/(d*g - c*h)) + PolyLo
g[2, -((d*g - c*h)/(h*(c + d*x)))]/(d*g - c*h))))/h + (2*b*p*r*(-((p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*(Log[
a + b*x]/(b*g - a*h) - Log[g + h*x]/(b*g - a*h))) + p*r*(-((Log[a + b*x]*L
og[1 + (b*g - a*h)/(h*(a + b*x))])/(b*g - a*h)) + PolyLog[2, -((b*g - a*h)
/(h*(a + b*x)))]/(b*g - a*h)) + q*r*((Log[-((d*(a + b*x))/(b*c - a*d))]*Lo
g[c + d*x])/(b*g - a*h) - (Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(b
*g - a*h) + PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(b*g - a*h) - PolyLog[2,
 -((h*(c + d*x))/(d*g - c*h))]/(b*g - a*h))))/h
```

3.40.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

3.40. $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx$

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.40.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}}{\left(hx+g\right)^{2}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

3.40.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx=\int \frac{\log \left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^2*x^2 + 2*g*h*x + g^2),
 x)
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**2,x)
```

output
```
Timed out
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 745, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx$$
$$= \frac{2\left(\frac{bfp\log(bx+a)}{bg-ah} + \frac{dfq\log(dx+c)}{dg-ch} - \frac{(adfhq-(dfg(p+q)-cfhp)b)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)r\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{fh}$$
$$- \frac{\left(\frac{2\left(bcf^2hpq-adf^2hpq\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{(dgh-ch^2)a-(dg^2-cgh)b} + \frac{2\left(adf^2hpq+\left(cf^2hp^2-\left(p^2+pq\right)df^2g\right)b\right)\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+\cdots\right.\right.}{(dgh-ch^2)a-(dg^2-cgh)b}\cdots}{\cdots}$$
$$- \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)h}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="maxim
a")
```

output
```
2*(b*f*p*log(b*x + a)/(b*g - a*h) + d*f*q*log(d*x + c)/(d*g - c*h) - (a*d*
f*h*q - (d*f*g*(p + q) - c*f*h*p)*b)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*
g^2 - c*g*h)*b))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) - (2*(b*c*f^
2*h*p*q - a*d*f^2*h*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*
b) + 2*(a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*(log(b*x +
a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))
/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) + 2*(a*d*f^2*h*q^2 + (c*f^2*h*p*q
 - (p*q + q^2)*d*f^2*g)*b)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1
) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h
)*b) - ((d*f^2*g*p^2 - c*f^2*h*p^2)*b*log(b*x + a)^2 + 2*(b*d*f^2*g*p*q -
a*d*f^2*h*p*q)*log(b*x + a)*log(d*x + c) + (b*d*f^2*g*q^2 - a*d*f^2*h*q^2)
*log(d*x + c)^2 + 2*((a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*
b)*log(b*x + a) + (a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q + q^2)*d*f^2*g)*b)*
log(d*x + c))*log(h*x + g))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*r^2/(
f^2*h) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)*h)
```

3.40.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^2, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(g+h\,x)^2}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2, x)
```

3.41 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3} dx$

3.41.1 Optimal result

Integrand size = 31, antiderivative size = 1304

$$
\begin{aligned}
&\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx \\
&= -\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)}+\frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)}-\frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} \\
&-\frac{bdpqr^2\log(c+dx)}{h(bg-ah)(dg-ch)}+\frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)}-\frac{dq^2r^2(c+dx)\log(c+dx)}{(dg-ch)^2(g+hx)} \\
&+\frac{b^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)^2}+\frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)^2} \\
&-\frac{bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)(g+hx)} \\
&-\frac{dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)(g+hx)} \\
&-\frac{b^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)^2} \\
&-\frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)^2} \\
&-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}+\frac{b^2p^2r^2\log(g+hx)}{h(bg-ah)^2} \\
&+\frac{2bdpqr^2\log(g+hx)}{h(bg-ah)(dg-ch)}+\frac{d^2q^2r^2\log(g+hx)}{h(dg-ch)^2} \\
&+\frac{b^2pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)}{h(bg-ah)^2} \\
&+\frac{d^2qr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)}{h(dg-ch)^2} \\
&-\frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)^2}-\frac{b^2pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)^2} \\
&-\frac{b^2p^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2}-\frac{d^2q^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} \\
&+\frac{b^2p^2r^2\,\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2}+\frac{d^2pqr^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)^2} \\
&-\frac{d^2pqr^2\,\text{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)^2}+\frac{d^2q^2r^2\,\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} \\
&+\frac{b^2pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{h(bg-ah)^2}-\frac{b^2pqr^2\,\text{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)}{h(bg-ah)^2}
\end{aligned}
$$

output
```
b^2*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^2+d^2*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^2+
b^2*p^2*r^2*polylog(2,(a*h-b*g)/h/(b*x+a))/h/(-a*h+b*g)^2+d^2*q^2*r^2*poly
log(2,(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)^2-d^2*p*q*r^2*ln(b*x+a)*ln(b*(h*x+
g)/(-a*h+b*g))/h/(-c*h+d*g)^2-b^2*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g
))/h/(-a*h+b*g)^2+d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)/(h*x+g)+b*p*q*r^2*ln(d*
x+c)/h/(-a*h+b*g)/(h*x+g)+b^2*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/
h/(-a*h+b*g)^2+d^2*p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)
^2+2*b*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/(-c*h+d*g)-1/2*ln(e*(f*(b*x+a)^p*(
d*x+c)^q)^r)^2/h/(h*x+g)^2-d^2*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)^2+b^2*p*r*(p*r*ln(b*x+a)+q*r
*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^2+d^2*q
*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)
/h/(-c*h+d*g)^2-b^2*p^2*r^2*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b
*g)^2-d^2*q^2*r^2*ln(d*x+c)*ln(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)^2+d^2*
p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)^2-d^2*p*q*r^2*polylo
g(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)^2-d*q^2*r^2*(d*x+c)*ln(d*x+c)/(-c*
h+d*g)^2/(h*x+g)+b^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/h/(-a*h+b*g)^
2-b^2*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(-a*h+b*g)^2-b*p^2*r^2*(b
*x+a)*ln(b*x+a)/(-a*h+b*g)^2/(h*x+g)-b*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)/(h*x+g)-d*q*r*(p*r*ln(b*x+a...
```

3.41.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12086 vs. $2(1304) = 2608$.

Time = 2.89 (sec) , antiderivative size = 12086, normalized size of antiderivative = 9.27

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

output
```
Result too large to show
```

3.41.3 Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 1048, normalized size of antiderivative = 0.80, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2984, 2993, 54, 2009, 2858, 27, 2789, 2751, 16, 2779, 2838, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2984}$$

$$\frac{bpr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)^2}dx}{h}+\frac{dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(c+dx)(g+hx)^2}dx}{h}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2993}$$

$$\frac{bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{1}{(a+bx)(g+hx)^2}dx\right)+qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)}\right.}{h}$$
$$\frac{dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{1}{(c+dx)(g+hx)^2}dx\right)+pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)}\right.}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \left(\frac{b^2}{(bg-ah)^2(a+bx)}-\frac{hb}{(bg-ah)^2(g+hx)}-\right.\right.\right.}{h}$$
$$\frac{dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \left(\frac{d^2}{(dg-ch)^2(c+dx)}-\frac{hd}{(dg-ch)^2(g+hx)}-\right.\right.\right.}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx+pr\int \frac{\log(a+bx)}{(a+bx)(g+hx)^2}dx-\left(\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}\right)\left(-\log\left(e(f(a+bx)^p\right.\right.\right.\right.}{h}$$
$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+qr\int \frac{\log(c+dx)}{(c+dx)(g+hx)^2}dx-\left(\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}\right)\left(-\log\left(e(f(a+bx)^p\right.\right.\right.\right.}{h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2858}$$

$$\frac{bpr\left(\frac{pr\int \frac{b^2\log(a+bx)}{(a+bx)\left(b\left(g-\frac{ah}{b}\right)+h(a+bx)\right)^2}d(a+bx)}{b}+qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx-\left(\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}\right)(-\log\right.\right.}{h}$$

$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+\frac{qr\int \frac{d^2\log(c+dx)}{(c+dx)\left(d\left(g-\frac{ch}{d}\right)+h(c+dx)\right)^2}d(c+dx)}{d}-\left(\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}\right)(-\log\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx+bpr\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))^2}d(a+bx)-\left(\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}\right)(-\right.\right.}{h}$$

$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+dqr\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))^2}d(c+dx)-\left(\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}\right)(-\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2789}$$

$$\frac{bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx+bpr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)}{bg-ah}-\frac{h\int \frac{\log(a+bx)}{(bg-ah+h(a+bx))^2}d(a+bx)}{bg-ah}\right)-\left(\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log}{(b}\right.\right.\right.}{h}$$

$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+dqr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)}{dg-ch}-\frac{h\int \frac{\log(c+dx)}{(dg-ch+h(c+dx))^2}d(c+dx)}{dg-ch}\right)-\left(\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log}{(d}\right.\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \text{2751}$$

3.41. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$

$$\frac{bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx+bpr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(h(a+bx)-ah+bg)}-\frac{\int \frac{1}{bg-ah+h(a+bx)}d(a+bx)}{bg-ah}\right)}{bg-ah}\right)-\cdots}{h\cdots}$$

$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+dqr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(h(c+dx)-ch+dg)}-\frac{\int \frac{1}{dg-ch+h(c+dx)}d(c+dx)}{dg-ch}\right)}{dg-ch}\right)-\cdots}{h\cdots}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx+bpr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(h(a+bx)-ah+bg)}-\frac{\log(h(a+bx)-ah+bg)}{h(bg-ah)}\right)}{bg-ah}\right)-\left(\left(\frac{\cdots}{(g+\cdots}\right.\right.}{h}$$

$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+dqr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(h(c+dx)-ch+dg)}-\frac{\log(h(c+dx)-ch+dg)}{h(dg-ch)}\right)}{dg-ch}\right)-\left(\left(\frac{\cdots}{(g+\cdots}\right.\right.}{h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \; 2779$$

$$\frac{bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx+bpr\left(\frac{\frac{\int \frac{\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{a+bx}d(a+bx)}{bg-ah}-\frac{\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(h(a+bx)-ah+bg)}-\frac{\log(h(a+bx)-a\cdots}{h(bg-ah)\cdots}\right.}{bg-ah}\cdots\right.\right.}{\cdots}$$

$$\frac{dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx+dqr\left(\frac{\frac{\int \frac{\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{c+dx}d(c+dx)}{dg-ch}-\frac{\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(h(c+dx)-ch+dg)}-\frac{\log(h(c+dx)-c\cdots}{h(dg-ch)\cdots}\right.}{dg-ch}\cdots\right.\right.}{\cdots}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$$\Big\downarrow \; 2838$$

3.41. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$

$$dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}dx-\left(\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(\right.\right.\right.$$

$$bpr\left(qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}dx-\left(\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}\right)\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(\right.\right.\right.$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$\Big\downarrow$ 2865

$$bpr\left(qr\int \left(\frac{\log(c+dx)b^2}{(bg-ah)^2(a+bx)}-\frac{h\log(c+dx)b}{(bg-ah)^2(g+hx)}-\frac{h\log(c+dx)}{(bg-ah)(g+hx)^2}\right)dx-\left(\left(\frac{1}{(g+hx)(bg-ah)}+\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}\right)(-\log\right.\right.$$

$$dqr\left(pr\int \left(\frac{\log(a+bx)d^2}{(dg-ch)^2(c+dx)}-\frac{h\log(a+bx)d}{(dg-ch)^2(g+hx)}-\frac{h\log(a+bx)}{(dg-ch)(g+hx)^2}\right)dx-\left(\left(\frac{1}{(g+hx)(dg-ch)}+\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}\right)(-\log\right.\right.$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}$$

$\Big\downarrow$ 2009

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}+$$

$$dqr\left(-\left(\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{d\log(c+dx)}{(dg-ch)^2}-\frac{d\log(g+hx)}{(dg-ch)^2}+\frac{1}{(dg-ch)(g+hx)}\right.\right.\right.$$

$$bpr\left(-\left(\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{b\log(a+bx)}{(bg-ah)^2}-\frac{b\log(g+hx)}{(bg-ah)^2}+\frac{1}{(bg-ah)(g+hx)}\right.\right.\right.$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

3.41. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$

output
```
-1/2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g + h*x)^2) + (d*q*r*(-((p
*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
*(1/((d*g - c*h)*(g + h*x)) + (d*Log[c + d*x])/(d*g - c*h)^2 - (d*Log[g +
h*x])/(d*g - c*h)^2)) + p*r*(-((b*Log[a + b*x])/((b*g - a*h)*(d*g - c*h)))
 + Log[a + b*x]/((d*g - c*h)*(g + h*x)) + (d*Log[a + b*x]*Log[(b*(c + d*x)
)/(b*c - a*d)])/(d*g - c*h)^2 + (b*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))
 - (d*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(d*g - c*h)^2 + (d*Poly
Log[2, -((d*(a + b*x))/(b*c - a*d))])/(d*g - c*h)^2 - (d*PolyLog[2, -((h*(
a + b*x))/(b*g - a*h))])/(d*g - c*h)^2) + d*q*r*(-((h*(((c + d*x)*Log[c +
d*x])/((d*g - c*h)*(d*g - c*h + h*(c + d*x))) - Log[d*g - c*h + h*(c + d*x
)]/(h*(d*g - c*h))))/(d*g - c*h)) + (-((Log[c + d*x]*Log[1 + (d*g - c*h)/(
h*(c + d*x))])/(d*g - c*h)) + PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))]/(d*
g - c*h))/(d*g - c*h))))/h + (b*p*r*(-((p*r*Log[a + b*x] + q*r*Log[c + d*x
] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*(1/((b*g - a*h)*(g + h*x)) + (b*
Log[a + b*x])/(b*g - a*h)^2 - (b*Log[g + h*x])/(b*g - a*h)^2)) + b*p*r*(-(
(h*(((a + b*x)*Log[a + b*x])/((b*g - a*h)*(b*g - a*h + h*(a + b*x))) - Log
[b*g - a*h + h*(a + b*x)]/(h*(b*g - a*h))))/(b*g - a*h)) + (-((Log[a + b*x
]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(b*g - a*h)) + PolyLog[2, -((b*g - a
*h)/(h*(a + b*x)))]/(b*g - a*h))/(b*g - a*h)) + q*r*(-((d*Log[c + d*x])/((
b*g - a*h)*(d*g - c*h))) + Log[c + d*x]/((b*g - a*h)*(g + h*x)) + (b*Lo...
```

3.41.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.41. $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx$

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.41.4 Maple [F]

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}}{\left(hx+g\right)^{3}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

3.41.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{\left(hx+g\right)^3}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^3*x^3 + 3*g*h^2*x^2 + 3
*g^2*h*x + g^3), x)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**3,x)
```

output
```
Timed out
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1857, normalized size of antiderivative = 1.42

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="maxim
a")
```

output
```
(b^2*f*p*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + d^2*f*q*log(d*x +
c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + (2*a*b*d^2*f*g*h*q - a^2*d^2*f*h^2*q
- (d^2*f*g^2*(p + q) - 2*c*d*f*g*h*p + c^2*f*h^2*p)*b^2)*log(h*x + g)/((d^
2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^
2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (a*d*f*h*q - (
d*f*g*(p + q) - c*f*h*p)*b)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b +
 ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*r*log(((b*x + a)^p*(d*x
 + c)^q*f)^r*e)/(f*h) + 1/2*(2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*
q - (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q)*b^2)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h^2 - c*h
^3)*a^2 - 2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) - 2*(2*a*b*d^
2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*f^2*h^2*p^2
 - (p^2 + p*q)*d^2*f^2*g^2)*b^2)*(log(b*x + a)*log((b*h*x + a*h)/(b*g - a*
h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d
*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) + 2*(2*a*b*d^2*f^2*g*h*q^2
- a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q + q^2)
*d^2*f^2*g^2)*b^2)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilo
g(-(d*h*x + c*h)/(d*g - c*h)))/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2
- 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h +
 c^2*g^2*h^2)*b^2) - 2*(a*d^2*f^2*h*q^2 + (c*d*f^2*h*p*q - (p*q + q^2)*...
```

3.41.8 Giac [F]

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^3, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(g+h\,x)^3}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3, x)
```

3.42 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4} dx$

3.42.1 Optimal result

Integrand size = 31, antiderivative size = 1957

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4} dx = \text{Too large to display}$$

output
```
b^3*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^3+d^3*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^3-
2/3*b*d^2*p*q*r^2*ln(b*x+a)/h/(-a*h+b*g)/(-c*h+d*g)^2-1/3*b^2*d*p*q*r^2*ln
(b*x+a)/h/(-a*h+b*g)^2/(-c*h+d*g)-1/3*b*d^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)
/(-c*h+d*g)^2-2/3*b^2*d*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)^2/(-c*h+d*g)-2/3*b*
d*p*q*r^2/h/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r)^2/h/(h*x+g)^3+b*d^2*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/(-c*h+d*g)^2+b^2*d*
p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)^2/(-c*h+d*g)-2/3*d^3*p*q*r^2*ln(b*x+a)*ln(b
*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)^3-2/3*b^3*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)
/(-c*h+d*g))/h/(-a*h+b*g)^3+1/3*d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)/(h*x+g)^2
+2/3*d^2*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)^2/(h*x+g)+1/3*b*p*q*r^2*ln(d*x+c)/
h/(-a*h+b*g)/(h*x+g)^2+2/3*b^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)^2/(h*x+g)+2/
3*b^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^3+2/3*d^3*p
*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^3-2/3*d^3*q*r*ln(d*
x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h
+d*g)^3+2/3*b^3*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)
^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^3+2/3*d^3*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-c*h+d*g)^3-2/3*b^3*p^2*r^2*
ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)^3-1/3*b^3*p^2*r^2*ln(b*x
+a)/h/(-a*h+b*g)^3-1/3*d^3*q^2*r^2*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*b^2*p^2*r^
2/h/(-a*h+b*g)^2/(h*x+g)-1/3*d^2*q^2*r^2/h/(-c*h+d*g)^2/(h*x+g)+2/3*b^3...
```

3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47127 vs. 2(1957) = 3914.

Time = 6.30 (sec) , antiderivative size = 47127, normalized size of antiderivative = 24.08

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

output
```
Result too large to show
```

3.42.3 Rubi [A] (verified)

Time = 4.46 (sec) , antiderivative size = 1656, normalized size of antiderivative = 0.85, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {2984, 2993, 54, 2009, 2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$

$$\Big\downarrow \text{2984}$$

$$\frac{2bpr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)^3}dx}{3h}+\frac{2dqr\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(c+dx)(g+hx)^3}dx}{3h}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$$\Big\downarrow \text{2993}$$

$$\frac{2bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{1}{(a+bx)(g+hx)^3}dx\right)+qr\int \frac{\log(c+dx)}{(a+bx)(g+h}\right.}{3h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int \frac{1}{(c+dx)(g+hx)^3}dx\right)+pr\int \frac{\log(a+bx)}{(c+dx)(g+h}\right.}{3h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{2bpr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int\left(\frac{b^3}{(bg-ah)^3(a+bx)}-\frac{hb^2}{(bg-ah)^3(g+hx)}-\right.\right.\right.}{3h}$$
$$\frac{2dqr\left(-\left(\left(-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+pr\log(a+bx)+qr\log(c+dx)\right)\int\left(\frac{d^3}{(dg-ch)^3(c+dx)}-\frac{hd^2}{(dg-ch)^3(g+hx)}-\right.\right.\right.}{3h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\Big\downarrow$ 2009

$$\frac{2bpr\left(qr\int\frac{\log(c+dx)}{(a+bx)(g+hx)^3}dx+pr\int\frac{\log(a+bx)}{(a+bx)(g+hx)^3}dx-\left(\left(\frac{b^2\log(a+bx)}{(bg-ah)^3}-\frac{b^2\log(g+hx)}{(bg-ah)^3}+\frac{b}{(g+hx)(bg-ah)^2}+\frac{1}{2(g+hx)^2(bg-ah)}\right)\right.\right.}{3h}$$
$$\frac{2dqr\left(pr\int\frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+qr\int\frac{\log(c+dx)}{(c+dx)(g+hx)^3}dx-\left(\left(\frac{d^2\log(c+dx)}{(dg-ch)^3}-\frac{d^2\log(g+hx)}{(dg-ch)^3}+\frac{d}{(g+hx)(dg-ch)^2}+\frac{1}{2(g+hx)^2(dg-ch)}\right.\right.\right.}{3h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\Big\downarrow$ 2858

$$\frac{2bpr\left(\frac{pr\int\frac{b^3\log(a+bx)}{(a+bx)\left(b\left(g-\frac{ah}{b}\right)+h(a+bx)\right)^3}d(a+bx)}{b}+qr\int\frac{\log(c+dx)}{(a+bx)(g+hx)^3}dx-\left(\left(\frac{b^2\log(a+bx)}{(bg-ah)^3}-\frac{b^2\log(g+hx)}{(bg-ah)^3}+\frac{b}{(g+hx)(bg-ah)^2}+2(\right.\right.\right.}{3h}$$
$$\frac{2dqr\left(pr\int\frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+\frac{qr\int\frac{d^3\log(c+dx)}{(c+dx)\left(d\left(g-\frac{ch}{d}\right)+h(c+dx)\right)^3}d(c+dx)}{d}-\left(\left(\frac{d^2\log(c+dx)}{(dg-ch)^3}-\frac{d^2\log(g+hx)}{(dg-ch)^3}+\frac{d}{(g+hx)(dg-ch)^2}+\frac{}{2(}\right.\right.\right.}{3h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\Big\downarrow$ 27

$$\frac{2bpr\left(b^2pr\int\frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))^3}d(a+bx)+qr\int\frac{\log(c+dx)}{(a+bx)(g+hx)^3}dx-\left(\frac{b^2\log(a+bx)}{(bg-ah)^3}-\frac{b^2\log(g+hx)}{(bg-ah)^3}+\frac{b}{(g+hx)(bg-ah)^2}+\right.\right.}{3h}$$
$$\frac{2dqr\left(pr\int\frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+d^2qr\int\frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))^3}d(c+dx)-\left(\frac{d^2\log(c+dx)}{(dg-ch)^3}-\frac{d^2\log(g+hx)}{(dg-ch)^3}+\frac{d}{(g+hx)(dg-ch)^2}\right.\right.}{3h}$$
$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\Big\downarrow$ 2789

3.42. $\int\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$

$$\frac{2bpr\left(b^2pr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))^2}d(a+bx)}{bg-ah}-\frac{h\int \frac{\log(a+bx)}{(bg-ah+h(a+bx))^3}d(a+bx)}{bg-ah}\right)+qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)^3}dx-\left(\frac{b^2\log(a+bx)}{(bg-ah)^3}-\frac{b^2\log}{(b}\right.\right.}{3h}$$

$$\frac{2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+d^2qr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))^2}d(c+dx)}{dg-ch}-\frac{h\int \frac{\log(c+dx)}{(dg-ch+h(c+dx))^3}d(c+dx)}{dg-ch}\right)-\left(\frac{d^2\log(c+dx)}{(dg-ch)^3}-\frac{d^2\log}{(d}\right.\right.}{3h}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\downarrow$ 2756

$$\frac{2bpr\left(b^2pr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))^2}d(a+bx)}{bg-ah}-\frac{h\left(\frac{\int \frac{1}{(a+bx)(bg-ah+h(a+bx))^2}d(a+bx)}{2h}-\frac{\log(a+bx)}{2h(h(a+bx)-ah+bg)^2}\right)}{bg-ah}\right)+qr\int \frac{\log(c+dx)}{(a+bx)(g+hx)}\right.}{}$$

$$\frac{2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+d^2qr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))^2}d(c+dx)}{dg-ch}-\frac{h\left(\frac{\int \frac{1}{(c+dx)(dg-ch+h(c+dx))^2}d(c+dx)}{2h}-\frac{\log(c+dx)}{2h(h(c+dx)-ch+dg)^2}\right.}{dg-ch}\right.\right.}{}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\downarrow$ 54

$$\frac{2bpr\left(b^2pr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))^2}d(a+bx)}{bg-ah}-\frac{h\left(\frac{\int\left(-\frac{h}{(bg-ah)^2(bg-ah+h(a+bx))}-\frac{h}{(bg-ah)(bg-ah+h(a+bx))^2}+\frac{1}{(bg-ah)^2(a+bx)}\right)d(a+bx)}{2h}-\right.}{bg-ah}\right.\right.}{}$$

$$\frac{2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+d^2qr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))^2}d(c+dx)}{dg-ch}-\frac{h\left(\frac{\int\left(-\frac{h}{(dg-ch)^2(dg-ch+h(c+dx))}-\frac{h}{(dg-ch)(dg-ch+h(c+dx))}\right.}{2h}\right.}{dg-ch}\right.\right.}{}$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\downarrow$ 2009

3.42. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$

$$2bpr\left(b^2pr\left(\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))^2}d(a+bx)}{bg-ah}-\frac{h\left(\frac{\frac{1}{(bg-ah)(h(a+bx)-ah+bg)}+\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(h(a+bx)-ah+bg)}{(bg-ah)^2}}{2h}-\frac{\log(a+bx)}{2h(h(a+bx)-ah+bg)^2}\right)}{bg-ah}\right)\right)$$

$$2dqr\left(pr\int \frac{\log(a+bx)}{(c+dx)(g+hx)^3}dx+d^2qr\left(\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))^2}d(c+dx)}{dg-ch}-\frac{h\left(\frac{\frac{1}{(dg-ch)(h(c+dx)-ch+dg)}+\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(h(c+dx)-ch+dg}{(dg-ch)^2}}{2h}\right.}{dg-ch}\right.\right.$$

$$\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}$$

$\downarrow$ 2789

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$2bpr\left(pr\left(\frac{\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)}{bg-ah}-\frac{h\int \frac{\log(a+bx)}{(bg-ah+h(a+bx))^2}d(a+bx)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(bg-ah+h(a+bx))}{(bg-ah)^2}+\frac{1}{(bg-ah)(bg-ah+h(a+bx))}}{2h}-\right.}{bg-ah}\right.\right.$$

$$2dqr\left(qr\left(\frac{\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)}{dg-ch}-\frac{h\int \frac{\log(c+dx)}{(dg-ch+h(c+dx))^2}d(c+dx)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(dg-ch+h(c+dx))}{(dg-ch)^2}+\frac{1}{(dg-ch)(dg-ch+h(c+dx))}}{2h}-\right.}{dg-ch}\right.\right.$$

$\downarrow$ 2751

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$2bpr\left(pr\left(\frac{\frac{\int \frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(bg-ah+h(a+bx))}-\frac{\int \frac{1}{bg-ah+h(a+bx)}d(a+bx)}{bg-ah}\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(bg-ah+h(a+bx))}{(bg-ah)^2}}{2h}\right.}{}\right.\right.$$

$$2dqr\left(qr\left(\frac{\frac{\int \frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(dg-ch+h(c+dx))}-\frac{\int \frac{1}{dg-ch+h(c+dx)}d(c+dx)}{dg-ch}\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(dg-ch+h(c+dx))}{(dg-ch)^2}}{2h}\right.}{}\right.\right.$$

$\downarrow$ 16

3.42. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$\frac{2bpr\left(pr\left(\frac{\frac{\int\frac{\log(a+bx)}{(a+bx)(bg-ah+h(a+bx))}d(a+bx)}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(bg-ah+h(a+bx))}-\frac{\log(bg-ah+h(a+bx))}{h(bg-ah)}\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(bg-ah+h(a+bx))}{(bg-ah)^2}+\frac{}{(bg-}}{2h}\right.}{bg}\right.\right.}{}$$

$$\frac{2dqr\left(qr\left(\frac{\frac{\int\frac{\log(c+dx)}{(c+dx)(dg-ch+h(c+dx))}d(c+dx)}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(dg-ch+h(c+dx))}-\frac{\log(dg-ch+h(c+dx))}{h(dg-ch)}\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(dg-ch+h(c+dx))}{(dg-ch)^2}+\frac{}{(dg-}}{2h}\right.}{dg}\right.\right.}{}$$

$\downarrow$ 2779

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$\frac{2bpr\left(pr\left(\frac{\frac{\frac{\int\frac{\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{a+bx}d(a+bx)}{bg-ah}-\frac{\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(bg-ah+h(a+bx))}-\frac{\log(bg-ah+h(a+bx))}{h(bg-ah)}\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(b}{}\right.}{}\right.\right.}{}$$

$$\frac{2dqr\left(qr\left(\frac{\frac{\frac{\int\frac{\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{c+dx}d(c+dx)}{dg-ch}-\frac{\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(dg-ch+h(c+dx))}-\frac{\log(dg-ch+h(c+dx))}{h(dg-ch)}\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(d}{}\right.}{}\right.\right.}{}$$

$\downarrow$ 2838

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$\frac{2dqr\left(qr\left(\frac{\frac{\frac{\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{dg-ch}-\frac{\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(dg-ch+h(c+dx))}-\frac{\log(dg-ch+h(c+dx))}{h(dg-ch)}\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(dg-ch}{(dg-}\right.}{}\right.\right.}{}$$

$$\frac{2bpr\left(pr\left(\frac{\frac{\frac{\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{bg-ah}-\frac{\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(bg-ah+h(a+bx))}-\frac{\log(bg-ah+h(a+bx))}{h(bg-ah)}\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(bg-ah}{(bg-}\right.}{}\right.\right.}{}$$

$\downarrow$ 2865

3.42. $\int\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$\frac{2dqr\left(qr\left(\frac{\frac{\frac{\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{dg-ch}-\frac{\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(dg-ch+h(c+dx))}-\frac{\log(dg-ch+h(c+dx))}{h(dg-ch)}\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(dg-c\ldots}{(dg-\ldots}\right.}{\ldots}\right.\right.}{\ldots}$$

$$\frac{2bpr\left(pr\left(\frac{\frac{\frac{\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{bg-ah}-\frac{\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(bg-ah+h(a+bx))}-\frac{\log(bg-ah+h(a+bx))}{h(bg-ah)}\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(bg-a\ldots}{(bg-\ldots}\right.}{\ldots}\right.\right.}{\ldots}$$

$\downarrow$ 2009

$$-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+$$

$$\frac{2dqr\left(qr\left(\frac{\frac{\frac{\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{dg-ch}-\frac{\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{(c+dx)\log(c+dx)}{(dg-ch)(dg-ch+h(c+dx))}-\frac{\log(dg-ch+h(c+dx))}{h(dg-ch)}\right)}{dg-ch}}{dg-ch}-\frac{h\left(\frac{\log(c+dx)}{(dg-ch)^2}-\frac{\log(dg-c\ldots}{(dg-\ldots}\right.}{\ldots}\right.\right.}{\ldots}$$

$$\frac{2bpr\left(pr\left(\frac{\frac{\frac{\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{bg-ah}-\frac{\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{(a+bx)\log(a+bx)}{(bg-ah)(bg-ah+h(a+bx))}-\frac{\log(bg-ah+h(a+bx))}{h(bg-ah)}\right)}{bg-ah}}{bg-ah}-\frac{h\left(\frac{\log(a+bx)}{(bg-ah)^2}-\frac{\log(bg-a\ldots}{(bg-\ldots}\right.}{\ldots}\right.\right.}{\ldots}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

3.42. $\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$

output
```
-1/3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g + h*x)^3) + (2*d*q*r*(-(
(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
])*(1/(2*(d*g - c*h)*(g + h*x)^2) + d/((d*g - c*h)^2*(g + h*x)) + (d^2*Log
[c + d*x])/(d*g - c*h)^3 - (d^2*Log[g + h*x])/(d*g - c*h)^3)) + p*r*(-1/2*
b/((b*g - a*h)*(d*g - c*h)*(g + h*x)) - (b*d*Log[a + b*x])/((b*g - a*h)*(d
*g - c*h)^2) - (b^2*Log[a + b*x])/(2*(b*g - a*h)^2*(d*g - c*h)) + Log[a +
b*x]/(2*(d*g - c*h)*(g + h*x)^2) + (d*Log[a + b*x])/((d*g - c*h)^2*(g + h*
x)) + (d^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(d*g - c*h)^3 + (b
*d*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)^2) + (b^2*Log[g + h*x])/(2*(b*g
- a*h)^2*(d*g - c*h)) - (d^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/
(d*g - c*h)^3 + (d^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d*g - c*h)
^3 - (d^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(d*g - c*h)^3) + d^2*q
*r*(-((h*(-1/2*Log[c + d*x]/(h*(d*g - c*h + h*(c + d*x))^2) + (1/((d*g - c
*h)*(d*g - c*h + h*(c + d*x))) + Log[c + d*x]/(d*g - c*h)^2 - Log[d*g - c*
h + h*(c + d*x)]/(d*g - c*h)^2)/(2*h)))/(d*g - c*h)) + (-((h*(((c + d*x)*L
og[c + d*x])/((d*g - c*h)*(d*g - c*h + h*(c + d*x))) - Log[d*g - c*h + h*(
c + d*x)]/(h*(d*g - c*h))))/(d*g - c*h)) + (-((Log[c + d*x]*Log[1 + (d*g -
 c*h)/(h*(c + d*x))])/(d*g - c*h)) + PolyLog[2, -((d*g - c*h)/(h*(c + d*x)
))]/(d*g - c*h))/(d*g - c*h))/(d*g - c*h))))/(3*h) + (2*b*p*r*(-((p*r*Log[
a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*(1/...
```

3.42.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

3.42. $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx$

rule 2984
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Simp[b*p*r*(
s/(h*(m + 1)))   Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
]^(s - 1)/(a + b*x)), x], x] - Simp[d*q*r*(s/(h*(m + 1)))   Int[(g + h*x)^(
m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[s, 0] && NeQ[m, -1]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.42.4 Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^{p}(dx+c)^{q}\right)^{r}\right)^{2}}{(hx+g)^{4}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

3.42.5 Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)^4}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^4*x^4 + 4*g*h^3*x^3 + 6
*g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**4,x)
```

output
```
Timed out
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4732 vs. 2(1875) = 3750.

Time = 0.80 (sec) , antiderivative size = 4732, normalized size of antiderivative = 2.42

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="maxim
a")
```

output
```
1/3*(2*b^3*f*p*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3
*h^3) + 2*d^3*f*q*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 -
c^3*h^3) - 2*(3*a*b^2*d^3*f*g^2*h*q - 3*a^2*b*d^3*f*g*h^2*q + a^3*d^3*f*h^
3*q - (d^3*f*g^3*(p + q) - 3*c*d^2*f*g^2*h*p + 3*c^2*d*f*g*h^2*p - c^3*f*h
^3*p)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 -
c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*
h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^
4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3)
+ ((3*d^2*f*g*h^2*q - c*d*f*h^3*q)*a^2 - (d^2*f*g^2*h*(p + 6*q) - 2*c*d*f*
g*h^2*(p + q) + c^2*f*h^3*p)*a*b - (c*d*f*g^2*h*(6*p + q) - 3*d^2*f*g^3*(p
 + q) - 3*c^2*f*g*h^2*p)*b^2 - 2*(2*a*b*d^2*f*g*h^2*q - a^2*d^2*f*h^3*q -
(d^2*f*g^2*h*(p + q) - 2*c*d*f*g*h^2*p + c^2*f*h^3*p)*b^2)*x)/((d^2*g^4*h^
2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*
g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 -
 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*
a*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h
^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2
*g^2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x))*r*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) + 1/3*(2*(3*a*b^2*d^3*f^2*g^2*h*p*q
- 3*a^2*b*d^3*f^2*g*h^2*p*q + a^3*d^3*f^2*h^3*p*q - (3*c*d^2*f^2*g^2*h*...
```

3.42.8 Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)^4}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^4, x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2}{\left(g+h\,x\right)^4}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4, x)
```

3.43 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.43.1 Optimal result

Integrand size = 40, antiderivative size = 42

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

output
```
-(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1+n)/b/c/(1+n)
```

3.43.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

input
```
Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
-((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

3.43.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2973, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7237}$$

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

input
```
Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
-((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

3.43.3.1 Defintions of rubi rules used

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.43.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^n}{-x^2c^2+1}dx$$

input
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{bcn+bc}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algori
thm="fricas")
```

output
```
-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)*(b*log(sqrt(-c*x + 1)/sqrt(c*x
+ 1)) + a)^n/(b*c*n + b*c)
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(32) = 64$.

Time = 78.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \begin{cases} -\frac{a^n \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ a^n x & \text{for } c=0 \\ -\frac{\begin{cases} \frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n+1}}{n+1} & \text{for } n\neq -1 \\ \log\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right) & \text{otherwise} \end{cases}}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
Piecewise((-a**n*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a
**n*x, Eq(c, 0)), (-Piecewise(((a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**
(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))
, True))/(b*c), True))
```

3.43.7 Maxima [F]

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algori
thm="maxima")
```

output
```
-integrate((b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(-\frac{1}{2}\,b\log\left(cx+1\right)+\frac{1}{2}\,b\log\left(-cx+1\right)+a\right)^{n+1}}{bc(n+1)}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algori
thm="giac")
```

output
```
-(-1/2*b*log(c*x + 1) + 1/2*b*log(-c*x + 1) + a)^(n + 1)/(b*c*(n + 1))
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\int \frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.44 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.44.1 Optimal result

Integrand size = 40, antiderivative size = 37

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

output
```
-1/4*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c
```

3.44.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

input
```
Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
-1/4*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c)
```

3.44.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2976, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2976}$$

$$-2c\int \frac{(cx+1)\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)^3}{4c^2(1-cx)}d\frac{1-cx}{cx+1}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(cx+1)\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)^3}{1-cx}d\frac{1-cx}{cx+1}}{2c}$$

$$\Big\downarrow \text{2739}$$

$$-\frac{\int \frac{(1-cx)^3}{(cx+1)^3}d\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{15}$$

$$-\frac{(1-cx)^4}{4bc(cx+1)^4}$$

input
```
Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
-1/4*(1 - c*x)^4/(b*c*(1 + c*x)^4)
```

3.44.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.44.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^{3}}{-x^{2}c^{2}+1}dx$$

input
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

output
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(31) = 62.

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.73

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{b^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4+4\,ab^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+6\,a^2b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+4\,a^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{4\,c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algori
thm="fricas")
```

output
```
-1/4*(b^3*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^4 + 4*a*b^2*log(sqrt(-c*x + 1)
/sqrt(c*x + 1))^3 + 6*a^2*b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 4*a^3*lo
g(sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.

Time = 4.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \begin{cases} -\frac{a^3\,\mathrm{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ a^3x & \text{for } c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^4}{4bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
Piecewise((-a**3*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a
**3*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**4/(4*b*c),
True))
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(31) = 62.

Time = 0.24 (sec) , antiderivative size = 526, normalized size of antiderivative = 14.22

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \frac{1}{2}b^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + \frac{3}{2}ab^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + \frac{3}{2}a^2b\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + \frac{1}{64}\left(\frac{24\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c}+\frac{8\left(\log(cx+1)^3-\right.}{} \right. + \frac{1}{8}ab^2\left(\frac{6\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log(cx+1)^3-3}{}\right. + \frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right) + \frac{3\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)a^2b}{8c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algori
thm="maxima")
```

output
```
1/2*b^3*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)
)^3 + 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 + 3/2*a^2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x +
1)/sqrt(c*x + 1)) + 1/64*(24*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1)
 + log(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2/c + 8*(log(c*x + 1)
^3 - 3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c
*x - 1)^3)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^4 - 4*log(c
*x + 1)^3*log(c*x - 1) + 6*log(c*x + 1)^2*log(c*x - 1)^2 - 4*log(c*x + 1)*
log(c*x - 1)^3 + log(c*x - 1)^4)/c)*b^3 + 1/8*a*b^2*(6*(log(c*x + 1)^2 - 2
*log(c*x + 1)*log(c*x - 1) + log(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x +
 1))/c + (log(c*x + 1)^3 - 3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*
log(c*x - 1)^2 - log(c*x - 1)^3)/c) + 1/2*a^3*(log(c*x + 1)/c - log(c*x -
1)/c) + 3/8*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1)^2
)*a^2*b/c
```

3.44.8 Giac [F]

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algori
thm="giac")
```

output
```
integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.45 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.45.1 Optimal result

Integrand size = 40, antiderivative size = 37

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

output
```
-1/3*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

input
```
Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-1/3*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c)
```

3.45.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2976, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2976}$$

$$-2c\int \frac{(cx+1)\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)^2}{4c^2(1-cx)}d\frac{1-cx}{cx+1}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(cx+1)\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)^2}{1-cx}d\frac{1-cx}{cx+1}}{2c}$$

$$\Big\downarrow \text{2739}$$

$$-\frac{\int \frac{(1-cx)^2}{(cx+1)^2}d\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{15}$$

$$-\frac{(1-cx)^3}{3bc(cx+1)^3}$$

input
```
Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-1/3*(1 - c*x)^3/(b*c*(1 + c*x)^3)
```

3.45.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.45.4 Maple [F]

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^2}{-x^2c^2+1}dx$$

input
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

output
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{b^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{3\,c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algori
thm="fricas")
```

output
```
-1/3*(b^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b*log(sqrt(-c*x + 1)/s
qrt(c*x + 1))^2 + 3*a^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 3.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \begin{cases} -\frac{a^2\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ a^2x & \text{for } c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{3bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
Piecewise((-a**2*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a
**2*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**3/(3*b*c),
True))
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(31) = 62$.

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 7.24

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \frac{1}{2}b^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + ab\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + \frac{1}{24}b^2\left(\frac{6\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log(cx+1)^3-3\right. + \frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right) + \frac{\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)ab}{4c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algori
thm="maxima")
```

output
```
1/2*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)
)^2 + a*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x +
1)) + 1/24*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x
- 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^3 - 3*log(c*x
+ 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1)^3)/c) +
 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/4*(log(c*x + 1)^2 - 2*log(c
*x + 1)*log(c*x - 1) + log(c*x - 1)^2)*a*b/c
```

3.45.8 Giac [F]

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algori
thm="giac")
```

output
```
integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.46 $\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.46.1 Optimal result

Integrand size = 38, antiderivative size = 37

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

output
```
-1/2*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c
```

3.46.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

input
```
Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-1/2*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c)
```

3.46.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2976, 27, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{2976}$$

$$-2c\int \frac{(cx+1)\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)}{4c^2(1-cx)}d\frac{1-cx}{cx+1}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(cx+1)\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)}{1-cx}d\frac{1-cx}{cx+1}}{2c}$$

$$\Big\downarrow \text{2738}$$

$$-\frac{\left(a+b\log\left(\sqrt{\frac{1-cx}{cx+1}}\right)\right)^2}{2bc}$$

input
```
Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-1/2*(a + b*Log[Sqrt[(1 - c*x)/(1 + c*x)]])^2/(b*c)
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.46.4 Maple [F]

$$\int \frac{a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)}{-x^2c^2+1}dx$$

input
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

output
```
int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,a\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{2\,c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorith
m="fricas")
```

output
```
-1/2*(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*log(sqrt(-c*x + 1)/sqrt(
c*x + 1)))/c
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.

Time = 3.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \begin{cases} -\frac{a\,\mathrm{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ ax & \text{for } c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{2bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
Piecewise((-a*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*x,
 Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**2/(2*b*c), True))
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(31) = 62$.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.84

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{1}{2}b\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + \frac{1}{2}a\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right) + \frac{\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)b}{8c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorith
m="maxima")
```

output
```
1/2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(log(c*x + 1)^2 - 2*log(c*
x + 1)*log(c*x - 1) + log(c*x - 1)^2)*b/c
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.32

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{b\log(cx+1)^2}{8c}+\frac{b\log(cx-1)^2}{8c} + \frac{1}{4}\left(\frac{b\log(cx+1)}{c}-\frac{b\log(cx-1)}{c}\right)\log(-cx+1) + \frac{a\log(cx+1)}{2c}-\frac{a\log(cx-1)}{2c}$$

input
```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorith
m="giac")
```

output
```
-1/8*b*log(c*x + 1)^2/c + 1/8*b*log(c*x - 1)^2/c + 1/4*(b*log(c*x + 1)/c -
 b*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*a*log(c*x + 1)/c - 1/2*a*log(c*x -
1)/c
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.47 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.47.1 Optimal result

Integrand size = 40, antiderivative size = 34

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

output
```
-ln(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/b/c
```

3.47.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
-(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

3.47.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2973, 7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \text{7235}$$

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
-(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

3.47.3.1 Defintions of rubi rules used

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

3.47.4 Maple [F]

$$\int \frac{1}{(-x^2c^2+1)\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}{bc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algori
thm="fricas")
```

output
```
-log(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(b*c)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \begin{cases} \frac{x}{a} & \text{for } b=0 \wedge c=0 \\ \frac{-\frac{\log\left(x-\frac{1}{c}\right)}{2c}+\frac{\log\left(x+\frac{1}{c}\right)}{2c}}{a} & \text{for } b=0 \\ \frac{x}{a} & \text{for } c=0 \\ -\frac{\log\left(\frac{a}{b}+\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/c)
/(2*c))/a, Eq(b, 0)), (x/a, Eq(c, 0)), (-log(a/b + log(sqrt(-c*x + 1)/sqrt
(c*x + 1)))/(b*c), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(-\frac{b\log(cx+1)-b\log(-cx+1)-2\,a}{2\,b}\right)}{bc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algori
thm="maxima")
```

output
```
-log(-1/2*(b*log(c*x + 1) - b*log(-c*x + 1) - 2*a)/b)/(b*c)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(-b\log\left(cx+1\right)+b\log\left(-cx+1\right)+2\,a\right)}{bc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algori
thm="giac")
```

output
```
-log(-b*log(c*x + 1) + b*log(-c*x + 1) + 2*a)/(b*c)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

input
```
int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.48 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

3.48.1 Optimal result

Integrand size = 40, antiderivative size = 34

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

output
```
1/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

3.48.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2973, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

3.48.3.1 Defintions of rubi rules used

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.48.4 Maple [F]

$$\int \frac{1}{\left(-x^2c^2+1\right)\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \frac{1}{b^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+abc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algo
rithm="fricas")
```

output
```
1/(b^2*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a*b*c)
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(26) = 52.

Time = 74.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for } b=0 \wedge c=0 \\ \frac{-\frac{\log\left(x-\frac{1}{c}\right)}{2c}+\frac{\log\left(x+\frac{1}{c}\right)}{2c}}{a^2} & \text{for } b=0 \\ \frac{x}{a^2} & \text{for } c=0 \\ \frac{1}{abc+b^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

output
```
Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1
/c)/(2*c))/a**2, Eq(b, 0)), (x/a**2, Eq(c, 0)), (1/(a*b*c + b**2*c*log(sqr
t(-c*x + 1)/sqrt(c*x + 1))), True))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algo
rithm="maxima")
```

output
```
-2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algo
rithm="giac")
```

output
```
-2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.49 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$

3.49.1 Optimal result

Integrand size = 40, antiderivative size = 37

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx=\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

output
```
1/2/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
```

3.49.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx=\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

output
```
1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

3.49.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2973, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}dx$$
$$\Big\downarrow \text{7237}$$
$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

output
```
1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

3.49.3.1 Defintions of rubi rules used

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.49.4 Maple [F]

$$\int \frac{1}{\left(-x^{2} c^{2}+1\right) \left(a+b \ln \left(\frac{\sqrt{-x c+1}}{\sqrt{x c+1}}\right)\right)^{3}}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = \frac{1}{2\left(b^3c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2bc\right)}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algo
rithm="fricas")
```

output
```
1/2/(b^3*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b^2*c*log(sqrt(-c*x +
 1)/sqrt(c*x + 1)) + a^2*b*c)
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3,x)
```

output
```
Timed out
```

3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$$
$$= \frac{2}{b^3c\log(cx+1)^2+b^3c\log(-cx+1)^2-4\,ab^2c\log(cx+1)+4\,a^2bc-2\,(b^3c\log(cx+1)-2\,ab^2c)\log(-cx+1)}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algo
rithm="maxima")
```

output
```
2/(b^3*c*log(c*x + 1)^2 + b^3*c*log(-c*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) +
 4*a^2*b*c - 2*(b^3*c*log(c*x + 1) - 2*a*b^2*c)*log(-c*x + 1))
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.30

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$$
$$= \frac{2}{b^3c\log(cx+1)^2-2\,b^3c\log(cx+1)\log(-cx+1)+b^3c\log(-cx+1)^2-4\,ab^2c\log(cx+1)+4\,ab^2c\log(-cx+1)+4\,a^2bc}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algo
rithm="giac")
```

output
```
2/(b^3*c*log(c*x + 1)^2 - 2*b^3*c*log(c*x + 1)*log(-c*x + 1) + b^3*c*log(-
c*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) + 4*a*b^2*c*log(-c*x + 1) + 4*a^2*b*c)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = -\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)), x)
```

3.50 $\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.50.1 Optimal result

Integrand size = 34, antiderivative size = 30

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/a
```

3.50.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

input
```
Integrate[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-1/2*Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/a
```

3.50.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2973, 2976, 27, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2} \, dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{\log \left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$
$$\Big\downarrow \text{2976}$$
$$-2a\int \frac{(ax+1)\log \left(\sqrt{\frac{1-ax}{ax+1}}\right)}{4a^2(1-ax)}d\frac{1-ax}{ax+1}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{(ax+1)\log \left(\sqrt{\frac{1-ax}{ax+1}}\right)}{1-ax}d\frac{1-ax}{ax+1}}{2a}$$
$$\Big\downarrow \text{2738}$$
$$-\frac{\log^2\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{2a}$$

input

```
Int[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output

```
-1/2*Log[Sqrt[(1 - a*x)/(1 + a*x)]]^2/a
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.50.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.03

method	result
parts	$\frac{\ln\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\ln(ax+1)}{2a}-\frac{\ln\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\ln(ax-1)}{2a}+\frac{\ln(ax-1)^2}{8a}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{4a}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4a}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}\right.\right.}{}$

input
```
int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*ln(a*x+1)-1/2*ln((-a*x+1)^(1/2)/(a*
x+1)^(1/2))/a*ln(a*x-1)+1/8/a*ln(a*x-1)^2-1/4/a*dilog(1/2*a*x+1/2)-1/4/a*l
n(a*x-1)*ln(1/2*a*x+1/2)-1/2/a*(1/2*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*
x+1/2)-1/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x+1)^2)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{2\,a}$$

input
```
integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
```

output
```
-1/2*log(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/a
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(24) = 48.

Time = 2.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{atan}^2\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{a^2\sqrt{-\frac{1}{a^2}}}$$

input
```
integrate(ln((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-atan(x/sqrt(-1/a**2))**2/(2*a) - log(sqrt(-a*x + 1)/sqrt(a*x + 1))*atan(x
/sqrt(-1/a**2))/(a**2*sqrt(-1/a**2))
```

3.50.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(24) = 48.

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{1}{2}\left(\frac{\log\left(ax+1\right)}{a} - \frac{\log\left(ax-1\right)}{a}\right)\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) + \frac{\log\left(ax-1\right)^2}{8\,a} + \frac{\log\left(ax+1\right)^2 - 2\,\log\left(ax+1\right)\log\left(ax-1\right)}{8\,a}$$

input
```
integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

output
```
1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(sqrt(-a*x + 1)/sqrt(a*x + 1)) +
1/8*log(a*x - 1)^2/a + 1/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1))/
a
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{1}{4}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log(-ax+1) - \frac{\log(ax+1)^2}{8a} + \frac{\log(ax-1)^2}{8a}$$

input
```
integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
```

output
```
1/4*(log(a*x + 1)/a - log(a*x - 1)/a)*log(-a*x + 1) - 1/8*log(a*x + 1)^2/a
 + 1/8*log(a*x - 1)^2/a
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\ln\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
-int(log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.51 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{gk+hkx}\,dx$

3.51.1 Optimal result

Integrand size = 48, antiderivative size = 410

$$\begin{aligned}
&\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} \\
&\quad+\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} \\
&\quad-\frac{pr(s+t\log\left(i(g+hx)^n\right))^2\operatorname{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk} \\
&\quad-\frac{qr(s+t\log\left(i(g+hx)^n\right))^2\operatorname{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk} \\
&\quad+\frac{2nprt(s+t\log\left(i(g+hx)^n\right))\operatorname{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk} \\
&\quad+\frac{2nqrt(s+t\log\left(i(g+hx)^n\right))\operatorname{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{hk} \\
&\quad-\frac{2n^2prt^2\operatorname{PolyLog}\left(4,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{2n^2qrt^2\operatorname{PolyLog}\left(4,\frac{d(g+hx)}{dg-ch}\right)}{hk}
\end{aligned}$$

output
```
-1/3*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-1/3*q*r
*ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t+1/3*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)
^n))^2*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))^2*pol
ylog(2,d*(h*x+g)/(-c*h+d*g))/h/k+2*n*p*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3
,b*(h*x+g)/(-a*h+b*g))/h/k+2*n*q*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,d*(h*
x+g)/(-c*h+d*g))/h/k-2*n^2*p*r*t^2*polylog(4,b*(h*x+g)/(-a*h+b*g))/h/k-2*n
^2*q*r*t^2*polylog(4,d*(h*x+g)/(-c*h+d*g))/h/k
```

3.51.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 958 vs. $2(410) = 820$.

Time = 1.93 (sec) , antiderivative size = 958, normalized size of antiderivative = 2.34

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx =$$

$$-\frac{3prs^2\log\left(\frac{h(a+bx)}{-bg+ah}\right)\log(g+hx)+3qrs^2\log\left(\frac{h(c+dx)}{-dg+ch}\right)\log(g+hx)-3s^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\mathrm{lo}}{}$$

input
```
Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])
^2)/(g*k + h*k*x),x]
```

output
```
-1/3*(3*p*r*s^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] + 3*q*r*s^2
*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] - 3*s^2*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]*Log[g + h*x] - 3*n*p*r*s*t*Log[(h*(a + b*x))/(-(b*g) +
a*h)]*Log[g + h*x]^2 - 3*n*q*r*s*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g
 + h*x]^2 + 3*n*s*t*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 +
n^2*p*r*t^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x]^3 + n^2*q*r*t^2
*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^3 - n^2*t^2*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^3 + 6*p*r*s*t*Log[(h*(a + b*x))/(-(b*g
) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] + 6*q*r*s*t*Log[(h*(c + d*x))/(-
(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 6*s*t*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 3*n*p*r*t^2*Log[(h*(a +
 b*x))/(-(b*g) + a*h)]*Log[g + h*x]^2*Log[i*(g + h*x)^n] - 3*n*q*r*t^2*Log
[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^2*Log[i*(g + h*x)^n] + 3*n*t^2
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2*Log[i*(g + h*x)^n] +
3*p*r*t^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n
]^2 + 3*q*r*t^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g +
h*x)^n]^2 - 3*t^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]*Log[i*
(g + h*x)^n]^2 + 3*p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g + h*x
))/(b*g - a*h)] + 3*q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (d*(g + h*
x))/(d*g - c*h)] - 6*n*p*r*s*t*PolyLog[3, (b*(g + h*x))/(b*g - a*h)] - ...
```

3.51.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2985, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(t\log (i(g+hx)^n)+s)^2 \log (e(f(a+bx)^p(c+dx)^q)^r)}{gk+hkx} \, dx$$

$$\Big\downarrow \text{2985}$$

$$-\frac{bpr\int \frac{(s+t\log(i(g+hx)^n))^3}{a+bx}dx}{3hknt}-\frac{dqr\int \frac{(s+t\log(i(g+hx)^n))^3}{c+dx}dx}{3hknt}+\frac{(t\log (i(g+hx)^n)+s)^3 \log (e(f(a+bx)^p(c+dx)^q)^r)}{3hknt}$$

$$\Big\downarrow \text{2843}$$

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^3}{b}-\frac{3hnt\int\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))^2}{g+hx}dx}{b}\right)}{3hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^3}{d}-\frac{3hnt\int\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))^2}{g+hx}dx}{d}\right)}{3hknt}+\frac{(t\log(i(g+hx)^n)+s)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{3hknt}$$

$$\Big\downarrow \ 2881$$

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^3}{b}-\frac{3nt\int\frac{(s+t\log(i(g+hx)^n))^2\log\left(-\frac{\left(a-\frac{bg}{h}\right)h+b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)}{b}\right)}{3hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^3}{d}-\frac{3nt\int\frac{(s+t\log(i(g+hx)^n))^2\log\left(-\frac{\left(c-\frac{dg}{h}\right)h+d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)}{d}\right)}{3hknt}+\frac{(t\log(i(g+hx)^n)+s)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{3hknt}$$

$$\Big\downarrow \ 2821$$

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^3}{b}-\frac{3nt\left(2nt\int\frac{(s+t\log(i(g+hx)^n))\operatorname{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)-\operatorname{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)(t\log(i(g+h\right.}{b}\right.}{3hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^3}{d}-\frac{3nt\left(2nt\int\frac{(s+t\log(i(g+hx)^n))\operatorname{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)-\operatorname{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)(t\log(i(g+hx\right.}{d}\right.}{3hknt}+\frac{(t\log(i(g+hx)^n)+s)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{3hknt}$$

$$\Big\downarrow \ 2830$$

3.51. $\int\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{gk+hkx}\,dx$

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^3}{b}-\frac{3nt\left(2nt\left(\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)-nt\int\frac{\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)\right)-\text{Po}\right.}{b}\right.}{3hknt}$$

$$\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^3}{d}-\frac{3nt\left(2nt\left(\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)-nt\int\frac{\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)\right)-\text{Poly}\right.}{d}\right.}{3hknt}$$

$$\frac{(t\log(i(g+hx)^n)+s)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{3hknt}$$

$$\Big\downarrow \; 7143$$

$$\frac{(t\log(i(g+hx)^n)+s)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{3hknt}-$$

$$\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^3}{b}-\frac{3nt\left(2nt\left(\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)-nt\,\text{PolyLog}\left(4,\frac{b(g+hx)}{bg-ah}\right)\right)-\text{PolyLog}\left(2,\frac{b(g+}{bg-}\right.\right.}{b}\right.}{3hknt}$$

$$\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^3}{d}-\frac{3nt\left(2nt\left(\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)-nt\,\text{PolyLog}\left(4,\frac{d(g+hx)}{dg-ch}\right)\right)-\text{PolyLog}\left(2,\frac{d(g+}{dg-}\right.\right.}{d}\right.}{3hknt}$$

input
```
Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^2)/(g
*k + h*k*x),x]
```

output
```
(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^3)/(3*h*k
*n*t) - (b*p*r*((Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^
n])^3)/b - (3*n*t*(-((s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g + h*x))
/(b*g - a*h)]) + 2*n*t*((s + t*Log[i*(g + h*x)^n])*PolyLog[3, (b*(g + h*x)
)/(b*g - a*h)] - n*t*PolyLog[4, (b*(g + h*x))/(b*g - a*h)])))/b))/(3*h*k*n
*t) - (d*q*r*((Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n]
)^3)/d - (3*n*t*(-((s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (d*(g + h*x))/(
d*g - c*h)]) + 2*n*t*((s + t*Log[i*(g + h*x)^n])*PolyLog[3, (d*(g + h*x))/
(d*g - c*h)] - n*t*PolyLog[4, (d*(g + h*x))/(d*g - c*h)])))/d))/(3*h*k*n*t
)
```

3.51. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{gk+hkx}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2985
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Simp[b*p*(r/(k*n*
t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Si
mp[d*q*(r/(k*n*t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*
x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}
, x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.51.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)\left(s+t\ln \left(i(hx+g)^n\right)\right)^2}{hkx+gk}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

3.51.5 Fricas [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \int \frac{\left(t\log \left((hx+g)^n i\right)+s\right)^2\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x
+g*k),x, algorithm="fricas")
```

output
```
integral((t^2*log((h*x + g)^n*i)^2 + 2*s*t*log((h*x + g)^n*i) + s^2)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))**2/(h*
k*x+g*k),x)
```

output
```
Timed out
```

3.51.7 Maxima [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \int \frac{(t\log\left((hx+g)^n i\right)+s)^2\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x
+g*k),x, algorithm="maxima")
```

output
```
1/3*((n^2*t^2*log(h*x + g)^3 + 3*t^2*log(h*x + g)*log((h*x + g)^n)^2 - 3*(
n*t^2*log(i) + n*s*t)*log(h*x + g)^2 + 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^
2)*log(h*x + g) - 3*(n*t^2*log(h*x + g)^2 - 2*(t^2*log(i) + s*t)*log(h*x +
 g))*log((h*x + g)^n))*log(((b*x + a)^p)^r) + (n^2*t^2*log(h*x + g)^3 + 3*
t^2*log(h*x + g)*log((h*x + g)^n)^2 - 3*(n*t^2*log(i) + n*s*t)*log(h*x + g
)^2 + 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)*log(h*x + g) - 3*(n*t^2*log(h*
x + g)^2 - 2*(t^2*log(i) + s*t)*log(h*x + g))*log((h*x + g)^n))*log(((d*x
+ c)^q)^r))/(h*k) - integrate(-1/3*(3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)
*h*log(e) + (r*t^2*log(i)^2 + 2*r*s*t*log(i) + r*s^2)*h*log(f))*b*d*x^2 -
((p*r + q*r)*b*d*h*n^2*t^2*x^2 + b*c*g*n^2*p*r*t^2 + a*d*g*n^2*q*r*t^2 + (
a*d*h*n^2*q*r*t^2 + (c*h*n^2*p*r*t^2 + (p*r + q*r)*d*g*n^2*t^2)*b)*x)*log(
h*x + g)^3 + 3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (r*t^2*log(
i)^2 + 2*r*s*t*log(i) + r*s^2)*h*log(f))*a*c + 3*(((p*r + q*r)*n*t^2*log(i
) + (p*r*s + q*r*s)*n*t)*b*d*h*x^2 + (n*p*r*t^2*log(i) + n*p*r*s*t)*b*c*g
+ (n*q*r*t^2*log(i) + n*q*r*s*t)*a*d*g + ((n*q*r*t^2*log(i) + n*q*r*s*t)*a
*d*h + (((p*r + q*r)*n*t^2*log(i) + (p*r*s + q*r*s)*n*t)*d*g + (n*p*r*t^2*
log(i) + n*p*r*s*t)*c*h)*b)*x)*log(h*x + g)^2 + 3*((h*r*t^2*log(f) + h*t^2
*log(e))*b*d*x^2 + (h*r*t^2*log(f) + h*t^2*log(e))*a*c + ((h*r*t^2*log(f)
+ h*t^2*log(e))*b*c + (h*r*t^2*log(f) + h*t^2*log(e))*a*d)*x - ((p*r + q*r
)*b*d*h*t^2*x^2 + b*c*g*p*r*t^2 + a*d*g*q*r*t^2 + (a*d*h*q*r*t^2 + (c*h...
```

3.51.8 Giac [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \int \frac{(t\log\left((hx+g)^n i\right)+s)^2\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x
+g*k),x, algorithm="giac")
```

output
```
integrate((t*log((h*x + g)^n*i) + s)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e
)/(h*k*x + g*k), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \int \frac{\ln \left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,{\left(s+t\,\ln \left(i\,{\left(g+h\,x\right)}^n\right)\right)}^2}{g\,k+h\,k\,x}\,dx$$

input
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g
*k + h*k*x),x)
```

output
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g
*k + h*k*x), x)
```

3.52 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))}{gk+hkx}\,dx$

3.52.1 Optimal result

Integrand size = 46, antiderivative size = 306

$$\begin{aligned}
&\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad+\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad-\frac{pr\left(s+t\log\left(i(g+hx)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk} \\
&\quad-\frac{qr\left(s+t\log\left(i(g+hx)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk} \\
&\quad+\frac{nprt\operatorname{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk}+\frac{nqrt\operatorname{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{hk}
\end{aligned}$$

output
```
-1/2*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-1/2*q*r
*ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t+1/2*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)
^n))*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))*polylog
(2,d*(h*x+g)/(-c*h+d*g))/h/k+n*p*r*t*polylog(3,b*(h*x+g)/(-a*h+b*g))/h/k+n
*q*r*t*polylog(3,d*(h*x+g)/(-c*h+d*g))/h/k
```

3.52.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.42

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx$$
$$= \frac{-2prs\log \left(\frac{h(a+bx)}{-bg+ah}\right)\log(g+hx)-2qrs\log \left(\frac{h(c+dx)}{-dg+ch}\right)\log(g+hx)+2s\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g}{}$$

input
```
Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])
)/(g*k + h*k*x),x]
```

output
```
(-2*p*r*s*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] - 2*q*r*s*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] + 2*s*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]*Log[g + h*x] + n*p*r*t*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g +
h*x]^2 + n*q*r*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^2 - n*t*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 - 2*p*r*t*Log[(h*(a + b*
x))/(-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*q*r*t*Log[(h*(c +
d*x))/(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] + 2*t*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*p*r*(s + t*Log[
i*(g + h*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] - 2*q*r*(s + t*Log[i
*(g + h*x)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] + 2*n*p*r*t*PolyLog[3
, (b*(g + h*x))/(b*g - a*h)] + 2*n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c
*h)])/(2*h*k)
```

3.52.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2985, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(t\log \left(i(g+hx)^n\right)+s\right)\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx}\,dx$$
$$\Big\downarrow \; 2985$$

$$-\frac{bpr\int \frac{(s+t\log(i(g+hx)^n))^2}{a+bx}dx}{2hknt}-\frac{dqr\int \frac{(s+t\log(i(g+hx)^n))^2}{c+dx}dx}{2hknt}+\frac{(t\log\left(i(g+hx)^n\right)+s)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2hknt}$$

$\downarrow$ 2843

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^2}{b}-\frac{2hnt\int \frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))}{g+hx}dx}{b}\right)}{2hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^2}{d}-\frac{2hnt\int \frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))}{g+hx}dx}{d}\right)}{2hknt}+\frac{(t\log\left(i(g+hx)^n\right)+s)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2hknt}$$

$\downarrow$ 2881

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^2}{b}-\frac{2nt\int \frac{(s+t\log(i(g+hx)^n))\log\left(-\frac{\left(a-\frac{bg}{h}\right)h+b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)}{b}\right)}{2hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^2}{d}-\frac{2nt\int \frac{(s+t\log(i(g+hx)^n))\log\left(-\frac{\left(c-\frac{dg}{h}\right)h+d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)}{d}\right)}{2hknt}+\frac{(t\log\left(i(g+hx)^n\right)+s)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2hknt}$$

$\downarrow$ 2821

3.52. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))}{gk+hkx}\,dx$

$$-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^2}{b}-\frac{2nt\left(nt\int\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)-\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)\right)}{b}\right)}{2hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^2}{d}-\frac{2nt\left(nt\int\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)-\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)\right)}{d}\right)}{2hknt}+\frac{(t\log(i(g+hx)^n)+s)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2hknt}$$

$$\Big\downarrow \; 7143$$

$$\frac{(t\log(i(g+hx)^n)+s)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2hknt}-\frac{bpr\left(\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)^2}{b}-\frac{2nt\left(nt\,\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)-\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)(t\log(i(g+hx)^n)+s)\right)}{b}\right)}{2hknt}-\frac{dqr\left(\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)^2}{d}-\frac{2nt\left(nt\,\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)-\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)(t\log(i(g+hx)^n)+s)\right)}{d}\right)}{2hknt}$$

input
```
Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))/(g*k
 + h*k*x),x]
```

output
```
(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*k
*n*t) - (b*p*r*((Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^
n])^2)/b - (2*n*t*(-((s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g + h*x))/(
b*g - a*h)]) + n*t*PolyLog[3, (b*(g + h*x))/(b*g - a*h)]))/b))/(2*h*k*n*t)
 - (d*q*r*((Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^2
)/d - (2*n*t*(-((s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d*(g + h*x))/(d*g -
 c*h)]) + n*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)]))/d))/(2*h*k*n*t)
```

3.52. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))}{gk+hkx}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2985
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Simp[b*p*(r/(k*n*
t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Si
mp[d*q*(r/(k*n*t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*
x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}
, x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.52.4 Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^{p} (dx+c)^{q}\right)^{r}\right) \left(s+t\ln \left(i(hx+g)^{n}\right)\right)}{hkx+gk}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)
```

3.52.5 Fricas [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx = \int \frac{\left(t\log \left((hx+g)^n i\right)+s\right)\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g
*k),x, algorithm="fricas")
```

output
```
integral((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(
h*k*x + g*k), x)
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))/(h*k*x
+g*k),x)
```

output
```
Timed out
```

3.52.7 Maxima [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx = \int \frac{(t\log\left((hx+g)^n i\right)+s)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g
*k),x, algorithm="maxima")
```

output
```
-1/2*((n*t*log(h*x + g)^2 - 2*t*log(h*x + g)*log((h*x + g)^n) - 2*(t*log(i
) + s)*log(h*x + g))*log(((b*x + a)^p)^r) + (n*t*log(h*x + g)^2 - 2*t*log(
h*x + g)*log((h*x + g)^n) - 2*(t*log(i) + s)*log(h*x + g))*log(((d*x + c)^
q)^r))/(h*k) - integrate(-1/2*(2*((t*log(i) + s)*h*log(e) + (r*t*log(i) +
r*s)*h*log(f))*b*d*x^2 + 2*((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h
*log(f))*a*c + ((p*r + q*r)*b*d*h*n*t*x^2 + b*c*g*n*p*r*t + a*d*g*n*q*r*t
+ (a*d*h*n*q*r*t + (c*h*n*p*r*t + (p*r + q*r)*d*g*n*t)*b)*x)*log(h*x + g)^
2 + 2*(((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h*log(f))*b*c + ((t*l
og(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h*log(f))*a*d)*x - 2*((p*r*s + q*
r*s + (p*r + q*r)*t*log(i))*b*d*h*x^2 + (p*r*t*log(i) + p*r*s)*b*c*g + (q*
r*t*log(i) + q*r*s)*a*d*g + ((q*r*t*log(i) + q*r*s)*a*d*h + ((p*r*s + q*r*
s + (p*r + q*r)*t*log(i))*d*g + (p*r*t*log(i) + p*r*s)*c*h)*b)*x)*log(h*x
+ g) + 2*((h*r*t*log(f) + h*t*log(e))*b*d*x^2 + (h*r*t*log(f) + h*t*log(e)
)*a*c + ((h*r*t*log(f) + h*t*log(e))*b*c + (h*r*t*log(f) + h*t*log(e))*a*d
)*x - ((p*r + q*r)*b*d*h*t*x^2 + b*c*g*p*r*t + a*d*g*q*r*t + (a*d*h*q*r*t
+ (c*h*p*r*t + (p*r + q*r)*d*g*t)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b
*d*h^2*k*x^3 + a*c*g*h*k + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*
g*h*k + (d*g*h*k + c*h^2*k)*a)*x), x)
```

3.52.8 Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx = \int \frac{(t\log\left((hx+g)^n i\right)+s)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g
*k),x, algorithm="giac")
```

output
```
integrate((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/
(h*k*x + g*k), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx$$
$$=\int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\,\left(s+t\,\ln \left(i\left(g+h\,x\right)^n\right)\right)}{g\,k+h\,k\,x}\,dx$$

input
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
 + h*k*x),x)
```

output
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
 + h*k*x), x)
```

3.53 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{gk+hkx}\,dx$

3.53.1 Optimal result

Integrand size = 32, antiderivative size = 172

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} - \frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

output
```
-p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*k*x+g*k)/h/k-q*r*ln(-h*(d*x+c)/(-c*h+d
*g))*ln(h*k*x+g*k)/h/k+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*k*x+g*k)/h/k-p
*r*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))
/h/k
```

3.53.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx$$
$$= \frac{-pr\log(a+bx)\log(g+hx)-qr\log(c+dx)\log(g+hx)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)+p}{}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

output
```
(-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g +
 h*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*
r*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/
(-(d*g) + c*h)])/(h*k)
```

3.53.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2980, 2841, 27, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx$$
$$\Big\downarrow \text{ 2980}$$
$$-\frac{bpr\int \frac{\log(gk+hxk)}{a+bx}dx}{hk}-\frac{dqr\int \frac{\log(gk+hxk)}{c+dx}dx}{hk}+\frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk}$$
$$\Big\downarrow \text{ 2841}$$

$$-\frac{bpr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{hk\int\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{k(g+hx)}dx}{b}\right)}{hk}-\frac{dqr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{hk\int\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{k(g+hx)}dx}{d}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bpr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{h\int\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{g+hx}dx}{b}\right)}{hk}-\frac{dqr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{h\int\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{g+hx}dx}{d}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk}$$

$$\Big\downarrow \text{2840}$$

$$-\frac{bpr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)}{b}\right)}{hk}-\frac{dqr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)}{d}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk}-\frac{bpr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}+\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}\right)}{hk}-\frac{dqr\left(\frac{\log(gk+hkx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}\right)}{hk}$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

3.53. $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{gk+hkx}\,dx$

output
```
(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g*k + h*k*x])/(h*k) - (b*p*r*((L
og[-((h*(a + b*x))/(b*g - a*h))]*Log[g*k + h*k*x])/b + PolyLog[2, (b*(g +
h*x))/(b*g - a*h)]/b))/(h*k) - (d*q*r*((Log[-((h*(c + d*x))/(d*g - c*h))]*
Log[g*k + h*k*x])/d + PolyLog[2, (d*(g + h*x))/(d*g - c*h)]/d))/(h*k)
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2980
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Simp[b*p*(r/h)   Int[Log[g + h*x]/(a + b
*x), x], x] - Simp[d*q*(r/h)   Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

3.53.4 Maple [A] (verified)

Time = 51.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

method	result
parts	$\frac{\ln(e(f(bx+a)^p(dx+c)^q)^r)\ln(hx+g)}{kh} - \frac{r\left(bph\left(\frac{\operatorname{dilog}\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}+\frac{\ln(hx+g)\ln\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}\right)+dqh\left(\frac{\operatorname{dilog}\left(\frac{d(hx+g)+ch}{ch-dg}\right.}{d}\right.\right.}{k\,h^2}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x,method=_RETURNVERBOSE)
```

output
```
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/k*ln(h*x+g)/h-1/k/h^2*r*(b*p*h*(dilog(((h*
x+g)*b+a*h-b*g)/(a*h-b*g))/b+ln(h*x+g)*ln(((h*x+g)*b+a*h-b*g)/(a*h-b*g))/b
)+d*q*h*(dilog((d*(h*x+g)+c*h-d*g)/(c*h-d*g))/d+ln(h*x+g)*ln((d*(h*x+g)+c*
h-d*g)/(c*h-d*g))/d))
```

3.53.5 Fricas [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="frica
s")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k),x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{hk}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{hk}\right)r}{f} - \frac{(fp\log(bx+a)+fq\log(dx+c))r\log(hkx+gk)}{fhk} + \frac{\log(hkx+gk)\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{hk}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="maxim
a")
```

output
```
((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(
b*g - a*h)))*f*p/(h*k) + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1)
+ dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/(h*k))*r/f - (f*p*log(b*x + a) +
f*q*log(d*x + c))*r*log(h*k*x + g*k)/(f*h*k) + log(h*k*x + g*k)*log(((b*x
+ a)^p*(d*x + c)^q*f)^r*e)/(h*k)
```

3.53.8 Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{hkx+gk}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="giac"
)
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{gk+hkx}\,dx=\int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{g\,k+h\,k\,x}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x), x)
```

3.54 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(gk+hkx)(s+t\log(i(g+hx)^n))}\,dx$

3.54.1 Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx=\text{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)},x\right)$$

output
```
Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)
^n)),x)
```

3.54.2 Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])),x]
```

output
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])), x]
```

3.54.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(t\log\left(i(g+hx)^n\right)+s\right)}dx$$

$$\Big\downarrow \; 2987$$

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(t\log\left(i(g+hx)^n\right)+s\right)}dx$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g +
h*x)^n])),x]
```

output
```
$Aborted
```

3.54.3.1 Defintions of rubi rules used

rule 2987
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]^(u_.)*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))
/((j_.) + (k_.)*(x_)), x_Symbol] :> Unintegrable[(Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^u*(s + t*Log[i*(g + h*x)^n])^m)/(j + k*x), x] /; FreeQ[{a, b, c
, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r, u}, x] && NeQ[b*c - a*d, 0]
```

3.54.4 Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hkx+gk\right)\left(s+t\ln\left(i\left(hx+g\right)^{n}\right)\right)}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

3.54.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log \left(\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{(hkx+gk)(t\log \left((hx+g)^n i\right)+s)}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^
n)),x, algorithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s*x + g*k*s + (h*k*t*x
+ g*k*t)*log((h*x + g)^n*i)), x)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)
**n)),x)
```

output
```
Timed out
```

3.54.7 Maxima [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log \left(\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{(hkx+gk)(t\log \left((hx+g)^n i\right)+s)}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^
n)),x, algorithm="maxima")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x
+ g)^n*i) + s)), x)
```

3.54.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log \left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\right)}{(hkx+gk)(t\log \left((hx+g)^n i\right)+s)}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^
n)),x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x
+ g)^n*i) + s)), x)
```

3.54.9 Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{\left(g\,k+h\,k\,x\right)\,\left(s+t\,\ln \left(i\left(g+h\,x\right)^n\right)\right)}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g +
h*x)^n))),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g +
h*x)^n))), x)
```

3.55 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2}\,dx$

3.55.1 Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)
^n))^2,x)
```

3.55.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])^2),x]
```

output
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])^2), x]
```

3.55.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(t\log \left(i(g+hx)^n\right)+s\right)^2}dx$$

$$\Big\downarrow \text{2987}$$

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(t\log \left(i(g+hx)^n\right)+s\right)^2}dx$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g +
h*x)^n])^2),x]
```

output
```
$Aborted
```

3.55.3.1 Defintions of rubi rules used

rule 2987
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]^(u_.)*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))
/((j_.) + (k_.)*(x_)), x_Symbol] :> Unintegrable[(Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^u*(s + t*Log[i*(g + h*x)^n])^m)/(j + k*x), x] /; FreeQ[{a, b, c
, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r, u}, x] && NeQ[b*c - a*d, 0]
```

3.55.4 Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hkx+gk\right)\left(s+t\ln \left(i\left(hx+g\right)^{n}\right)\right)^{2}}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

3.55.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{(hkx+gk)(t\log\left(\left(hx+g\right)^n i\right)+s)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^
n))^2,x, algorithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s^2*x + g*k*s^2 + (h*k*
t^2*x + g*k*t^2)*log((h*x + g)^n*i)^2 + 2*(h*k*s*t*x + g*k*s*t)*log((h*x +
 g)^n*i)), x)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)
**n))**2,x)
```

output
```
Timed out
```

3.55.7 Maxima [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 222, normalized size of antiderivative = 4.62

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)(t\log \left((hx+g)^n i\right)+s)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^
n))^2,x, algorithm="maxima")
```

output
```
-(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e))/(h*k*n*
t^2*log((h*x + g)^n) + (k*n*t^2*log(i) + k*n*s*t)*h) + integrate((b*c*p*r
+ a*d*q*r + (p*r + q*r)*b*d*x)/((k*n*t^2*log(i) + k*n*s*t)*b*d*h*x^2 + (k*
n*t^2*log(i) + k*n*s*t)*a*c*h + ((k*n*t^2*log(i) + k*n*s*t)*b*c*h + (k*n*t
^2*log(i) + k*n*s*t)*a*d*h)*x + (b*d*h*k*n*t^2*x^2 + a*c*h*k*n*t^2 + (b*c*
h*k*n*t^2 + a*d*h*k*n*t^2)*x)*log((h*x + g)^n)), x)
```

3.55.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)(t\log \left((hx+g)^n i\right)+s)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^
n))^2,x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x
+ g)^n*i) + s)^2), x)
```

3.55.9 Mupad [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{(g\,k+h\,k\,x)\,\left(s+t\,\ln\left(i\,(g+h\,x)^n\right)\right)^2}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g +
h*x)^n))^2),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g +
h*x)^n))^2), x)
```

3.56 $\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

3.56.1 Optimal result

Integrand size = 39, antiderivative size = 328

$$
\begin{aligned}
&\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx \\
&= -\frac{pr\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu}+\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu} \\
&\quad -\frac{qr\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu}-pr\log^3\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad -qr\log^3\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)+3prtu\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right) \\
&\quad +3qrtu\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right) \\
&\quad -6prt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(4,-\frac{bx}{a}\right) \\
&\quad -6qrt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(4,-\frac{dx}{c}\right) \\
&\quad +6prt^3u^3\operatorname{PolyLog}\left(5,-\frac{bx}{a}\right)+6qrt^3u^3\operatorname{PolyLog}\left(5,-\frac{dx}{c}\right)
\end{aligned}
$$

output
```
-1/4*p*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+b*x/a)/t/u+1/4*ln(i*(j*(h*x)^t)^u)^4*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/4*q*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+d*x/
c)/t/u-p*r*ln(i*(j*(h*x)^t)^u)^3*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)
^3*polylog(2,-d*x/c)+3*p*r*t*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-b*x/a)+3*q
*r*t*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-d*x/c)-6*p*r*t^2*u^2*ln(i*(j*(h*x)
^t)^u)*polylog(4,-b*x/a)-6*q*r*t^2*u^2*ln(i*(j*(h*x)^t)^u)*polylog(4,-d*x/
c)+6*p*r*t^3*u^3*polylog(5,-b*x/a)+6*q*r*t^3*u^3*polylog(5,-d*x/c)
```

3.56.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1241 vs. $2(328) = 656$.

Time = 1.09 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.78

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,
x]
```

output
```
p*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[a + b*x] - p*r*t^3*u^3*Log[h*x]^4*Log[a
+ b*x] - 3*p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x]
 + 3*p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + 3*p*r*t*u*
Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - 3*p*r*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*L
og[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[a + b*x] + (p*r*t^3*
u^3*Log[h*x]^4*Log[1 + (b*x)/a])/4 - p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)
^t)^u]*Log[1 + (b*x)/a] + (3*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log
[1 + (b*x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a] +
q*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[c + d*x] - q*r*t^3*u^3*Log[h*x]^4*Log[c
+ d*x] - 3*q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x]
 + 3*q*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t*u*
Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - 3*q*r*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*L
og[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] - t^3*u^3*L
og[x]*Log[h*x]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t^3*u^3*Log[h*x
]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/4 + 3*t^2*u^2*Log[x]*Log[h*x]^2*
Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*t^2*u^2*Log[
h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 3*t*u*L
og[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)...
```

3.56.3 Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.205, Rules used = {2895, 2895, 2985, 2754, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}dx$$

$$\Big\downarrow \text{2985}$$

$$-\frac{bpr\int \frac{\log^4\left(i\left(j(hx)^t\right)^u\right)}{a+bx}dx}{4tu}-\frac{dqr\int \frac{\log^4\left(i\left(j(hx)^t\right)^u\right)}{c+dx}dx}{4tu}+\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}$$

$$\Big\downarrow \text{2754}$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{4tu\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(\frac{bx}{a}+1\right)}{x}dx}{b}\right)}{4tu}-\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{4tu\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(\frac{dx}{c}+1\right)}{x}dx}{d}\right)}{4tu}+\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}$$

$$\Big\downarrow \text{2821}$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{4tu\left(3tu\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^3\left(i\left(j(hx)^t\right)^u\right)\right)}{b}\right)}{4tu}-\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{4tu\left(3tu\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{dx}{c}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log^3\left(i\left(j(hx)^t\right)^u\right)\right)}{d}\right)}{4tu}+\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}$$

$$\Big\downarrow \text{2830}$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{4tu\left(3tu\left(\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)-2tu\int\frac{\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{bx}{a}\right)}{x}dx\right)-\text{PolyLog}\left(2,\right.}{b}}{4tu}$$

$$\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{4tu\left(3tu\left(\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)-2tu\int\frac{\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{dx}{c}\right)}{x}dx\right)-\text{PolyLog}\left(2,-\right.}{d}}{4tu}$$

$$\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}$$

$$\Big\downarrow \text{2830}$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{4tu\left(3tu\left(\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)-2tu\left(\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\int\frac{\text{PolyLog}\left(4\right.}{x}\right.\right.\right.}{b}}{4tu}$$

$$\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{4tu\left(3tu\left(\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)-2tu\left(\text{PolyLog}\left(4,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\int\frac{\text{PolyLog}\left(4,-\right.}{x}\right.\right.\right.}{d}}{4tu}$$

$$\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}-$$

$$\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{4tu\left(3tu\left(\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)-2tu\left(\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\,\text{PolyLog}\left(5,-\frac{bx}{a}\right.\right.\right.\right.}{b}}{4tu}$$

$$\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^4\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{4tu\left(3tu\left(\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)-2tu\left(\text{PolyLog}\left(4,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\,\text{PolyLog}\left(5,-\frac{d}{c}\right.\right.\right.\right.}{d}}{4tu}$$

input
```
Int[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

3.56. $\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

output
```
(Log[i*(j*(h*x)^t)^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*t*u) - (b
*p*r*((Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/b - (4*t*u*(-(Log[i*(j*(h*
x)^t)^u]^3*PolyLog[2, -((b*x)/a)]) + 3*t*u*(Log[i*(j*(h*x)^t)^u]^2*PolyLog
[3, -((b*x)/a)] - 2*t*u*(Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((b*x)/a)] - t*u
*PolyLog[5, -((b*x)/a)]))))/b))/(4*t*u) - (d*q*r*((Log[i*(j*(h*x)^t)^u]^4*
Log[1 + (d*x)/c])/d - (4*t*u*(-(Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -((d*x)/
c)]) + 3*t*u*(Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((d*x)/c)] - 2*t*u*(Log[i
*(j*(h*x)^t)^u]*PolyLog[4, -((d*x)/c)] - t*u*PolyLog[5, -((d*x)/c)]))))/d)
)/(4*t*u)
```

3.56.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 2985
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Simp[b*p*(r/(k*n*
t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Si
mp[d*q*(r/(k*n*t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*
x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}
, x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.56.4 Maple [F]

$$\int \frac{\ln \left(i\left(j(hx)^{t}\right)^{u}\right)^{3} \ln \left(e\left(f(bx+a)^{p}(dx+c)^{q}\right)^{r}\right)}{x}dx$$

input
```
int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

output
```
int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

3.56.5 Fricas [F]

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(\left(((hx)^t j)^u i\right)^3\right)}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, alg
orithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(i*(j*(h*x)**t)**u)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(((hx)^t j)^u i\right)^3}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, alg
orithm="maxima")
```

output
```
-1/4*(t^3*u^3*log(x)^4 - 4*(t^3*u^3*log(h) + t^2*u^3*log(j) + t^2*u^2*log(
i))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t^3*u^3*log(h)^2 + t*u^3*log(j
)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2 + 2*(t^2*u^3*log(j) + t^2*u^2*l
og(i))*log(h))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log
(i))*log(x))*log((x^t)^u)^2 - 4*(t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*l
og(i)*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))
*log(h)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log
(i)^2)*log(h))*log(x) - 4*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*lo
g(j) + t*u*log(i))*log(x)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log
(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(x))*log(
(x^t)^u))*log(((b*x + a)^p)^r) - 1/4*(t^3*u^3*log(x)^4 - 4*(t^3*u^3*log(h)
 + t^2*u^3*log(j) + t^2*u^2*log(i))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6
*(t^3*u^3*log(h)^2 + t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2
 + 2*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h))*log(x)^2 + 6*(t*u*log(x)^2
- 2*(t*u*log(h) + u*log(j) + log(i))*log(x))*log((x^t)^u)^2 - 4*(t^3*u^3*l
og(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(
t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 +
 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(x) - 4*(t^2*u^2*log(x)^
3 - 3*(t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*log(x)^2 + 3*(t^2*u^2*l
og(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(...
```

3.56.8 Giac [F]

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$=\int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, alg
orithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x
)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\,\left(j\,(h\,x)^t\right)^u\right)^3}{x}\,dx$$

input
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x,x)
```

output
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x, x)
```

3.57 $\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

3.57.1 Optimal result

Integrand size = 39, antiderivative size = 262

$$\begin{aligned}
&\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx \\
&= -\frac{pr\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu}+\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu} \\
&\quad -\frac{qr\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)}{3tu}-pr\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad -qr\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)+2prtu\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right) \\
&\quad +2qrtu\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right) \\
&\quad -2prt^2u^2\operatorname{PolyLog}\left(4,-\frac{bx}{a}\right)-2qrt^2u^2\operatorname{PolyLog}\left(4,-\frac{dx}{c}\right)
\end{aligned}$$

output
```
-1/3*p*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+b*x/a)/t/u+1/3*ln(i*(j*(h*x)^t)^u)^3*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/3*q*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+d*x/
c)/t/u-p*r*ln(i*(j*(h*x)^t)^u)^2*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)
^2*polylog(2,-d*x/c)+2*p*r*t*u*ln(i*(j*(h*x)^t)^u)*polylog(3,-b*x/a)+2*q*r
*t*u*ln(i*(j*(h*x)^t)^u)*polylog(3,-d*x/c)-2*p*r*t^2*u^2*polylog(4,-b*x/a)
-2*q*r*t^2*u^2*polylog(4,-d*x/c)
```

3.57.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 839 vs. 2(262) = 524.

Time = 0.55 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.20

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= -prt^2u^2\log(x)\log^2(hx)\log(a+bx)+prt^2u^2\log^3(hx)\log(a+bx)$$
$$+2prtu\log(x)\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx)$$
$$-2prtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx)-pr\log(x)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(a+bx)$$
$$+pr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(a+bx)-\frac{1}{3}prt^2u^2\log^3(hx)\log\left(1+\frac{bx}{a}\right)$$
$$+prtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)$$
$$-pr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)-qrt^2u^2\log(x)\log^2(hx)\log(c+dx)$$
$$+qrt^2u^2\log^3(hx)\log(c+dx)+2qrtu\log(x)\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx)$$
$$-2qrtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx)$$
$$-qr\log(x)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(c+dx)+qr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(c+dx)$$
$$+t^2u^2\log(x)\log^2(hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$-\frac{2}{3}t^2u^2\log^3(hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$-2tu\log(x)\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$+tu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$+\log(x)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$-\frac{1}{3}qrt^2u^2\log^3(hx)\log\left(1+\frac{dx}{c}\right)+qrtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)$$
$$-qr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)-pr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right)$$
$$-qr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{dx}{c}\right)+2prtu\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{bx}{a}\right)$$
$$+2qrtu\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{dx}{c}\right)$$
$$-2prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)-2qrt^2u^2\text{PolyLog}\left(4,-\frac{dx}{c}\right)$$

input
```
Integrate[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,
x]
```

output
```
-(p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[a + b*x]) + p*r*t^2*u^2*Log[h*x]^3*Log
[a + b*x] + 2*p*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] -
2*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - p*r*Log[x]*Log[i*
(j*(h*x)^t)^u]^2*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a
+ b*x] - (p*r*t^2*u^2*Log[h*x]^3*Log[1 + (b*x)/a])/3 + p*r*t*u*Log[h*x]^2*
Log[i*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^
2*Log[1 + (b*x)/a] - q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[c + d*x] + q*r*t^2*
u^2*Log[h*x]^3*Log[c + d*x] + 2*q*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^
u]*Log[c + d*x] - 2*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] -
 q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h
*x)^t)^u]^2*Log[c + d*x] + t^2*u^2*Log[x]*Log[h*x]^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r] - (2*t^2*u^2*Log[h*x]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r])/3 - 2*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] + t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r] + Log[x]*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r] - (q*r*t^2*u^2*Log[h*x]^3*Log[1 + (d*x)/c])/3 + q*r*t*u*Log[h*x]
^2*Log[i*(j*(h*x)^t)^u]*Log[1 + (d*x)/c] - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^
u]^2*Log[1 + (d*x)/c] - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((b*x)/a)]
- q*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*Log[i*(j*(
h*x)^t)^u]*PolyLog[3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]*Poly...
```

3.57.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2895, 2895, 2985, 2754, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}dx$$

$$\Big\downarrow \text{2985}$$

$$-\frac{bpr\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)}{a+bx}dx}{3tu}-\frac{dqr\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)}{c+dx}dx}{3tu}+\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu}$$

$$\Big\downarrow \text{2754}$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{3tu\int\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(\frac{bx}{a}+1\right)}{x}dx}{b}\right)}{3tu}-\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{3tu\int\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(\frac{dx}{c}+1\right)}{x}dx}{d}\right)}{3tu}+\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu}$$

$$\Big\downarrow \ 2821$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{3tu\left(2tu\int\frac{\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)}{x}dx-\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)\right)}{b}\right)}{3tu}-\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{3tu\left(2tu\int\frac{\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)}{x}dx-\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)\right)}{d}\right)}{3tu}+\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu}$$

$$\Big\downarrow \ 2830$$

$$-\frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{3tu\left(2tu\left(\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\int\frac{\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)}{x}dx\right)-\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)\right)}{b}\right)}{3tu}-\frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{3tu\left(2tu\left(\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\int\frac{\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right)}{x}dx\right)-\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)\right)}{d}\right)}{3tu}+\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu}$$

$$\Big\downarrow \ 7143$$

3.57. $\int\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}dx$

$$\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu} - \frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{b} - \frac{3tu\left(2tu\left(\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\,\text{PolyLog}\left(4,-\frac{bx}{a}\right)\right)-\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)\right)}{b}\right)}{3tu} - \frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^3\left(i\left(j(hx)^t\right)^u\right)}{d} - \frac{3tu\left(2tu\left(\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)-tu\,\text{PolyLog}\left(4,-\frac{dx}{c}\right)\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)\right)}{d}\right)}{3tu}$$

input
```
Int[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

output
```
(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*t*u) - (b
*p*r*((Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a])/b - (3*t*u*(-(Log[i*(j*(h*
x)^t)^u]^2*PolyLog[2, -((b*x)/a)]) + 2*t*u*(Log[i*(j*(h*x)^t)^u]*PolyLog[3
, -((b*x)/a)] - t*u*PolyLog[4, -((b*x)/a)])))/b))/(3*t*u) - (d*q*r*((Log[i
*(j*(h*x)^t)^u]^3*Log[1 + (d*x)/c])/d - (3*t*u*(-(Log[i*(j*(h*x)^t)^u]^2*P
olyLog[2, -((d*x)/c)]) + 2*t*u*(Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((d*x)/c)
] - t*u*PolyLog[4, -((d*x)/c)])))/d))/(3*t*u)
```

3.57.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 2985
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Simp[b*p*(r/(k*n*
t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Si
mp[d*q*(r/(k*n*t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*
x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}
, x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.57.4 Maple [F]

$$\int \frac{\ln\left(i{\left(j{\left(hx\right)}^{t}\right)}^{u}\right)^{2} \ln\left(e{\left(f{\left(bx+a\right)}^{p} \left(dx+c\right)^{q}\right)}^{r}\right)}{x}dx$$

input
```
int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

output
```
int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

3.57.5 Fricas [F]

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)\log\left(\left(\left(hx\right)^{t}j\right)^{u}i\right)^{2}}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, alg
orithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(i*(j*(h*x)**t)**u)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, alg
orithm="maxima")
```

output
```
1/3*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*log
(x)^2 + 3*log(x)*log((x^t)^u)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u
*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(x) -
 3*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log(x))*log((x^t)^u)
)*log(((b*x + a)^p)^r) + 1/3*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2
*log(j) + t*u*log(i))*log(x)^2 + 3*log(x)*log((x^t)^u)^2 + 3*(t^2*u^2*log(
h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*lo
g(h) + log(i)^2)*log(x) - 3*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log
(i))*log(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(-1/3*(3*((t^2*
u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*lo
g(i))*log(h) + log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2
*r*u*log(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h)
)*log(f))*b*d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^2)*b*d*x^2 + (b*c*p*r*t^2*u^
2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 + 3*((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2
*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e)
 + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2
 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*a*c + 3*((r*log(f) +
log(e))*b*d*x^2 + (r*log(f) + log(e))*a*c + ((r*log(f) + log(e))*b*c + (r*
log(f) + log(e))*a*d)*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*lo
g(x))*log((x^t)^u)^2 + 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*...
```

3.57.8 Giac [F]

$$\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$$
$$=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, alg
orithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x
)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\,\left(j\,(h\,x)^t\right)^u\right)^2}{x}\,dx$$

input
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x,x)
```

output
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x, x)
```

3.58 $\int \frac{\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

3.58.1 Optimal result

Integrand size = 37, antiderivative size = 194

$$\begin{aligned}
&\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx \\
&= -\frac{pr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu}+\frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu} \\
&\quad -\frac{qr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{2tu}-pr\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad -qr\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right) \\
&\quad +prtu\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)+qrtu\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right)
\end{aligned}$$

output
```
-1/2*p*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+b*x/a)/t/u+1/2*ln(i*(j*(h*x)^t)^u)^2*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/2*q*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+d*x/
c)/t/u-p*r*ln(i*(j*(h*x)^t)^u)*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)*p
olylog(2,-d*x/c)+p*r*t*u*polylog(3,-b*x/a)+q*r*t*u*polylog(3,-d*x/c)
```

3.58.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 451 vs. $2(194) = 388$.

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.32

$$
\begin{aligned}
&\int \frac{\log \left(i(j(hx)^t)^u\right) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x} \, dx \\
&= prtu \log(x) \log(hx) \log(a+bx) - prtu \log^2(hx) \log(a+bx) \\
&\quad - pr \log(x) \log \left(i\left(j(hx)^t\right)^u\right) \log(a+bx) + pr \log(hx) \log \left(i\left(j(hx)^t\right)^u\right) \log(a+bx) \\
&\quad + \frac{1}{2} prtu \log^2(hx) \log \left(1+\frac{bx}{a}\right) - pr \log(hx) \log \left(i\left(j(hx)^t\right)^u\right) \log \left(1+\frac{bx}{a}\right) \\
&\quad + qrtu \log(x) \log(hx) \log(c+dx) - qrtu \log^2(hx) \log(c+dx) \\
&\quad - qr \log(x) \log \left(i\left(j(hx)^t\right)^u\right) \log(c+dx) + qr \log(hx) \log \left(i\left(j(hx)^t\right)^u\right) \log(c+dx) \\
&\quad - tu \log(x) \log(hx) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad + \frac{1}{2} tu \log^2(hx) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad + \log(x) \log \left(i\left(j(hx)^t\right)^u\right) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad + \frac{1}{2} qrtu \log^2(hx) \log \left(1+\frac{dx}{c}\right) - qr \log(hx) \log \left(i\left(j(hx)^t\right)^u\right) \log \left(1+\frac{dx}{c}\right) \\
&\quad - pr \log \left(i\left(j(hx)^t\right)^u\right) \operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) - qr \log \left(i\left(j(hx)^t\right)^u\right) \operatorname{PolyLog}\left(2,-\frac{dx}{c}\right) \\
&\quad + prtu \operatorname{PolyLog}\left(3,-\frac{bx}{a}\right) + qrtu \operatorname{PolyLog}\left(3,-\frac{dx}{c}\right)
\end{aligned}
$$

input
```
Integrate[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

output
```
p*r*t*u*Log[x]*Log[h*x]*Log[a + b*x] - p*r*t*u*Log[h*x]^2*Log[a + b*x] - p
*r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^
t)^u]*Log[a + b*x] + (p*r*t*u*Log[h*x]^2*Log[1 + (b*x)/a])/2 - p*r*Log[h*x
]*Log[i*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] + q*r*t*u*Log[x]*Log[h*x]*Log[c +
d*x] - q*r*t*u*Log[h*x]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]*L
og[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - t*u*Log[x]*
Log[h*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (t*u*Log[h*x]^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r] + (q*r*t*u*Log[h*x]^2*Log[1 + (d*x)/c])/2 - q*r*Log
[h*x]*Log[i*(j*(h*x)^t)^u]*Log[1 + (d*x)/c] - p*r*Log[i*(j*(h*x)^t)^u]*Pol
yLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*
r*t*u*PolyLog[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

3.58.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2895, 2895, 2985, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(i\left(j(hx)^t\right)^u\right) \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log \left(i\left(j(hx)^t\right)^u\right) \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}dx$$

$$\Big\downarrow \text{2985}$$

$$-\frac{bpr\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)}{a+bx}dx}{2tu}-\frac{dqr\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)}{c+dx}dx}{2tu}+\frac{\log^2\left(i\left(j(hx)^t\right)^u\right) \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}$$

$$\Big\downarrow \text{2754}$$

$$-\frac{bpr\left(\frac{\log \left(\frac{bx}{a}+1\right) \log^2\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{2tu\int \frac{\log \left(i\left(j(hx)^t\right)^u\right) \log \left(\frac{bx}{a}+1\right)}{x}dx}{b}\right)}{2tu}-\frac{dqr\left(\frac{\log \left(\frac{dx}{c}+1\right) \log^2\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{2tu\int \frac{\log \left(i\left(j(hx)^t\right)^u\right) \log \left(\frac{dx}{c}+1\right)}{x}dx}{d}\right)}{2tu}+\frac{\log^2\left(i\left(j(hx)^t\right)^u\right) \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}$$

$$\Big\downarrow \text{2821}$$

$$-\frac{bpr\left(\frac{\log \left(\frac{bx}{a}+1\right) \log^2\left(i\left(j(hx)^t\right)^u\right)}{b}-\frac{2tu\left(tu\int \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{bx}{a}\right) \log \left(i\left(j(hx)^t\right)^u\right)\right)}{b}\right)}{2tu}-\frac{dqr\left(\frac{\log \left(\frac{dx}{c}+1\right) \log^2\left(i\left(j(hx)^t\right)^u\right)}{d}-\frac{2tu\left(tu\int \frac{\text{PolyLog}\left(2,-\frac{dx}{c}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{dx}{c}\right) \log \left(i\left(j(hx)^t\right)^u\right)\right)}{d}\right)}{2tu}+\frac{\log^2\left(i\left(j(hx)^t\right)^u\right) \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu} - \frac{bpr\left(\frac{\log\left(\frac{bx}{a}+1\right)\log^2\left(i\left(j(hx)^t\right)^u\right)}{b} - \frac{2tu\left(tu\,\text{PolyLog}\left(3,-\frac{bx}{a}\right)-\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)\right)}{b}\right)}{2tu} - \frac{dqr\left(\frac{\log\left(\frac{dx}{c}+1\right)\log^2\left(i\left(j(hx)^t\right)^u\right)}{d} - \frac{2tu\left(tu\,\text{PolyLog}\left(3,-\frac{dx}{c}\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)\right)}{d}\right)}{2tu}$$

input
```
Int[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

output
```
(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*t*u) - (b
*p*r*((Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/b - (2*t*u*(-(Log[i*(j*(h*
x)^t)^u]*PolyLog[2, -((b*x)/a)]) + t*u*PolyLog[3, -((b*x)/a)]))/b))/(2*t*u
) - (d*q*r*((Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x)/c])/d - (2*t*u*(-(Log[i*
(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)]) + t*u*PolyLog[3, -((d*x)/c)]))/d))/
(2*t*u)
```

3.58.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 2985
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Simp[b*p*(r/(k*n*
t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Si
mp[d*q*(r/(k*n*t*(m + 1)))   Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*
x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}
, x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.58.4 Maple [F]

$$\int \frac{\ln\left(i\left(j(hx)^{t}\right)^{u}\right)\ln\left(e\left(f(bx+a)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{x}dx$$

input
```
int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

output
```
int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

3.58.5 Fricas [F]

$$\int \frac{\log\left(i(j(hx)^{t})^{u}\right)\log\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^{p}(dx+c)^{q}f)^{r}\,e\right)\log\left(\left((hx)^{t}\,j\right)^{u}i\right)}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algor
ithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(i*(j*(h*x)**t)**u)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algor
ithm="maxima")
```

output
```
-1/2*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log(x) - 2*log(x)*
log((x^t)^u))*log(((b*x + a)^p)^r) - 1/2*(t*u*log(x)^2 - 2*(t*u*log(h) + u
*log(j) + log(i))*log(x) - 2*log(x)*log((x^t)^u))*log(((d*x + c)^q)^r) - i
ntegrate(-1/2*(2*((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h)
+ r*u*log(j) + r*log(i))*log(f))*b*d*x^2 + 2*((t*u*log(h) + u*log(j) + log
(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*a*c + ((p*r*t
*u + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((t*u
*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i
))*log(f))*b*c + ((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h)
+ r*u*log(j) + r*log(i))*log(f))*a*d)*x + 2*((r*log(f) + log(e))*b*d*x^2 +
 (r*log(f) + log(e))*a*c + ((r*log(f) + log(e))*b*c + (r*log(f) + log(e))*
a*d)*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u
) - 2*(((p*r*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q*r*u)*
log(j))*b*d*x^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c + (q*r
*t*u*log(h) + q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))/(b*d*x^3 + a*c*x
+ (b*c + a*d)*x^2), x)
```

3.58.8 Giac [F]

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)}{x}\,dx$$

input
```
integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algor
ithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\,\left(j\,(h\,x)^t\right)^u\right)}{x}\,dx$$

input
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x,x)
```

output
```
int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x, x)
```

3.59 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

3.59.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = & -pr\log(x)\log\left(1+\frac{bx}{a}\right)\\ & +\log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\\ & -qr\log(x)\log\left(1+\frac{dx}{c}\right)\\ & -pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\end{aligned}$$

output
```
-p*r*ln(x)*ln(1+b*x/a)+ln(x)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-q*r*ln(x)*ln(
1+d*x/c)-p*r*polylog(2,-b*x/a)-q*r*polylog(2,-d*x/c)
```

3.59.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = & \log(x)\left(-pr\log\left(1+\frac{bx}{a}\right)\right.\\ & \left.+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-qr\log\left(1+\frac{dx}{c}\right)\right)\\ & -pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\end{aligned}$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

output
```
Log[x]*(-(p*r*Log[1 + (b*x)/a]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - q
*r*Log[1 + (d*x)/c]) - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[2, -((d*x)
/c)]
```

3.59.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2980, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x} \, dx$$

$$\Big\downarrow \text{2980}$$

$$-bpr\int \frac{\log(x)}{a+bx}dx-dqr\int \frac{\log(x)}{c+dx}dx+\log(x)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)$$

$$\Big\downarrow \text{2754}$$

$$-bpr\left(\frac{\log(x)\log \left(\frac{bx}{a}+1\right)}{b}-\frac{\int \frac{\log \left(\frac{bx}{a}+1\right)}{x}dx}{b}\right)-dqr\left(\frac{\log(x)\log \left(\frac{dx}{c}+1\right)}{d}-\frac{\int \frac{\log \left(\frac{dx}{c}+1\right)}{x}dx}{d}\right)+\log(x)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)$$

$$\Big\downarrow \text{2838}$$

$$\log(x)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)-bpr\left(\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log \left(\frac{bx}{a}+1\right)}{b}\right)-dqr\left(\frac{\text{PolyLog}\left(2,-\frac{dx}{c}\right)}{d}+\frac{\log(x)\log \left(\frac{dx}{c}+1\right)}{d}\right)$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

output
```
Log[x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - b*p*r*((Log[x]*Log[1 + (b*x)
/a])/b + PolyLog[2, -((b*x)/a)]/b) - d*q*r*((Log[x]*Log[1 + (d*x)/c])/d +
PolyLog[2, -((d*x)/c)]/d)
```

3.59.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2980
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Simp[b*p*(r/h)   Int[Log[g + h*x]/(a + b
*x), x], x] - Simp[d*q*(r/h)   Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

3.59.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

method	result
parts	$\ln\left(x\right)\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)-\frac{r\left(bfp\left(\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln\left(x\right)\ln\left(\frac{bx+a}{a}\right)}{b}\right)+dfq\left(\frac{\operatorname{dilog}\left(\frac{dx+c}{c}\right)}{d}+\frac{\ln\left(x\right)\ln\left(\frac{dx+c}{c}\right)}{d}\right)\right)}{f}$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-r/f*(b*f*p*(dilog((b*x+a)/a)/b+ln(x)
*ln((b*x+a)/a)/b)+d*f*q*(dilog((d*x+c)/c)/d+ln(x)*ln((d*x+c)/c)/d))
```

3.59.5 Fricas [F]

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} \, dx = \int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{x} \, dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

3.59.6 Sympy [F]

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} \, dx = \int \frac{\log \left(e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right)}{x} \, dx$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

output
```
Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/x, x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x} \, dx = -\frac{(fp\log (bx+a) + fq\log (dx+c))r\log (x)}{f} + \log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)\log (x) + \frac{\left(\left(\log (bx+a)\log \left(-\frac{bx+a}{a}+1\right) + \mathrm{Li}_2\left(\frac{bx+a}{a}\right)\right)fp + \left(\log (dx+c)\log \left(-\frac{dx+c}{c}+1\right) + \mathrm{Li}_2\left(\frac{dx+c}{c}\right)\right)fq\right)r}{f}$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="maxima")
```

output
```
-(f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(x)/f + log(((b*x + a)^p*(d*x
+ c)^q*f)^r*e)*log(x) + ((log(b*x + a)*log(-(b*x + a)/a + 1) + dilog((b*x
+ a)/a))*f*p + (log(d*x + c)*log(-(d*x + c)/c + 1) + dilog((d*x + c)/c))*f
*q)*r/f
```

3.59.8 Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{x}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{x}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x,x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x, x)
```

3.60 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x\log(i(j(hx)^t)^u)}\,dx$

3.60.1 Optimal result

Integrand size = 39, antiderivative size = 39

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx=\text{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

output
```
CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

3.60.2 Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

output
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x
]
```

3.60.3 Rubi [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}dx$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

output
```
$Aborted
```

3.60.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.60.4 Maple [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e{\left(f{\left(b x +a \right)}^{p} {\left(d x +c \right)}^{q}\right)}^{r}\right)}{x \ln \left(i {\left(j {\left(h x \right)}^{t}\right)}^{u}\right)}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

3.60.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{x\log \left(\left(\left(hx\right)^t j\right)^u i\right)}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algor
ithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u),x)
```

output
```
Timed out
```

3.60.7 Maxima [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{x\log \left(\left(\left(hx\right)^t j\right)^u i\right)}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algor
ithm="maxima")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x
)
```

3.60.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(\left(\left(bx+a\right)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left(\left(hx\right)^t j\right)^u i\right)}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algor
ithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x
)
```

3.60.9 Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{x\,\ln \left(i\left(j\left(h\,x\right)^t\right)^u\right)}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)), x)
```

3.61 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x\log^2(i(j(hx)^t)^u)}\,dx$

3.61.1 Optimal result

Integrand size = 39, antiderivative size = 39

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \operatorname{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

output
```
CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

3.61.2 Mathematica [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

input
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
x]
```

output
```
Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
 x]
```

3.61.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x \log^2\left(i\left(j(hx)^t\right)^u\right)} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x \log^2\left(i\left(j(hx)^t\right)^u\right)}dx$$

input
```
Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x]
```

output
```
$Aborted
```

3.61.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.61.4 Maple [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e(f(bx+a)^p \left(dx+c\right)^q)^r\right)}{x\ln \left(i \left(j \left(hx\right)^t\right)^u\right)^2}dx$$

input
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

output
```
int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

3.61.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, alg
orithm="fricas")
```

output
```
integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2),
x)
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u)**2,x)
```

output
```
Timed out
```

3.61.7 Maxima [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.92

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, alg
orithm="maxima")
```

output
```
-(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e))/(t^2*u^
2*log(h) + t*u^2*log(j) + t*u*log(i) + t*u*log((x^t)^u)) + integrate((b*c*
p*r + a*d*q*r + (p*r + q*r)*b*d*x)/((t^2*u^2*log(h) + t*u^2*log(j) + t*u*l
og(i))*b*d*x^2 + (t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*a*c + ((t^2*
u^2*log(h) + t*u^2*log(j) + t*u*log(i))*b*c + (t^2*u^2*log(h) + t*u^2*log(
j) + t*u*log(i))*a*d)*x + (b*d*t*u*x^2 + a*c*t*u + (b*c*t*u + a*d*t*u)*x)*
log((x^t)^u)), x)
```

3.61.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{x\log\left(\left(\left(hx\right)^t j\right)^u i\right)^2}\,dx$$

input
```
integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, alg
orithm="giac")
```

output
```
integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2),
 x)
```

3.61.9 Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\ln \left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)}{x\ln\left(i\left(j\left(h\,x\right)^t\right)^u\right)^2}\,dx$$

input
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2),x)
```

output
```
int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2), x)
```

3.62 $\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

3.62.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \text{Int}\left(\frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x},x\right)$$

output
```
Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

3.62.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

input
```
Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
```

output
```
Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

3.62.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{x(bc-ad)}\right)}{x}dx$$
$$\Big\downarrow \ 2987$$
$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{x(bc-ad)}\right)}{x}dx$$

input
```
Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
```

output
```
$Aborted
```

3.62.3.1 Defintions of rubi rules used

rule 2987
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]^(u_.)*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))
/((j_.) + (k_.)*(x_)), x_Symbol] :> Unintegrable[(Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^u*(s + t*Log[i*(g + h*x)^n])^m)/(j + k*x), x] /; FreeQ[{a, b, c
, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r, u}, x] && NeQ[b*c - a*d, 0]
```

3.62.4 Maple [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln(x)\ln\left(\frac{bx+a}{(-ad+cb)x}\right)^3}{x}dx$$

input
```
int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

output
```
int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

3.62.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="fricas")
```

output
```
integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

3.62.6 Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \frac{3a\int \frac{\log(x)^2\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)^2}{ax+bx^2}\,dx}{2} + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^3}{2}$$

input
```
integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**3/x,x)
```

output
```
3*a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))**2/(
a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**3/2
```

3.62.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 9.71

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)^3*log(x)^2 - integrate(1/2*(2*(b*x + a)*log(x)^4 + 6*(b*x
*log(b*c - a*d) + a*log(b*c - a*d))*log(x)^3 + 3*((3*b*x + 2*a)*log(x)^2 +
 2*(b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a)^2 + 6*(b*x
*log(b*c - a*d)^2 + a*log(b*c - a*d)^2)*log(x)^2 - 6*((b*x + a)*log(x)^3 +
 2*(b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x)^2 + (b*x*log(b*c - a*d)^
2 + a*log(b*c - a*d)^2)*log(x))*log(b*x + a) + 2*(b*x*log(b*c - a*d)^3 + a
*log(b*c - a*d)^3)*log(x))/(b*x^2 + a*x), x)
```

3.62.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="giac")
```

output
```
integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

3.62.9 Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^3\ln(x)}{x}\,dx$$

input
```
int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x,x)
```

output
```
int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x, x)
```

3.63 $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

3.63.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \text{Int}\left(\frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x},x\right)$$

output
```
Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

3.63.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

input
```
Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
```

output
```
Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

3.63.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{x(bc-ad)}\right)}{x}\,dx$$
$$\Big\downarrow \; 2987$$
$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{x(bc-ad)}\right)}{x}\,dx$$

input
```
Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
```

output
```
$Aborted
```

3.63.3.1 Defintions of rubi rules used

rule 2987
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]^(u_.)*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))
/((j_.) + (k_.)*(x_)), x_Symbol] :> Unintegrable[(Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^u*(s + t*Log[i*(g + h*x)^n])^m)/(j + k*x), x] /; FreeQ[{a, b, c
, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r, u}, x] && NeQ[b*c - a*d, 0]
```

3.63.4 Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(x\right)\ln\left(\frac{bx+a}{\left(-ad+cb\right)x}\right)^{2}}{x}dx$$

input
```
int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

output
```
int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

3.63.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log{(x)}\log{\left(\frac{bx+a}{(bc-ad)x}\right)}^2}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="fricas")
```

output
```
integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

3.63.6 Sympy [N/A]

Not integrable

Time = 6.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = a\int \frac{\log{(x)}^2\log{\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}}{ax+bx^2}\,dx + \frac{\log{(x)}^2\log{\left(\frac{a+bx}{x(-ad+bc)}\right)}^2}{2}$$

input
```
integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**2/x,x)
```

output
```
a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))/(a*x +
 b*x**2), x) + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**2/2
```

3.63.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 5.50

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)^2*log(x)^2 - integrate(-((b*x + a)*log(x)^3 + 2*(b*x*log(
b*c - a*d) + a*log(b*c - a*d))*log(x)^2 - ((3*b*x + 2*a)*log(x)^2 + 2*(b*x
*log(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a) + (b*x*log(b*c -
a*d)^2 + a*log(b*c - a*d)^2)*log(x))/(b*x^2 + a*x), x)
```

3.63.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="giac")
```

output
```
integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

3.63.9 Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^2\,\ln(x)}{x}\,dx$$

input
```
int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x,x)
```

output
```
int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x, x)
```

3.63. $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

3.64 $\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

3.64.1 Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\operatorname{PolyLog}\left(2,-\frac{a}{bx}\right)+\operatorname{PolyLog}\left(3,-\frac{a}{bx}\right)$$

output
```
-1/2*ln(1+a/b/x)*ln(x)^2+1/2*ln(b/(-a*d+b*c)+a/(-a*d+b*c)/x)*ln(x)^2+ln(x)
*polylog(2,-a/b/x)+polylog(3,-a/b/x)
```

3.64.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \frac{1}{6}\log^2(x)\left(\log(x)-3\log\left(1+\frac{bx}{a}\right)+3\log\left(\frac{a+bx}{bcx-adx}\right)\right) - \log(x)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)+\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)$$

input
```
Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

output
```
(Log[x]^2*(Log[x] - 3*Log[1 + (b*x)/a] + 3*Log[(a + b*x)/(b*c*x - a*d*x)])
)/6 - Log[x]*PolyLog[2, -((b*x)/a)] + PolyLog[3, -((b*x)/a)]
```

3.64.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2827, 2822, 27, 2005, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\log\left(\frac{a+bx}{x(bc-ad)}\right)}{x}dx$$

$$\Big\downarrow \text{ 2827}$$

$$\int \frac{\log(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)}{x}dx$$

$$\Big\downarrow \text{ 2822}$$

$$\frac{a\int \frac{(bc-ad)\log^2(x)}{\left(\frac{a}{x}+b\right)x^2}dx}{2(bc-ad)}+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}a\int \frac{\log^2(x)}{\left(\frac{a}{x}+b\right)x^2}dx+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{1}{2}a\int \frac{\log^2(x)}{x(a+bx)}dx+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)$$

$$\Big\downarrow \text{ 2779}$$

$$\frac{1}{2}a\left(\frac{2\int \frac{\log\left(\frac{a}{bx}+1\right)\log(x)}{x}dx}{a}-\frac{\log^2(x)\log\left(\frac{a}{bx}+1\right)}{a}\right)+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{1}{2}a\left(\frac{2\left(\log(x)\text{PolyLog}\left(2,-\frac{a}{bx}\right)-\int \frac{\text{PolyLog}\left(2,-\frac{a}{bx}\right)}{x}dx\right)}{a}-\frac{\log^2(x)\log\left(\frac{a}{bx}+1\right)}{a}\right)+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)+\frac{1}{2}a\left(\frac{2\left(\text{PolyLog}\left(3,-\frac{a}{bx}\right)+\log(x)\,\text{PolyLog}\left(2,-\frac{a}{bx}\right)\right)}{a}-\frac{\log^2(x)\log\left(\frac{a}{bx}+1\right)}{a}\right)$$

input
```
Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

output
```
(Log[b/(b*c - a*d) + a/((b*c - a*d)*x)]*Log[x]^2)/2 + (a*(-((Log[1 + a/(b*
x)]*Log[x]^2)/a) + (2*(Log[x]*PolyLog[2, -(a/(b*x))] + PolyLog[3, -(a/(b*x
))]))/a))/2
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2827
```
Int[Log[(d_.)*(u_)^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((g_.
)*(x_))^(q_.), x_Symbol] :> Int[(g*x)^q*Log[d*ExpandToSum[u, x]^r]*(a + b*L
og[c*x^n])^p, x] /; FreeQ[{a, b, c, d, g, r, n, p, q}, x] && BinomialQ[u, x
] &&  !BinomialMatchQ[u, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.64.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

method	result
default	$\frac{\ln\left(-\frac{bx+a}{(ad-cb)x}\right)\ln(x)^2}{2}+\frac{\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(-\frac{\ln(x)^3}{3}+\ln(x)^2\ln\left(1+\frac{xb}{a}\right)+2\ln(x)\operatorname{Li}_2\left(-\frac{xb}{a}\right)-2\operatorname{Li}_3\left(-\frac{xb}{a}\right)\right)}{ad-cb}$
risch	$\frac{\ln(x)^2\ln(bx+a)}{2}-\frac{\ln(x)^3}{3}-\frac{\left(2i\pi\operatorname{csgn}\left(\frac{i(bx+a)}{x(ad-cb)}\right)^2+i\pi\operatorname{csgn}(i(bx+a))\operatorname{csgn}\left(\frac{i}{ad-cb}\right)\operatorname{csgn}\left(\frac{i(bx+a)}{ad-cb}\right)-i\pi\operatorname{csgn}(i(bx+a))\operatorname{csgn}\left(\frac{i(bx}{ad}\right.\right.}{}$

input
```
int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-(b*x+a)/(a*d-b*c)/x)*ln(x)^2+(-1/2*a*d+1/2*c*b)/(a*d-b*c)*(-1/3*ln
(x)^3+ln(x)^2*ln(1+x/a*b)+2*ln(x)*polylog(2,-x/a*b)-2*polylog(3,-x/a*b))
```

3.64.5 Fricas [F]

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx=\int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="fricas")
```

output
```
integral(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

3.64.6 Sympy [F]

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \frac{a\int\frac{\log(x)^2}{ax+bx^2}\,dx}{2} + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)}{2}$$

input
```
integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)
```

output
```
a*Integral(log(x)**2/(a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a
*d + b*c)))/2
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="maxima")
```

output
```
Exception raised: TypeError >> unable to make sense of Maxima expression '
li[2]' in Sage
```

3.64.8 Giac [F]

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}\,dx$$

input
```
integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="giac")
```

output
```
integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)\,\ln\left(x\right)}{x}\,dx$$

input
```
int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x,x)
```

output
```
int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x, x)
```

3.65 $\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx$

3.65.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \mathrm{Int}\left(\frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)}, x\right)$$

output
```
Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

3.65.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

input
```
Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
```

output
```
Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

3.65.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{x(bc-ad)}\right)}dx$$

$$\Big\downarrow \text{2987}$$

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{x(bc-ad)}\right)}dx$$

input
```
Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
```

output
```
$Aborted
```

3.65.3.1 Defintions of rubi rules used

rule 2987
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]^(u_.)*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))
/((j_.) + (k_.)*(x_)), x_Symbol] :> Unintegrable[(Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^u*(s + t*Log[i*(g + h*x)^n])^m)/(j + k*x), x] /; FreeQ[{a, b, c
, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r, u}, x] && NeQ[b*c - a*d, 0]
```

3.65.4 Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln(x)}{x\ln\left(\frac{bx+a}{(-ad+cb)x}\right)}dx$$

input
```
int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

output
```
int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

3.65.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)}\, dx = \int \frac{\log(x)}{x \log\left(\frac{bx+a}{(bc-ad)x}\right)}\, dx$$

input
```
integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="fricas")
```

output
```
integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

3.65.6 Sympy [N/A]

Not integrable

Time = 27.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)}\, dx = \int \frac{\log(x)}{x \log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)}\, dx$$

input
```
integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

output
```
Integral(log(x)/(x*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x)
```

3.65.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)}\, dx = \int \frac{\log(x)}{x \log\left(\frac{bx+a}{(bc-ad)x}\right)}\, dx$$

input
```
integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="maxima")
```

output
```
integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

3.65.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}\,dx$$

input
```
integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="giac")
```

output
```
integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

3.65.9 Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\ln(x)}{x\,\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)}\,dx$$

input
```
int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))),x)
```

output
```
int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))), x)
```

3.66 $\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$

3.66.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \text{Int}\left(\frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}, x\right)$$

output
```
Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

3.66.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

input
```
Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
```

output
```
Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

3.66.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{x(bc-ad)}\right)} dx$$
$$\Big\downarrow \ 2987$$
$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{x(bc-ad)}\right)} dx$$

input
```
Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
```

output
```
$Aborted
```

3.66.3.1 Defintions of rubi rules used

rule 2987
```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]^(u_.)*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))
/((j_.) + (k_.)*(x_)), x_Symbol] :> Unintegrable[(Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^u*(s + t*Log[i*(g + h*x)^n])^m)/(j + k*x), x] /; FreeQ[{a, b, c
, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r, u}, x] && NeQ[b*c - a*d, 0]
```

3.66.4 Maple [N/A]

Not integrable

Time = 33.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln(x)}{x \ln\left(\frac{bx+a}{(-ad+cb)x}\right)^2} dx$$

input
```
int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

output
```
int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

3.66.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\log\left(x\right)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}\,dx$$

input
```
integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="fricas")
```

output
```
integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

3.66.6 Sympy [N/A]

Not integrable

Time = 41.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.50

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \frac{a\log\left(x\right)+bx\log\left(x\right)}{a\log\left(\frac{a+bx}{x(-ad+bc)}\right)} - \frac{\int \frac{b}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx + \int \frac{a}{x\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx + \int \frac{b\log\left(x\right)}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx}{a}$$

input
```
integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)**2,x)
```

output
```
(a*log(x) + b*x*log(x))/(a*log((a + b*x)/(x*(-a*d + b*c)))) - (Integral(b/
log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x) + Integral(a/(x*log(a/(
-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x) + Integral(b*log(x)/log(a/(-a
*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x))/a
```

3.66.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.96

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}\,dx$$

input
```
integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="maxima")
```

output
```
-(b*x + a)*log(x)/(a*log(b*c - a*d) - a*log(b*x + a) + a*log(x)) - integra
te(-(b*x*log(x) + b*x + a)/(a*x*log(b*c - a*d) - a*x*log(b*x + a) + a*x*lo
g(x)), x)
```

3.66.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}\,dx$$

input
```
integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="giac")
```

output
```
integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

3.66.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\ln(x)}{x\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^2}\,dx$$

input
```
int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c))))^2),x)
```

output
```
int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c))))^2), x)
```

3.67 $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

3.67.1 Optimal result

Integrand size = 45, antiderivative size = 620

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{6mn^3\text{PolyLog}\left(5,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{6mn^3\text{PolyLog}\left(5,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

output
```
1/4*m*ln(e*((b*x+a)/(d*x+c))^n)^4*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/
4*ln(e*((b*x+a)/(d*x+c))^n)^4*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/4*m*ln(e*((b*
x+a)/(d*x+c))^n)^4*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/
n+m*ln(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-
m*ln(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x
+c))/(-a*d+b*c)-3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,d*(b*x+a)/b/(d
*x+c))/(-a*d+b*c)+3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,(-c*g+d*f)*(
b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)+6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*po
lylog(4,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*
polylog(4,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)-6*m*n^3*polylo
g(5,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)+6*m*n^3*polylog(5,(-c*g+d*f)*(b*x+a)/(
-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

3.67.2 Mathematica [F]

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$$

input
```
Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)
*(c + d*x)),x]
```

output
```
Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)
*(c + d*x)), x]
```

3.67.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2989, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2989}$$

$$\frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}-\frac{gm\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}dx}{4n(bc-ad)}$$

$\big\downarrow$ 2953

$$\frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}-\frac{gm\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{4n}$$

$\big\downarrow$ 2804

$$\frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}-\frac{gm\int\left(\frac{d\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{4n}$$

$\big\downarrow$ 2009

$$\frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}-\frac{gm\left(\frac{24n^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g(bc-ad)}-\frac{12n^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g(bc-ad)}+\frac{4n\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}}{g(bc-ad)}\right.}{}$$

input
```
Int[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*(c +
d*x)),x]
```

3.67. $\int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}dx$

output
```
(Log[e*((a + b*x)/(c + d*x))^n]^4*Log[h*(f + g*x)^m])/(4*(b*c - a*d)*n) -
(g*m*(-((Log[e*((a + b*x)/(c + d*x))^n]^4*Log[1 - (d*(a + b*x))/(b*(c + d*
x))])/((b*c - a*d)*g)) + (Log[e*((a + b*x)/(c + d*x))^n]^4*Log[1 - ((d*f -
 c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) - (4*n*Log[e*((
a + b*x)/(c + d*x))^n]^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/((b*c -
a*d)*g) + (4*n*Log[e*((a + b*x)/(c + d*x))^n]^3*PolyLog[2, ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) + (12*n^2*Log[e*((a + b
*x)/(c + d*x))^n]^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*
g) - (12*n^2*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[3, ((d*f - c*g)*(a +
 b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) - (24*n^3*Log[e*((a + b*x
)/(c + d*x))^n]*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g) +
 (24*n^3*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[4, ((d*f - c*g)*(a + b*x))
/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) + (24*n^4*PolyLog[5, (d*(a + b*
x))/(b*(c + d*x))])/((b*c - a*d)*g) - (24*n^4*PolyLog[5, ((d*f - c*g)*(a +
 b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g)))/(4*n)
```

3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2989
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Simp[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d)))   Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &
& EqQ[p + q, 0] && NeQ[s, -1]
```

3.67. $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

3.67.4 Maple [F]

$$\int \frac{\ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3} \ln \left(h (gx+f)^{m}\right)}{(bx+a)\,(dx+c)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

3.67.5 Fricas [F]

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
 algorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/(b*d*x^2 + a*
c + (b*c + a*d)*x), x)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**3*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x
)
```

output
```
Timed out
```

3.67.7 Maxima [F]

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
 algorithm="maxima")
```

output
```
-1/4*(n^3*log(b*x + a)^4 + n^3*log(d*x + c)^4 - 4*n^2*log(b*x + a)^3*log(e
) + 6*n*log(b*x + a)^2*log(e)^2 - 4*(n^3*log(b*x + a) - n^2*log(e))*log(d*
x + c)^3 - 4*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^3 + 4*(log(b*x
 + a) - log(d*x + c))*log((d*x + c)^n)^3 - 4*log(b*x + a)*log(e)^3 + 6*(n^
3*log(b*x + a)^2 - 2*n^2*log(b*x + a)*log(e) + n*log(e)^2)*log(d*x + c)^2
+ 6*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log
(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n)^2 + 6*(n*log(b*x + a)^
2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*(log(b
*x + a) - log(d*x + c))*log((b*x + a)^n) - 2*log(b*x + a)*log(e))*log((d*x
 + c)^n)^2 - 4*(n^3*log(b*x + a)^3 - 3*n^2*log(b*x + a)^2*log(e) + 3*n*log
(b*x + a)*log(e)^2 - log(e)^3)*log(d*x + c) - 4*(n^2*log(b*x + a)^3 - n^2*
log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*log(e
))*log(d*x + c)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*
log(b*x + a)*log(e) + log(e)^2)*log(d*x + c))*log((b*x + a)^n) + 4*(n^2*lo
g(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log
(b*x + a) - n*log(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log
((b*x + a)^n)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*lo
g(b*x + a)*log(e) + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d
*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(
e))*log((b*x + a)^n))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) + ...
```

3.67.8 Giac [F]

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
 algorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/((b*x + a)*(
d*x + c)), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(h\,(f+g\,x)^m\right)\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{(a+b\,x)\,\left(c+d\,x\right)}\,dx$$

input
```
int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c +
d*x)),x)
```

output
```
int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c +
d*x)), x)
```

3.68 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

3.68.1 Optimal result

Integrand size = 45, antiderivative size = 496

$$\begin{aligned}\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx &= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} \\ &+ \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{3(bc-ad)n} \\ &- \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} \\ &+ \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} \\ &- \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\ &- \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} \\ &+ \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\ &+ \frac{2mn^2\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} \\ &- \frac{2mn^2\operatorname{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}\end{aligned}$$

output
```
1/3*m*ln(e*((b*x+a)/(d*x+c))^n)^3*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/
3*ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/3*m*ln(e*((b*
x+a)/(d*x+c))^n)^3*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/
n+m*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-
m*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x
+c))/(-a*d+b*c)-2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,d*(b*x+a)/b/(d*x
+c))/(-a*d+b*c)+2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,(-c*g+d*f)*(b*x+
a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)+2*m*n^2*polylog(4,d*(b*x+a)/b/(d*x+c))/(
-a*d+b*c)-2*m*n^2*polylog(4,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b
*c)
```

3.68.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25557 vs. $2(496) = 992$.

Time = 7.15 (sec) , antiderivative size = 25557, normalized size of antiderivative = 51.53

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)
*(c + d*x)),x]
```

output
```
Result too large to show
```

3.68.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2989, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(h(f+gx)^m\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2989}$$

$$\frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}-\frac{gm\int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}dx}{3n(bc-ad)}$$

$$\Big\downarrow \; 2953$$

$$\frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}-\frac{gm\int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{3n}$$

$$\Big\downarrow \; 2804$$

$$\frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}-\frac{gm\int\left(\frac{d\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3n}$$

$$\Big\downarrow \; 2009$$

$$\frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}-$$

$$gm\left(-\frac{6n^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g(bc-ad)}+\frac{3n\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g(bc-ad)}+\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+b}{(c+d}\right.}{g(bc-ad)}\right.$$

input
```
Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*(c +
d*x)),x]
```

output
```
(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/(3*(b*c - a*d)*n) -
(g*m*(-((Log[e*((a + b*x)/(c + d*x))^n]^3*Log[1 - (d*(a + b*x))/(b*(c + d*
x))])/((b*c - a*d)*g)) + (Log[e*((a + b*x)/(c + d*x))^n]^3*Log[1 - ((d*f -
 c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) - (3*n*Log[e*((
a + b*x)/(c + d*x))^n]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/((b*c -
a*d)*g) + (3*n*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) + (6*n^2*Log[e*((a + b*
x)/(c + d*x))^n]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g)
- (6*n^2*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, ((d*f - c*g)*(a + b*x))
/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) - (6*n^3*PolyLog[4, (d*(a + b*x
))/(b*(c + d*x))])/((b*c - a*d)*g) + (6*n^3*PolyLog[4, ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g)))/(3*n)
```

3.68. $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}dx$

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2989
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Simp[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d)))   Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &
& EqQ[p + q, 0] && NeQ[s, -1]
```

3.68.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\ln\left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

3.68.5 Fricas [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
 algorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/(b*d*x^2 + a*
c + (b*c + a*d)*x), x)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x
)
```

output
```
Timed out
```

3.68.7 Maxima [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
 algorithm="maxima")
```

output
```
1/3*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) +
 3*(n^2*log(b*x + a) - n*log(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*
x + c))*log((b*x + a)^n)^2 + 3*(log(b*x + a) - log(d*x + c))*log((d*x + c)
^n)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)
*log(e) + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2
- 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b
*x + a)^n) + 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) -
log(e))*log(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) -
2*log(b*x + a)*log(e))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) - in
tegrate(-1/3*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m
*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (
b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c
)^3 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d
*g*m*n*log(e))*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log
(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m
*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*
c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2
 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m
+ (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*f*log(h
) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c...
```

3.68.8 Giac [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
 algorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/((b*x + a)*(
d*x + c)), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(h\,\left(f+g\,x\right)^m\right)\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c +
d*x)),x)
```

output
```
int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c +
d*x)), x)
```

3.69 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

3.69.1 Optimal result

Integrand size = 43, antiderivative size = 371

$$\begin{aligned}\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx &= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} \\ &+ \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} \\ &- \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} \\ &+ \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} \\ &- \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\ &- \frac{mn\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} \\ &+ \frac{mn\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}\end{aligned}$$

output
```
1/2*m*ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/
2*ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/2*m*ln(e*((b*
x+a)/(d*x+c))^n)^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/
n+m*ln(e*((b*x+a)/(d*x+c))^n)*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*
ln(e*((b*x+a)/(d*x+c))^n)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/(-a*d+b*c)-m*n*polylog(3,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)+m*n*polylog(3,(-
c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

3.69.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1842 vs. $2(371) = 742$.

Time = 0.88 (sec) , antiderivative size = 1842, normalized size of antiderivative = 4.96

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(
c + d*x)),x]
```

output
```
(m*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*f -
 c*g)*(a + b*x))]^2 - 2*m*n*Log[a/b + x]^2*Log[f + g*x] + 2*m*n*Log[a/b +
x]*Log[c/d + x]*Log[f + g*x] - 2*m*n*Log[c/d + x]^2*Log[f + g*x] + 2*m*n*L
og[a/b + x]*Log[a + b*x]*Log[f + g*x] - 2*m*n*Log[c/d + x]*Log[a + b*x]*Lo
g[f + g*x] + 2*m*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[f +
g*x] + 2*m*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*
Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[a + b*x
]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[c +
 d*x]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[c + d*x]*Log[f + g*x] + 2*m*Lo
g[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 2*m*n*Log[a/b + x
]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x] + m*n*Log[a/b + x]^2*Log[(b*
(f + g*x))/(b*f - a*g)] - 2*m*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*
Log[(b*(f + g*x))/(b*f - a*g)] - 2*m*n*Log[a/b + x]*Log[(g*(c + d*x))/(-(d
*f) + c*g)]*Log[(b*(f + g*x))/(b*f - a*g)] + m*n*Log[(g*(c + d*x))/(-(d*f)
 + c*g)]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*m*n*Log[(g*(c + d*x))/(-(d*f
) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f +
 g*x))/(b*f - a*g)] + m*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*
x))]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*m*n*Log[a/b + x]*Log[c/d + x]*Lo
g[(d*(f + g*x))/(d*f - c*g)] + m*n*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f -
 c*g)] + 2*m*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*...
```

3.69.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2989, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right) \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \text{2989}$$

$$\frac{\log \left(h(f+gx)^m\right) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}-\frac{gm\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}dx}{2n(bc-ad)}$$

$$\Big\downarrow \text{2953}$$

$$\frac{\log \left(h(f+gx)^m\right) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}-\frac{gm\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{2n}$$

$$\Big\downarrow \text{2804}$$

$$\frac{\log \left(h(f+gx)^m\right) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}-\frac{gm\int \left(\frac{d\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{2n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log \left(h(f+gx)^m\right) \log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}-\frac{gm\left(\frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g(bc-ad)}+\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{g(bc-ad)}-\frac{2n\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}\right.}{2n}$$

input
```
Int[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*
x)),x]
```

output
```
(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/(2*(b*c - a*d)*n) -
(g*m*(-((Log[e*((a + b*x)/(c + d*x))^n]^2*Log[1 - (d*(a + b*x))/(b*(c + d*
x))])/((b*c - a*d)*g)) + (Log[e*((a + b*x)/(c + d*x))^n]^2*Log[1 - ((d*f -
 c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) - (2*n*Log[e*((
a + b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*
d)*g) + (2*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g) + (2*n^2*PolyLog[3, (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*g) - (2*n^2*PolyLog[3, ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)*g)))/(2*n)
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2989
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Simp[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d)))   Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &
& EqQ[p + q, 0] && NeQ[s, -1]
```

3.69.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

3.69.5 Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, a
lgorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c
+ (b*c + a*d)*x), x)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, a
lgorithm="maxima")
```

output
```
-1/2*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*lo
g(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) + 2*(log(b*x
 + a) - log(d*x + c))*log((d*x + c)^n) - 2*log(b*x + a)*log(e))*log((g*x +
 f)^m)/(b*c - a*d) + integrate(1/2*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)
*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x
+ a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x +
 c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*l
og(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a)
 + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(
e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x +
a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*
log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) +
 (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x +
a)^n) - 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x -
 (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x
^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((d*x + c)^n))/(a*b
*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g
- (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)
*x), x)
```

3.69.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, a
lgorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*
x + c)), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\, dx = \int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)\,\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\, dx$$

input
```
int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*
x)),x)
```

output
```
int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*
x)), x)
```

3.70 $\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.70.1 Optimal result

Integrand size = 45, antiderivative size = 45

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{b\text{Int}\left(\frac{\log(h(f+gx)^m)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad} - \frac{d\text{Int}\left(\frac{\log(h(f+gx)^m)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}$$

output
```
b*Unintegrable(ln(h*(g*x+f)^m)/(b*x+a)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+
b*c)-d*Unintegrable(ln(h*(g*x+f)^m)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)/(
-a*d+b*c)
```

3.70.2 Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*
x))^n]),x]
```

output
```
Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*
x))^n]), x]
```

3.70.3 Rubi [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{b\log \left(h(f+gx)^m\right)}{(a+bx)(bc-ad)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d\log \left(h(f+gx)^m\right)}{(c+dx)(bc-ad)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\int \frac{\log(h(f+gx)^m)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx}{bc-ad}-\frac{d\int \frac{\log(h(f+gx)^m)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx}{bc-ad}$$

input
```
Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]
),x]
```

output
```
$Aborted
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.70.4 Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

3.70.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx = \int \frac{\log\left((gx+f)^{m}h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, a
lgorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
 a)/(d*x + c))^n)), x)
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

3.70.7 Maxima [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, a
lgorithm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x +
c))^n)), x)
```

3.70.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, a
lgorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x +
c))^n)), x)
```

3.70.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log (h(f+gx)^m)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x)
),x)
```

output
```
int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x)
), x)
```

3.71 $\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.71.1 Optimal result

Integrand size = 45, antiderivative size = 45

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\log(h(f+gx)^m)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{gm\text{Int}\left(\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{(bc-ad)n}$$

output
```
-ln(h*(g*x+f)^m)/(-a*d+b*c)/n/ln(e*((b*x+a)/(d*x+c))^n)+g*m*Unintegrable(1
/(g*x+f)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b*c)/n
```

3.71.2 Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*
x))^n]^2),x]
```

output
```
Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*
x))^n]^2), x]
```

3.71.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2989, 2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{2989}$$

$$\frac{gm\int \frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx}{n(bc-ad)}-\frac{\log \left(h(f+gx)^m\right)}{n(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

$$\Big\downarrow \text{2955}$$

$$\frac{gm\int \frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx}{n(bc-ad)}-\frac{\log \left(h(f+gx)^m\right)}{n(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

input
```
Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]
^2),x]
```

output
```
$Aborted
```

3.71.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

rule 2989
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Simp[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d)))   Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &
& EqQ[p + q, 0] && NeQ[s, -1]
```

3.71.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}dx$$

input
```
int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

output
```
int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

3.71.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\log \left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)\log^{2}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx = \int \frac{\log \left((gx+f)^{m}h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
 algorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
 a)/(d*x + c))^n)^2), x)
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n)**2,x
)
```

output
```
Timed out
```

3.71.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.02

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
 algorithm="maxima")
```

output
```
g*m*integrate(1/(b*c*f*n*log(e) - a*d*f*n*log(e) + (b*c*g*n*log(e) - a*d*g
*n*log(e))*x + (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((b*x + a)^n
) - (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((d*x + c)^n)), x) - (l
og((g*x + f)^m) + log(h))/(b*c*n*log(e) - a*d*n*log(e) + (b*c*n - a*d*n)*l
og((b*x + a)^n) - (b*c*n - a*d*n)*log((d*x + c)^n))
```

3.71.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log (h(f+gx)^m)}{(a+bx)(c+dx)\log ^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
 algorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x +
c))^n)^2), x)
```

3.71.9 Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log (h(f+gx)^m)}{(a+bx)(c+dx)\log ^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{{\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*
x)),x)
```

output
```
int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*
x)), x)
```

3.72 $\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

3.72.1 Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\frac{b\text{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

output
```
b*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/
(-a*d+b*c)-d*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(d*x+c)/ln((b*x+a)/(d*
x+c))^2,x)/(-a*d+b*c)
```

3.72.2 Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

input
```
Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/
(c + d*x)]^2),x]
```

output
```
Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/
(c + d*x)]^2), x]
```

3.72.3 Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{b\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(bc-ad)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)(bc-ad)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}$$

input
```
Int[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d
*x)]^2),x]
```

output
```
$Aborted
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.72.4 Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\ln \left(1+\frac{-bx-a}{dx+c}\right)}{\left(bx+a\right)\left(dx+c\right)\ln \left(\frac{bx+a}{dx+c}\right)^{2}}dx$$

input
```
int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

output
```
int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

3.72.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log ^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\log \left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log \left(\frac{bx+a}{dx+c}\right)^{2}}\,dx$$

input
```
integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x
, algorithm="fricas")
```

output
```
integral(log(-((b - d)*x + a - c)/(d*x + c))/((b*d*x^2 + a*c + (b*c + a*d)
*x)*log((b*x + a)/(d*x + c))^2), x)
```

3.72.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> '>' not supported between instances of 'Pol
y' and 'int'
```

3.72.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.21

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

input
```
integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x
, algorithm="maxima")
```

output
```
-(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
 - a*d)*log(d*x + c)) - integrate(-1/(((b*d - d^2)*x^2 + a*c - c^2 + (b*c
+ a*d - 2*c*d)*x)*log(b*x + a) - ((b*d - d^2)*x^2 + a*c - c^2 + (b*c + a*d
 - 2*c*d)*x)*log(d*x + c)), x)
```

3.72.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

input
```
integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x
, algorithm="giac")
```

output
```
integrate(log(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)
/(d*x + c))^2), x)
```

3.72.9 Mupad [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\ln \left(1-\frac{a+b\,x}{c+d\,x}\right)}{{\ln \left(\frac{a+b\,x}{c+d\,x}\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c
+ d*x)),x)
```

output
```
int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c
+ d*x)), x)
```

3.73 $\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

3.73.1 Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\frac{b\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

output
```
b*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/
(-a*d+b*c)-d*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(d*x+c)/ln((b*x+a)/(d*
x+c))^2,x)/(-a*d+b*c)
```

3.73.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

input
```
Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/
(c + d*x)]^2),x]
```

output
```
Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/
(c + d*x)]^2), x]
```

3.73.3 Rubi [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{b\log \left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(bc-ad)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log \left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)(bc-ad)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\int \frac{\log \left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}-\frac{d\int \frac{\log \left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}$$

input
```
Int[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d
*x)]^2),x]
```

output
```
$Aborted
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.73.4 Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\ln \left(1+\frac{-dx-c}{bx+a}\right)}{\left(bx+a\right)\left(dx+c\right)\ln \left(\frac{bx+a}{dx+c}\right)^{2}}dx$$

input
```
int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

output
```
int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

3.73.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \frac{\log \left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\log \left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log \left(\frac{bx+a}{dx+c}\right)^{2}}\,dx$$

input
```
integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x
, algorithm="fricas")
```

output
```
integral(log(((b - d)*x + a - c)/(b*x + a))/((b*d*x^2 + a*c + (b*c + a*d)*
x)*log((b*x + a)/(d*x + c))^2), x)
```

3.73.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> '>' not supported between instances of 'Pol
y' and 'int'
```

3.73.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.23

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

input
```
integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x
, algorithm="maxima")
```

output
```
-(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c)) - integrate(-1/(((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*
b - d) - b*c)*x)*log(b*x + a) - ((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b - d
) - b*c)*x)*log(d*x + c)), x)
```

3.73.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

input
```
integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x
, algorithm="giac")
```

output
```
integrate(log(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)
/(d*x + c))^2), x)
```

3.73.9 Mupad [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c
+ d*x)),x)
```

output
```
int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c
+ d*x)), x)
```

3.74 $\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right.$

3.74.1 Optimal result

Integrand size = 87, antiderivative size = 45

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

output
```
-ln(1+(-b*x-a)/(d*x+c))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

3.74.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(-bc+ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

input
```
Integrate[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + L
og[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^
2),x]
```

output
```
Log[1 - (a + b*x)/(c + d*x)]/((-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)])
```

3.74.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log ^2\left(\frac{a+bx}{c+dx}\right)}+\frac{1}{(c+dx)(-a+x(d-b)+c)\log \left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{b\int \frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log ^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}-\frac{d\int \frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log ^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}+\int \frac{1}{(c+dx)(-a+c+(d-b)x)\log \left(\frac{a+bx}{c+dx}\right)}dx$$

input
```
Int[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Log[1 -
 (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

output
```
$Aborted
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(47) = 94$.

Time = 218.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.96

method	result	size
parallelrisch	$-\frac{-\ln \left(-\frac{bx-dx+a-c}{dx+c}\right)b^2d^4+2\ln \left(-\frac{bx-dx+a-c}{dx+c}\right)b^3d^3-\ln \left(-\frac{bx-dx+a-c}{dx+c}\right)b^4d^2}{\ln \left(\frac{bx+a}{dx+c}\right)(b-d)^2d^2(ad-cb)b^2}$	133

3.74. $\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log \left(\frac{a+bx}{c+dx}\right)}+\frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log ^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$

input
```
int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(
b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-ln(-(b*x-d*x+a-c)/(d*x+c))*b^2*d^4+2*ln(-(b*x-d*x+a-c)/(d*x+c))*b^3*d^3
-ln(-(b*x-d*x+a-c)/(d*x+c))*b^4*d^2)/ln((b*x+a)/(d*x+c))/(b-d)^2/d^2/(a*d-
b*c)/b^2
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

input
```
integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d
*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
-log(-((b - d)*x + a - c)/(d*x + c))/((b*c - a*d)*log((b*x + a)/(d*x + c))
)
```

3.74.6 Sympy [F(-2)]

Exception generated.

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x
+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> '>' not supported between instances of 'Pol
y' and 'int'
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=-\frac{\log\left(-(b-d)x-a+c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}$$

input
```
integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d
*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
 - a*d)*log(d*x + c))
```

3.74.8 Giac [F]

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=\int -\frac{1}{((b-d)x+a-c)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}+\frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d
*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(-1/(((b - d)*x + a - c)*(d*x + c)*log((b*x + a)/(d*x + c))) + lo
g(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^
2), x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)\,dx$$
$$=\frac{\ln\left(1-\frac{a+b\,x}{c+d\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

input
```
int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c
+ d*x)) - 1/(log((a + b*x)/(c + d*x))*(c + d*x)*(a - c + x*(b - d))),x)
```

output
```
log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

3.75 $\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)$

3.75.1 Optimal result

Integrand size = 88, antiderivative size = 45

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right) dx = -\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

output
```
-ln(1+(-d*x-c)/(b*x+a))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

3.75.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right) dx = -\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

input
```
Integrate[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) +
Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]
^2),x]
```

output
```
-(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

3.75.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\log \left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{1}{(a+bx)(a+x(b-d)-c)\log\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}-\int \frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}dx$$

input
```
Int[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + Log[1
- (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

output
```
$Aborted
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(47) = 94$.

Time = 220.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

method	result
parallelrisch	$-\frac{2\ln\left(\frac{bx-dx+a-c}{bx+a}\right)b^3d^3-\ln\left(\frac{bx-dx+a-c}{bx+a}\right)b^4d^2-\ln\left(\frac{bx-dx+a-c}{bx+a}\right)b^2d^4}{\ln\left(\frac{bx+a}{dx+c}\right)(b-d)^2(ad-cb)b^2d^2}$
risch	$\frac{2i\ln(bx-dx+a-c)}{(ad-cb)\left(\pi \operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)\operatorname{csgn}(i(bx+a))\operatorname{csgn}\left(\frac{i}{dx+c}\right)-\pi \operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2\operatorname{csgn}(i(bx+a))-\pi \operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2\operatorname{csgn}\left(\frac{i}{dx+c}\right)+\pi\right.}$

input
```
int(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a))/(b
*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*ln((b*x-d*x+a-c)/(b*x+a))*b^3*d^3-ln((b*x-d*x+a-c)/(b*x+a))*b^4*d^2-ln
((b*x-d*x+a-c)/(b*x+a))*b^2*d^4)/ln((b*x+a)/(d*x+c))/(b-d)^2/(a*d-b*c)/b^2
/d^2
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

input
```
integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*
x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
-log(((b - d)*x + a - c)/(b*x + a))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

3.75.6 Sympy [F(-2)]

Exception generated.

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+
a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> '>' not supported between instances of 'Pol
y' and 'int'
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left((b-d)x+a-c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}$$

input
```
integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*
x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c))
```

3.75.8 Giac [F]

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \int -\frac{1}{((b-d)x+a-c)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}+\frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

input
```
integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*
x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(-1/(((b - d)*x + a - c)*(b*x + a)*log((b*x + a)/(d*x + c))) + lo
g(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^
2), x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=\frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

input
```
int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c
+ d*x)) - 1/(log((a + b*x)/(c + d*x))*(a + b*x)*(a - c + x*(b - d))),x)
```

output
```
log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

3.76 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

3.76.1 Optimal result

Integrand size = 32, antiderivative size = 560

$$\begin{aligned}\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = & -\frac{anx}{2bg}+\frac{cnx}{2dg}+\frac{a^2n\log(a+bx)}{2b^2g}-\frac{nx^2\log(a+bx)}{2g} \\ & -\frac{c^2n\log(c+dx)}{2d^2g}+\frac{nx^2\log(c+dx)}{2g} \\ & +\frac{x^2\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{2g} \\ & -\frac{fn\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} \\ & +\frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2} \\ & -\frac{fn\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} \\ & +\frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} \\ & +\frac{f\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g^2} \\ & -\frac{fn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{fn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} \\ & +\frac{fn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}+\frac{fn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2}\end{aligned}$$

output
```
-1/2*a*n*x/b/g+1/2*c*n*x/d/g+1/2*a^2*n*ln(b*x+a)/b^2/g-1/2*n*x^2*ln(b*x+a)
/g-1/2*c^2*n*ln(d*x+c)/d^2/g+1/2*n*x^2*ln(d*x+c)/g+1/2*x^2*(n*ln(b*x+a)-ln
(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/g+1/2*f*(n*ln(b*x+a)-ln(e*((b*x+a)/(d
*x+c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)-x*
g^(1/2))/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1
/2))/(d*f^(1/2)+c*g^(1/2)))/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))
/(b*f^(1/2)-a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*
f^(1/2)-c*g^(1/2)))/g^2-1/2*f*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^
(1/2)))/g^2-1/2*f*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g^2+1
/2*f*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g^2+1/2*f*n*polyl
og(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g^2
```

3.76.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.82

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$
$$= \frac{-gx^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \frac{gn\left(a^2d^2\log(a+bx)-b\left(d(-bc+ad)x+bc^2\log(c+dx)\right)\right)}{b^2d^2} - f\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right) - f\,\mathrm{lo}}{}$$

input
```
Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

output
```
(-(g*x^2*Log[e*((a + b*x)/(c + d*x))^n]) + (g*n*(a^2*d^2*Log[a + b*x] - b*
(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^2) - f*Log[e*((a + b*x)
/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - f*Log[e*((a + b*x)/(c + d*x))^n]
*Log[Sqrt[f] + Sqrt[g]*x] + f*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*S
qrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] -
 Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])
] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) + f*n*(
(Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c +
 d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b
*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f]
+ Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^2)
```

3.76.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2993, 243, 49, 2009, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \frac{x^3}{f-gx^2}dx\right)+ n \int \frac{x^3 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^3 \log (c+dx)}{f-gx^2}dx$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2} \left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \frac{x^2}{f-gx^2}dx^2+ n \int \frac{x^3 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^3 \log (c+dx)}{f-gx^2}dx$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2} \left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \left(-\frac{f}{g \left(gx^2-f\right)}-\frac{1}{g}\right)dx^2+ n \int \frac{x^3 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^3 \log (c+dx)}{f-gx^2}dx$$

$$\Big\downarrow \text{2009}$$

$$n \int \frac{x^3 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^3 \log (c+dx)}{f-gx^2}dx- \frac{1}{2} \left(-\frac{f \log \left(f-gx^2\right)}{g^2}-\frac{x^2}{g}\right) \left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right)$$

$$\Big\downarrow \text{2863}$$

$$n \int \left(\frac{fx \log (a+bx)}{g \left(f-gx^2\right)}-\frac{x \log (a+bx)}{g}\right)dx-n \int \left(\frac{fx \log (c+dx)}{g \left(f-gx^2\right)}-\frac{x \log (c+dx)}{g}\right)dx- \frac{1}{2} \left(-\frac{f \log \left(f-gx^2\right)}{g^2}-\frac{x^2}{g}\right) \left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right)$$

$$\Big\downarrow \text{2009}$$

$$n\left(\frac{a^2\log(a+bx)}{2b^2g}-\frac{f\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{f\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g^2}-\frac{f\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2}-\frac{f}{}\right.$$
$$\frac{1}{2}\left(-\frac{f\log\left(f-gx^2\right)}{g^2}-\frac{x^2}{g}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)-$$
$$n\left(\frac{c^2\log(c+dx)}{2d^2g}-\frac{f\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}-\frac{f\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2g^2}-\frac{f\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2}-\frac{f}{}\right.$$

input
```
Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

output
```
-1/2*((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*(
-(x^2/g) - (f*Log[f - g*x^2])/g^2)) + n*(-1/2*(a*x)/(b*g) + x^2/(4*g) + (a
^2*Log[a + b*x])/(2*b^2*g) - (x^2*Log[a + b*x])/(2*g) - (f*Log[a + b*x]*Lo
g[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) - (f*Log[a +
 b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) - (f
*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) - (f*
PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2)) - n*(-1/
2*(c*x)/(d*g) + x^2/(4*g) + (c^2*Log[c + d*x])/(2*d^2*g) - (x^2*Log[c + d*
x])/(2*g) - (f*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*S
qrt[g])])/(2*g^2) - (f*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[
f] - c*Sqrt[g])])/(2*g^2) - (f*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f]
 - c*Sqrt[g]))])/(2*g^2) - (f*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] +
c*Sqrt[g])])/(2*g^2))
```

3.76.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76. $\int \frac{x^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.76.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)x^{2}}{2g}-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)f\ln\left(-g\,x^{2}+f\right)}{2g^{2}}-\frac{n\left(\frac{(ad-cb)\left(\frac{x}{bd}+\frac{c^{2}\ln(dx+c)}{d^{2}(ad-cb)}-\frac{a^{2}\ln(bx+a)}{b^{2}(ad-cb)}\right)}{g}+\frac{f(ad-cb)\left(\left(\frac{\ln(dx+c)\ln\left(-g\right.}{d}\right.\right.}{}\right.}{}$

input
```
int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(e*((b*x+a)/(d*x+c))^n)*x^2/g-1/2*ln(e*((b*x+a)/(d*x+c))^n)*f/g^2*l
n(-g*x^2+f)-1/2*n*((a*d-b*c)/g*(x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*
a^2/(a*d-b*c)*ln(b*x+a))+f*(a*d-b*c)/g^2*((ln(d*x+c)/d*ln(-g*x^2+f)+2/d*g*
(-1/2*ln(d*x+c)*(ln((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+ln(
(d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g-1/2*(dilog((d*(f*g)^
(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+dilog((d*(f*g)^(1/2)+(d*x+c)*g-c
*g)/(d*(f*g)^(1/2)-c*g)))/g))*d/(a*d-b*c)-(ln(b*x+a)/b*ln(-g*x^2+f)+2/b*g*
(-1/2*ln(b*x+a)*(ln((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+ln(
(b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g-1/2*(dilog((b*(f*g)^
(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+dilog((b*(f*g)^(1/2)+g*(b*x+a)-a
*g)/(b*(f*g)^(1/2)-a*g)))/g))*b/(a*d-b*c)))
```

3.76.5 Fricas [F]

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx = \int -\frac{x^3 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f} \, dx$$

input
```
integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

output
```
integral(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx = \text{Timed out}$$

input
```
integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

output
```
-integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.76.8 Giac [F]

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

output
```
integrate(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int \frac{x^3 \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

input
```
int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
```

output
```
int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.77 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

3.77.1 Optimal result

Integrand size = 32, antiderivative size = 550

$$\begin{aligned}
\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx
&= -\frac{n(a+bx)\log(a+bx)}{bg}+\frac{n(c+dx)\log(c+dx)}{dg} \\
&\quad+\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g} \\
&\quad-\frac{\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g^{3/2}} \\
&\quad-\frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad+\frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad+\frac{\sqrt{f}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad-\frac{\sqrt{f}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}}
\end{aligned}$$

output
```
-n*(b*x+a)*ln(b*x+a)/b/g+n*(d*x+c)*ln(d*x+c)/d/g+x*(n*ln(b*x+a)-ln(e*((b*x
+a)/(d*x+c))^n)-n*ln(d*x+c))/g-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(
e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*f^(1/2)/g^(3/2)-1/2*n*ln(b*x+a)*ln(b*(
f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(d*x+c)*
ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(b
*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*
n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2
)+1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-
1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2
*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n
*polylog(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx$$
$$= \frac{-\frac{2\sqrt{g}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{2(bc-ad)\sqrt{g}n\log(c+dx)}{bd} - \sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right) + \sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \mathrm{l}}{}$$

input
```
Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

output
```
((-2*Sqrt[g]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (2*(b*c - a*d)*
Sqrt[g]*n*Log[c + d*x])/(b*d) - Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log
[Sqrt[f] - Sqrt[g]*x] + Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f]
 + Sqrt[g]*x] + Sqrt[f]*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g]
)] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt[
g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - Po
lyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) - Sqrt[f]*n*(
(Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c +
 d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b
*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f]
+ Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^(3/2))
```

3.77.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2993, 262, 221, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \frac{x^2}{f-gx^2}dx\right)+ n \int \frac{x^2 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^2 \log (c+dx)}{f-gx^2}dx$$

$$\Big\downarrow \text{262}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \left(\frac{f \int \frac{1}{f-gx^2}dx}{g}-\frac{x}{g}\right)\right)+ n \int \frac{x^2 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^2 \log (c+dx)}{f-gx^2}dx$$

$$\Big\downarrow \text{221}$$

$$n \int \frac{x^2 \log (a+bx)}{f-gx^2}dx-n \int \frac{x^2 \log (c+dx)}{f-gx^2}dx- \left(\left(\frac{\sqrt{f}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{g^{3/2}}-\frac{x}{g}\right) \left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right)\right)$$

$$\Big\downarrow \text{2863}$$

$$n \int \left(\frac{f \log (a+bx)}{g \left(f-gx^2\right)}-\frac{\log (a+bx)}{g}\right)dx-n \int \left(\frac{f \log (c+dx)}{g \left(f-gx^2\right)}-\frac{\log (c+dx)}{g}\right)dx- \left(\left(\frac{\sqrt{f}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{g^{3/2}}-\frac{x}{g}\right) \left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\left(\left(\frac{\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{g^{3/2}}-\frac{x}{g}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)+$$
$$n\left(\frac{\sqrt{f}\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}+\frac{\sqrt{f}\log(a+b\right.$$
$$n\left(\frac{\sqrt{f}\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}+\frac{\sqrt{f}\log(c+d\right.$$

input
```
Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

output
```
-((-(x/g) + (Sqrt[f]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]])/g^(3/2))*(n*Log[a + b*x
] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])) + n*(x/g - ((a + b*x
)*Log[a + b*x])/(b*g) - (Sqrt[f]*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x)
)/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*Log[a + b*x]*Log[(b*(Sq
rt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*PolyL
og[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^(3/2)) - (Sqrt
[f]*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)))
- n*(x/g - ((c + d*x)*Log[c + d*x])/(d*g) - (Sqrt[f]*Log[c + d*x]*Log[(d*(
Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*Log
[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^(3/
2)) + (Sqrt[f]*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])
/(2*g^(3/2)) - (Sqrt[f]*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt
[g])])/(2*g^(3/2)))
```

3.77.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.77.4 Maple [F]

$$\int \frac{x^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)}{-g \,x^{2}+f}d x$$

input
```
int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

output
```
int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

3.77.5 Fricas [F]

$$\int \frac{x^2 \log \left(e \left(\frac{a+b x}{c+d x}\right)^n\right)}{f-g x^2} \, dx=\int -\frac{x^{2} \log \left(e \left(\frac{b x + a}{d x + c}\right)^{n}\right)}{g x^{2} - f} \,d x$$

input
```
integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

output
```
integral(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

output
```
Timed out
```

3.77.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1047 vs. 2(438) = 876.

Time = 0.40 (sec) , antiderivative size = 1047, normalized size of antiderivative = 1.90

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

output
```
-1/2*(2*b*c*(c^2/((b*c*d^3 - a*d^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*l
og(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*lo
g(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g))*d - 2*(c^3/((b*c*d^4
 - a*d^5)*g*x + (b*c^2*d^3 - a*c*d^4)*g) + a^3*log(b*x + a)/((b^4*c^2 - 2*
a*b^3*c*d + a^2*b^2*d^2)*g) + (2*b*c^3 - 3*a*c^2*d)*log(d*x + c)/((b^2*c^2
*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g) - x/(b*d^2*g))*b*d^2 + 2*a*(c^2/((b*c*d^3
 - a*d^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*log(b*x + a)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(d*x + c)/((b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*g))*d^2 - 2*a*c*d*(c/((b*c*d^2 - a*d^3)*g*x + (b*c
^2*d - a*c*d^2)*g) + a*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g) -
a*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g)) - 2*b*d*(a^2*log(b*x +
 a)/((b^3*c - a*b^2*d)*g) - c^2*log(d*x + c)/((b*c*d^2 - a*d^3)*g) + x/(b*
d*g)) + 2*b*c*(a*log(b*x + a)/((b^2*c - a*b*d)*g) - c*log(d*x + c)/((b*c*d
 - a*d^2)*g)) - (log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b
*sqrt(f) + a*sqrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) +
 a*sqrt(g))))*sqrt(f)/g^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*
x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g)) + 1) + dilog((b*sqrt(g)*x + b*sqrt(
f))/(b*sqrt(f) - a*sqrt(g))))*sqrt(f)/g^(3/2) + (log(sqrt(g)*x - sqrt(f))*
log((d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + dilog(-(d*sqr
t(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g))))*sqrt(f)/g^(3/2) - (log(sq...
```

3.77.8 Giac [F]

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x^2 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

output
```
integrate(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int \frac{x^2\,\ln \left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

input
```
int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
```

output
```
int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.78 $\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$

3.78.1 Optimal result

Integrand size = 30, antiderivative size = 403

$$\begin{aligned}\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx = &-\frac{n \log(a+bx) \log \left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g}+\frac{n \log(c+dx) \log \left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} \\ &-\frac{n \log(a+bx) \log \left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}+\frac{n \log(c+dx) \log \left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} \\ &+\frac{\left(n \log(a+bx)-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)-n \log(c+dx)\right) \log (f-gx^2)}{2g} \\ &-\frac{n \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{n \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} \\ &+\frac{n \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}+\frac{n \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g}\end{aligned}$$

output
```
1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g-1/2
*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))/g+1/2*n*ln(d*
x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))/g-1/2*n*ln(b*x+a)*ln(
b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))/g+1/2*n*ln(d*x+c)*ln(d*(f^(1/
2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/g-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(b
*f^(1/2)-a*g^(1/2)))/g-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2
)))/g+1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g+1/2*n*poly
log(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g
```

3.78.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.02

$$\int \frac{x \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx =$$

$$-\frac{-n \log \left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right) \log \left(\sqrt{f}-\sqrt{g}x\right)+\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log \left(\sqrt{f}-\sqrt{g}x\right)+n \log \left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right) \log \left(\sqrt{f}-\sqrt{\right.}{}$$

input
```
Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

output
```
-1/2*(-(n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - S
qrt[g]*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + n*L
og[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] -
 n*Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[
g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-(
(Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] - n
*PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog
[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(
Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f]
+ Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/g
```

3.78.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2993, 240, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n \log(a+bx)-n \log(c+dx)\right) \int \frac{x}{f-gx^2}dx\right)+ n\int \frac{x \log(a+bx)}{f-gx^2}dx-n\int \frac{x \log(c+dx)}{f-gx^2}dx$$

$$\Big\downarrow \; 240$$

$$n\int \frac{x\log(a+bx)}{f-gx^2}dx-n\int \frac{x\log(c+dx)}{f-gx^2}dx+\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g}$$

$$\Big\downarrow \; 2863$$

$$n\int \left(\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)}-\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{g}x+\sqrt{f}\right)}\right)dx-n\int \left(\frac{\log(c+dx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)}-\frac{\log(c+dx)}{2\sqrt{g}\left(\sqrt{g}x+\sqrt{f}\right)}\right)dx+\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g}$$

$$\Big\downarrow \; 2009$$

$$\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g}+n\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g}-\frac{\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g}-\frac{\log(a+bx)\log\left(\frac{b(\sqrt{f}+}{b\sqrt{f}-}\right.}{2g}\right.$$

$$n\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2g}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g}-\frac{\log(c+dx)\log\left(\frac{d(\sqrt{f}+}{d\sqrt{f}-}\right.}{2g}\right.$$

input
```
Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

output
```
((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f
- g*x^2])/(2*g) + n*(-1/2*(Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*S
qrt[f] + a*Sqrt[g])])/g - (Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*S
qrt[f] - a*Sqrt[g])])/(2*g) - PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f]
- a*Sqrt[g]))]/(2*g) - PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[
g])]/(2*g)) - n*(-1/2*(Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[
f] + c*Sqrt[g])])/g - (Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[
f] - c*Sqrt[g])])/(2*g) - PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*
Sqrt[g]))]/(2*g) - PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]
/(2*g))
```

3.78. $\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.78.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.09

method	result
parts	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(-g\,x^{2}+f\right)}{2g}+\frac{n(-ad+cb)\left(\frac{\frac{\ln(dx+c)\ln\left(-g\,x^{2}+f\right)}{d}+\frac{2g\left(-\frac{\ln(dx+c)\left(\ln\left(\frac{d\sqrt{fg}-(dx+c)g+cg}{d\sqrt{fg}+cg}\right)+\ln\left(\frac{d\sqrt{fg}+(dx+c)g}{d\sqrt{fg}-cg}\right)\right.}{2g}\right.}{}}{ad-cb}\right.}{}$

input
```
int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(e*((b*x+a)/(d*x+c))^n)/g*ln(-g*x^2+f)+1/2/g*n*(-a*d+b*c)*((ln(d*x+
c)/d*ln(-g*x^2+f)+2/d*g*(-1/2*ln(d*x+c)*(ln((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/
(d*(f*g)^(1/2)+c*g))+ln((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g))
)/g-1/2*(dilog((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+dilog((d
*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g))*d/(a*d-b*c)-(ln(b*x+
a)/b*ln(-g*x^2+f)+2/b*g*(-1/2*ln(b*x+a)*(ln((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/
(b*(f*g)^(1/2)+a*g))+ln((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g))
)/g-1/2*(dilog((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+dilog((b
*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g))*b/(a*d-b*c))
```

3.78.5 Fricas [F]

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

output
```
integral(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

output
```
-integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.78.8 Giac [F]

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

output
```
integrate(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int \frac{x\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2}\,dx$$

input
```
int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
```

output
```
int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.79 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

3.79.1 Optimal result

Integrand size = 29, antiderivative size = 291

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{n\,\text{PolyLog}\left(2,\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\,\text{PolyLog}\left(2,\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}$$

output
```
1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(
b*f^(1/2)-a*g^(1/2)))/f^(1/2)/g^(1/2)-1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(
b*x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2
)+1/2*n*polylog(2,(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b*f^(1/2)-a*g^(1/
2)))/f^(1/2)/g^(1/2)-1/2*n*polylog(2,(b*x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)
/(b*f^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.45

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$
$$= \frac{n\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}x\right)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right)-n\log\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}x\right)}{}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

output
```
(n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*
x] - Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[(Sqrt
[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[-
((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] +
Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt[g]
*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + n*PolyLog
[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - n*PolyLog[2, (d*(
Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f]
+ Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g
]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g])
```

3.79.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2976, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$

$$\Big\downarrow \text{2976}$$

$$(bc-ad)\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{-ga^2+b^2f-\frac{2(bdf-acg)(a+bx)}{c+dx}+\frac{(d^2f-c^2g)(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\int \left(\frac{(d^2f-c^2g)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)\sqrt{f}\sqrt{g}\left(-2\sqrt{f}\sqrt{g}(bc-ad)+2bdf-2acg-\frac{2(d^2f-c^2g)(a+bx)}{c+dx}\right)}+\frac{(d^2f-c^2}{(bc-ad)\sqrt{f}\sqrt{g}\left(-2\sqrt{f}\sqrt{g}(bc\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(d\sqrt{f}-c\sqrt{g})}{(c+dx)(b\sqrt{f}-a\sqrt{g})}\right)}{2\sqrt{f}\sqrt{g}(bc-ad)}-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(c\sqrt{g}+d\sqrt{f})}{(c+dx)(a\sqrt{g}+b\sqrt{f})}\right)}{2\sqrt{f}\sqrt{g}(bc-ad)}+\frac{n\,\text{PolyLog}\left(2,\frac{(}{(}\right.}{2\sqrt{f}\sqrt{g}}\right.$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

output
```
(b*c - a*d)*((Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] - c*Sqrt[
g])*(a + b*x))/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))]])/(2*(b*c - a*d)*Sqrt[f
]*Sqrt[g]) - (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] + c*Sqrt[
g])*(a + b*x))/((b*Sqrt[f] + a*Sqrt[g])*(c + d*x))]])/(2*(b*c - a*d)*Sqrt[f
]*Sqrt[g]) + (n*PolyLog[2, ((d*Sqrt[f] - c*Sqrt[g])*(a + b*x))/((b*Sqrt[f]
 - a*Sqrt[g])*(c + d*x))]])/(2*(b*c - a*d)*Sqrt[f]*Sqrt[g]) - (n*PolyLog[2,
 ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] + a*Sqrt[g])*(c + d*x))]])
/(2*(b*c - a*d)*Sqrt[f]*Sqrt[g]))
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.79.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{-g\,x^{2}+f}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

3.79.5 Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f-gx^{2}}\,dx = \int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{2}-f}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

output
```
integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f-gx^{2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

output
```
Timed out
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$
$$= \frac{\left(\log\left(\sqrt{g}x-\sqrt{f}\right)\log\left(\frac{b\sqrt{g}x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}+1\right)-\log\left(\sqrt{g}x+\sqrt{f}\right)\log\left(-\frac{b\sqrt{g}x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}+1\right)-\log\left(\sqrt{g}x-\sqrt{f}\right)\log\left(\frac{d}{c}\right.\right.}{}$$
$$-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\log\left(\frac{gx-\sqrt{fg}}{gx+\sqrt{fg}}\right)}{2\sqrt{fg}}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*(log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a
*sqrt(g)) + 1) - log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(
b*sqrt(f) - a*sqrt(g)) + 1) - log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x -
d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + log(sqrt(g)*x + sqrt(f))*log(-(d
*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog(-(b*sqrt(g)*x
 - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))) - dilog((b*sqrt(g)*x + b*sqrt(f))/(
b*sqrt(f) - a*sqrt(g))) - dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*
sqrt(g))) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*n/sq
rt(f*g) - 1/2*log(e*((b*x + a)/(d*x + c))^n)*log((g*x - sqrt(f*g))/(g*x +
sqrt(f*g)))/sqrt(f*g)
```

3.79.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

output
```
integrate(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2), x)
```

3.80 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx$

3.80.1 Optimal result

Integrand size = 32, antiderivative size = 518

$$\begin{aligned}\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx &= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}\\
&- \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f}\\
&- \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f}\\
&- \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}\\
&+ \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2f}\\
&- \frac{n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} - \frac{n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f}\\
&+ \frac{n\,\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)}{f} + \frac{n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}\\
&+ \frac{n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f} - \frac{n\,\mathrm{PolyLog}\left(2,1+\frac{dx}{c}\right)}{f}\end{aligned}$$

output
```
n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*
((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c)
)^n)-n*ln(d*x+c))*ln(-g*x^2+f)/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/
(b*f^(1/2)+a*g^(1/2)))/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/
2)+c*g^(1/2)))/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(
1/2)))/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/f
+n*polylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/c)/f-1/2*n*polylog(2,-(b*x+a)*g^
(1/2)/(b*f^(1/2)-a*g^(1/2)))/f-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+
a*g^(1/2)))/f+1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/f+1/
2*n*polylog(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/f
```

3.80.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.94

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\,(f-gx^2)}\,dx =$$

$$-\frac{2n\log(x)\log\left(1+\frac{bx}{a}\right)-2\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2n\log(x)\log\left(1+\frac{dx}{c}\right)-n\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\right.}{}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

output
```
-1/2*(2*n*Log[x]*Log[1 + (b*x)/a] - 2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n
] - 2*n*Log[x]*Log[1 + (d*x)/c] - n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a
*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[S
qrt[f] - Sqrt[g]*x] + n*Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*L
og[Sqrt[f] - Sqrt[g]*x] - n*Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[
g]))]*Log[Sqrt[f] + Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f
] + Sqrt[g]*x] + n*Log[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log
[Sqrt[f] + Sqrt[g]*x] + 2*n*PolyLog[2, -((b*x)/a)] - 2*n*PolyLog[2, -((d*x
)/c)] - n*PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] +
n*PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLo
g[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*
(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/f
```

3.80.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2993, 243, 47, 14, 16, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int \frac{1}{x\left(f-gx^2\right)}dx\right)+n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}dx-n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}dx$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int \frac{1}{x^2\left(f-gx^2\right)}dx^2+n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}dx-n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}dx$$

$$\Big\downarrow \text{47}$$

$$-\frac{1}{2}\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{g\int \frac{1}{f-gx^2}dx^2}{f}+\frac{\int \frac{1}{x^2}dx^2}{f}\right)+n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}dx-n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}dx$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{2}\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{g\int \frac{1}{f-gx^2}dx^2}{f}+\frac{\log\left(x^2\right)}{f}\right)+n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}dx-n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}dx$$

$$\Big\downarrow \text{16}$$

$$n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}dx-n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}dx-\frac{1}{2}\left(\frac{\log\left(x^2\right)}{f}-\frac{\log\left(f-gx^2\right)}{f}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)$$

$$\Big\downarrow \text{2863}$$

$$n\int\left(\frac{\log(a+bx)}{fx}-\frac{gx\log(a+bx)}{f\left(gx^2-f\right)}\right)dx-n\int\left(\frac{\log(c+dx)}{fx}-\frac{gx\log(c+dx)}{f\left(gx^2-f\right)}\right)dx-\frac{1}{2}\left(\frac{\log\left(x^2\right)}{f}-\frac{\log\left(f-gx^2\right)}{f}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\left(\frac{\log\left(x^2\right)}{f}-\frac{\log\left(f-gx^2\right)}{f}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)+$$
$$n\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2f}-\frac{\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f}-\frac{\log(a+bx)\log\left(\frac{b(\sqrt{f}+}{b\sqrt{f}-}\right.}{2f}\right.$$
$$n\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2f}-\frac{\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f}-\frac{\log(c+dx)\log\left(\frac{d(\sqrt{f}+}{d\sqrt{f}-}\right.}{2f}\right.$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

output
```
-1/2*((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*(
Log[x^2]/f - Log[f - g*x^2]/f)) + n*((Log[-((b*x)/a)]*Log[a + b*x])/f - (L
og[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f)
- (Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2
*f) - PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]/(2*f) - P
olyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]/(2*f) + PolyLog[2,
1 + (b*x)/a]/f) - n*((Log[-((d*x)/c)]*Log[c + d*x])/f - (Log[c + d*x]*Log[
(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (Log[c + d*x]*
Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*f) - PolyLog[2,
 -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]/(2*f) - PolyLog[2, (Sqrt[
g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]/(2*f) + PolyLog[2, 1 + (d*x)/c]/f)
```

3.80.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.80.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.09

method	result
parts	$\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln(x)}{f}-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln(-g\,x^{2}+f)}{2f}-\frac{n\left((ad-cb)\left(\left(\frac{\ln(dx+c)\ln\left(-g\,x^{2}+f\right)}{d}+\frac{2g\left(-\frac{\ln(dx+c)\left(\ln\left(\frac{d\sqrt{fg}-(dx+c)g+c\cdots}{d\sqrt{fg}+cg}\right)\cdots\right.}{2\cdots}\right.}{\cdots}\right.\right.}{\cdots}$

input

```
int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x,method=_RETURNVERBOSE)
```

output

```
ln(e*((b*x+a)/(d*x+c))^n)/f*ln(x)-1/2*ln(e*((b*x+a)/(d*x+c))^n)/f*ln(-g*x^
2+f)-1/2*n*((a*d-b*c)/f*((ln(d*x+c)/d*ln(-g*x^2+f)+2/d*g*(-1/2*ln(d*x+c)*(
ln((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+ln((d*(f*g)^(1/2)+(d
*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g-1/2*(dilog((d*(f*g)^(1/2)-(d*x+c)*g+c
*g)/(d*(f*g)^(1/2)+c*g))+dilog((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2
)-c*g)))/g))*d/(a*d-b*c)-(ln(b*x+a)/b*ln(-g*x^2+f)+2/b*g*(-1/2*ln(b*x+a)*(
ln((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+ln((b*(f*g)^(1/2)+g*
(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g-1/2*(dilog((b*(f*g)^(1/2)-g*(b*x+a)+a
*g)/(b*(f*g)^(1/2)+a*g))+dilog((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2
)-a*g)))/g))*b/(a*d-b*c))-2*(a*d-b*c)/f*(d/(a*d-b*c)*(dilog((d*x+c)/c)/d+l
n(x)*ln((d*x+c)/c)/d)-b/(a*d-b*c)*(dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/
b)))
```

3.80. $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{x(f-gx^{2})}\,dx$

3.80.5 Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="fricas")
```

output
```
integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^3 - f*x), x)
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(-g*x**2+f),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="maxima")
```

output
```
-integrate(log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

3.80.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="giac")
```

output
```
integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x\,\left(f-g\,x^2\right)}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)), x)
```

3.81 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$

3.81.1 Optimal result

Integrand size = 32, antiderivative size = 596

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx = \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\,\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}} - \frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}}$$

output
```
b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x+d*n*ln(d*x+c
)/c/f+n*ln(d*x+c)/f/x+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/
f/x-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln
(d*x+c))*g^(1/2)/f^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/
2)+a*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d
*f^(1/2)+c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/
2))/(b*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x
*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,-(b*x+a)*
g^(1/2)/(b*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,(b*x+a)*g^(
1/2)/(b*f^(1/2)+a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,-(d*x+c)*g^(1/
2)/(d*f^(1/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,(d*x+c)*g^(1/2)/
(d*f^(1/2)+c*g^(1/2)))*g^(1/2)/f^(3/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$$
$$= \frac{-\frac{2\sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} + \frac{2\sqrt{f}n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} - \sqrt{g}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right)+\sqrt{g}\,\mathrm{lo}}{}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

output
```
((-2*Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n])/x + (2*Sqrt[f]*n*((b*c - a*d)
*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - Sqrt[g]*Log[e*((a
+ b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + Sqrt[g]*Log[e*((a + b*x)/(
c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + Sqrt[g]*n*((Log[(Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] + a*Sqrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g
])])*Log[Sqrt[f] - Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sq
rt[f] + a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*
Sqrt[g])]) - Sqrt[g]*n*((Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g])
)] - Log[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sq
rt[g]*x] + PolyLog[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] -
 PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*f^(3/2
))
```

3.81.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2993, 264, 221, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$$
$$\Big\downarrow \; 2993$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int\frac{1}{x^2(f-gx^2)}dx\right)+$$
$$n\int\frac{\log(a+bx)}{x^2(f-gx^2)}dx-n\int\frac{\log(c+dx)}{x^2(f-gx^2)}dx$$

$\big\downarrow$ 264

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{g\int\frac{1}{f-gx^2}dx}{f}-\frac{1}{fx}\right)\right)+$$
$$n\int\frac{\log(a+bx)}{x^2(f-gx^2)}dx-n\int\frac{\log(c+dx)}{x^2(f-gx^2)}dx$$

$\big\downarrow$ 221

$$n\int\frac{\log(a+bx)}{x^2(f-gx^2)}dx-n\int\frac{\log(c+dx)}{x^2(f-gx^2)}dx-$$
$$\left(\left(\frac{\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{f^{3/2}}-\frac{1}{fx}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)$$

$\big\downarrow$ 2863

$$n\int\left(\frac{g\log(a+bx)}{f(f-gx^2)}+\frac{\log(a+bx)}{fx^2}\right)dx-n\int\left(\frac{g\log(c+dx)}{f(f-gx^2)}+\frac{\log(c+dx)}{fx^2}\right)dx-$$
$$\left(\left(\frac{\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{f^{3/2}}-\frac{1}{fx}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)$$

$\big\downarrow$ 2009

$$-\left(\left(\frac{\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{f^{3/2}}-\frac{1}{fx}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)+$$
$$n\left(\frac{\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}+\frac{\sqrt{g}\log(a+bx)}{}\right.$$
$$n\left(\frac{\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}+\frac{\sqrt{g}\log(c+dx)}{}\right.$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

3.81. $\int\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}dx$

output
```
-((-(1/(f*x)) + (Sqrt[g]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]])/f^(3/2))*(n*Log[a +
 b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])) + n*((b*Log[x])/
(a*f) - (b*Log[a + b*x])/(a*f) - Log[a + b*x]/(f*x) - (Sqrt[g]*Log[a + b*x
]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) + (S
qrt[g]*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])]
)/(2*f^(3/2)) + (Sqrt[g]*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*S
qrt[g]))])/(2*f^(3/2)) - (Sqrt[g]*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f
] + a*Sqrt[g])])/(2*f^(3/2))) - n*((d*Log[x])/(c*f) - (d*Log[c + d*x])/(c*
f) - Log[c + d*x]/(f*x) - (Sqrt[g]*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*
x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*Log[c + d*x]*Log[(d*(
Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*Pol
yLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f^(3/2)) - (Sq
rt[g]*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2))
)
```

3.81.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

3.81. $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.81.4 Maple [F]

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x^{2} \left(-g\,x^{2}+f\right)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

3.81.5 Fricas [F]

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2 \left(f-gx^2\right)}\,dx=\int -\frac{\log \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(gx^{2}-f)x^{2}}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="fricas")
```

output
```
integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^4 - f*x^2), x)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2 \left(f-gx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(-g*x**2+f),x)
```

output
```
Timed out
```

3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(484) = 968$.

Time = 0.40 (sec) , antiderivative size = 969, normalized size of antiderivative = 1.63

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$$
$$=\frac{1}{2}\left(2\,acd\left(\frac{b^2\log(bx+a)}{(ab^2c^2-2\,a^2bcd+a^3d^2)f}+\frac{d}{(bc^2d-acd^2)fx+(bc^3-ac^2d)f}-\frac{(2\,bcd-ad^2)\log(dx+c)}{(b^2c^4-2\,abc^3d+a^2c^2d^2)f}-\right.\right.$$
$$-\frac{1}{2}\left(\frac{g\log\left(\frac{gx-\sqrt{fg}}{gx+\sqrt{fg}}\right)}{\sqrt{fg}f}+\frac{2}{fx}\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*(2*a*c*d*(b^2*log(b*x + a)/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*f) + d
/((b*c^2*d - a*c*d^2)*f*x + (b*c^3 - a*c^2*d)*f) - (2*b*c*d - a*d^2)*log(d
*x + c)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*f) - log(x)/(a*c^2*f)) + 2*
b*d^2*(c/((b*c*d^2 - a*d^3)*f*x + (b*c^2*d - a*c*d^2)*f) + a*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) - a*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*f)) - 2*b*c*d*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*f) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d - a*d
^2)*f*x + (b*c^2 - a*c*d)*f)) - 2*a*d^2*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*f) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1
/((b*c*d - a*d^2)*f*x + (b*c^2 - a*c*d)*f)) - 2*b*c*(b*log(b*x + a)/((a*b*
c - a^2*d)*f) - d*log(d*x + c)/((b*c^2 - a*c*d)*f) - log(x)/(a*c*f)) + 2*b
*d*(log(b*x + a)/((b*c - a*d)*f) - log(d*x + c)/((b*c - a*d)*f)) + (log(sq
rt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g)) +
 1) + dilog(-(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*sqrt(g)/f
^(3/2) - (log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(
f) - a*sqrt(g)) + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt
(g))))*sqrt(g)/f^(3/2) - (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*sq
rt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*
sqrt(f) + c*sqrt(g))))*sqrt(g)/f^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(d
*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)...
```

3.81.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx = \int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x^2}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="giac")
```

output
```
integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x^2), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx = \int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x^2\,(f-g\,x^2)}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)), x)
```

3.82 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

3.82.1 Optimal result

Integrand size = 34, antiderivative size = 1046

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n\log(a+bx)}{2b^2 h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2}$$
$$+ \frac{c^2 n\log(c+dx)}{2d^2 h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2}$$
$$+ \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2}$$
$$- \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2h}$$
$$- \frac{g(g^2-3fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^3\sqrt{g^2-4fh}}$$
$$+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{(g^2-fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^3}$$
$$+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$

output
```
1/2*a*n*x/b/h-1/2*c*n*x/d/h-1/2*a^2*n*ln(b*x+a)/b^2/h+1/2*n*x^2*ln(b*x+a)/
h-g*n*(b*x+a)*ln(b*x+a)/b/h^2+1/2*c^2*n*ln(d*x+c)/d^2/h-1/2*n*x^2*ln(d*x+c
)/h+g*n*(d*x+c)*ln(d*x+c)/d/h^2+g*x*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)
-n*ln(d*x+c))/h^2-1/2*x^2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+
c))/h-1/2*(-f*h+g^2)*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*l
n(h*x^2+g*x+f)/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a
*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/
h^3-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h
+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*polyl
og(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)
/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g
^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*ln(b*x+
a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g
^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*ln(d*x+c)*ln(-d*(g+2*h
*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*
h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4
*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*p
olylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+
g^2)/(-4*f*h+g^2)^(1/2))/h^3-g*(-3*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g^2)
^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^3/(-4*f*h...
```

3.82.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 1240, normalized size of antiderivative = 1.19

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$

$$= \frac{h^2x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \frac{2gh(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{2(bc-ad)ghn\log(c+dx)}{bd} + \frac{h^2n(-a^2d^2\log(a+bx)+b(d(-bc+ad)x+bc^2\log(c+d}{b^2d^2}}{}$$

input
```
Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

output
```
(h^2*x^2*Log[e*((a + b*x)/(c + d*x))^n] - (2*g*h*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n])/b + (2*(b*c - a*d)*g*h*n*Log[c + d*x])/(b*d) + (h^2*n*(-(
a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2
*d^2) + (2*f*g*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h]
+ 2*h*x])/Sqrt[g^2 - 4*f*h] + (g^2 - f*h)*(1 - g/Sqrt[g^2 - 4*f*h])*Log[e*
((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*Log
[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x])/Sqrt[g^2 -
 4*f*h] + (g^2 - f*h)*(1 + g/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x)
)^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*n*((Log[(2*h*(a + b*x))
/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2
*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog
[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*
f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g +
Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(-g + Sqrt[g^2 - 4
*f*h])*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - L
og[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g
^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(
b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*
h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + (
2*f*g*h*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - ...
```

3.82.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 959, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2993, 1143, 2009, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int \frac{x^3}{hx^2+gx+f}dx\right)+ n\int \frac{x^3\log(a+bx)}{hx^2+gx+f}dx-n\int \frac{x^3\log(c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{1143}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int\left(-\frac{g}{h^2}+\frac{x}{h}+\frac{fg+(g^2-fh)x}{h^2(hx^2+gx+f)}\right)dx\right)+$$
$$n\int\frac{x^3\log(a+bx)}{hx^2+gx+f}dx-n\int\frac{x^3\log(c+dx)}{hx^2+gx+f}dx$$

$\downarrow$ 2009

$$n\int\frac{x^3\log(a+bx)}{hx^2+gx+f}dx-n\int\frac{x^3\log(c+dx)}{hx^2+gx+f}dx-$$
$$\left(\left(\frac{g(g^2-3fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h^3\sqrt{g^2-4fh}}+\frac{(g^2-fh)\log(f+gx+hx^2)}{2h^3}-\frac{gx}{h^2}+\frac{x^2}{2h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log\right.\right.$$

$\downarrow$ 2865

$$n\int\left(\frac{x\log(a+bx)}{h}+\frac{(fg+(g^2-fh)x)\log(a+bx)}{h^2(hx^2+gx+f)}-\frac{g\log(a+bx)}{h^2}\right)dx-$$
$$n\int\left(\frac{x\log(c+dx)}{h}+\frac{(fg+(g^2-fh)x)\log(c+dx)}{h^2(hx^2+gx+f)}-\frac{g\log(c+dx)}{h^2}\right)dx-$$
$$\left(\left(\frac{g(g^2-3fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h^3\sqrt{g^2-4fh}}+\frac{(g^2-fh)\log(f+gx+hx^2)}{2h^3}-\frac{gx}{h^2}+\frac{x^2}{2h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log\right.\right.$$

$\downarrow$ 2009

$$-\left(\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\left(\frac{x^2}{2h}-\frac{gx}{h^2}+\frac{g(g^2-3fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h^3\sqrt{g^2-4fh}}+\frac{(g^2-\right.\right.$$
$$n\left(-\frac{\log(a+bx)a^2}{2b^2h}+\frac{xa}{2bh}-\frac{x^2}{4h}+\frac{gx}{h^2}+\frac{x^2\log(a+bx)}{2h}-\frac{g(a+bx)\log(a+bx)}{bh^2}+\frac{\left(g^2-\frac{(g^2-3fh)g}{\sqrt{g^2-4fh}}-fh\right)\log(a+}{2}\right.$$
$$n\left(-\frac{\log(c+dx)c^2}{2d^2h}+\frac{xc}{2dh}-\frac{x^2}{4h}+\frac{gx}{h^2}+\frac{x^2\log(c+dx)}{2h}-\frac{g(c+dx)\log(c+dx)}{dh^2}+\frac{\left(g^2-\frac{(g^2-3fh)g}{\sqrt{g^2-4fh}}-fh\right)\log(c+}{2}\right.$$

input
```
Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

3.82. $\int \frac{x^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

output
```
-((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*(-((g
*x)/h^2) + x^2/(2*h) + (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f
*h]])/(h^3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h)*Log[f + g*x + h*x^2])/(2*h^3)
)) + n*((g*x)/h^2 + (a*x)/(2*b*h) - x^2/(4*h) - (a^2*Log[a + b*x])/(2*b^2*
h) + (x^2*Log[a + b*x])/(2*h) - (g*(a + b*x)*Log[a + b*x])/(b*h^2) + ((g^2
 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-((b*(g - S
qrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^3)
+ ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(
2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*
h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f*h +
 (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)) - n*((g*x)/h^2 + (c*x)/(2*d*h) - x^2
/(4*h) - (c^2*Log[c + d*x])/(2*d^2*h) + (x^2*Log[c + d*x])/(2*h) - (g*(c +
 d*x)*Log[c + d*x])/(d*h^2) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4
*f*h])*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[
g^2 - 4*f*h])*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*
h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h)
)/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^...
```

3.82.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.82.4 Maple [F]

$$\int \frac{x^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{h\,x^2+gx+f}dx$$

input
```
int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

output
```
int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

3.82.5 Fricas [F]

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

input
```
integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="frica
s")
```

output
```
integral(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

output
```
Timed out
```

3.82.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx + hx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for
 more deta
```

3.82.8 Giac [F]

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx + hx^2}\,dx = \int \frac{x^3 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f}\,dx$$

input
```
integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac"
)
```

output
```
integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx + hx^2}\,dx = \int \frac{x^3 \ln \left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2 + g\,x + f}\,dx$$

input
```
int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

output
```
int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.83 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

3.83.1 Optimal result

Integrand size = 34, antiderivative size = 831

$$\begin{aligned}
&\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx \\
&= \frac{n(a+bx)\log(a+bx)}{bh}-\frac{n(c+dx)\log(c+dx)}{dh} \\
&-\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h} \\
&+\frac{(g^2-2fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}} \\
&-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&-\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2} \\
&-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&-\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}
\end{aligned}$$

output
```
n*(b*x+a)*ln(b*x+a)/b/h-n*(d*x+c)*ln(d*x+c)/d/h-x*(n*ln(b*x+a)-ln(e*((b*x+
a)/(d*x+c))^n)-n*ln(d*x+c))/h+1/2*g*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)
-n*ln(d*x+c))*ln(h*x^2+g*x+f)/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g
^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(
1/2))/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(
-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(
2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2
)^(1/2))/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))
*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4
*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h
+g^2)^(1/2))/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h
-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*
polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-
4*f*h+g^2)^(1/2))/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)
^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2+(-2*f*h+g^2)*arctanh((2*
h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x
+c))/h^2/(-4*f*h+g^2)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 1105, normalized size of antiderivative = 1.33

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$

$$= \frac{2dh\sqrt{g^2-4fh}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2(bc-ad)h\sqrt{g^2-4fh}n\log(c+dx)-2bdfh\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log}{}$$

input
```
Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

output
```
(2*d*h*Sqrt[g^2 - 4*f*h]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*(b*c
 - a*d)*h*Sqrt[g^2 - 4*f*h]*n*Log[c + d*x] - 2*b*d*f*h*Log[e*((a + b*x)/(c
 + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + b*d*g*(g - Sqrt[g^2 - 4*f
*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + 2
*b*d*f*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x]
 - b*d*g*(g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sq
rt[g^2 - 4*f*h] + 2*h*x] + 2*b*d*f*h*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a
*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[
g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + S
qrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyL
og[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f
*h]))]) - b*d*g*(g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(-(b*g) +
2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sq
rt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g
+ Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Po
lyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 -
4*f*h]))]) - 2*b*d*f*h*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 -
4*f*h]))] - Log[(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[
g + Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*
h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g...
```

3.83.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 766, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2993, 1143, 2009, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \frac{x^2}{hx^2+gx+f}dx\right)+ n \int \frac{x^2 \log (a+bx)}{hx^2+gx+f}dx-n \int \frac{x^2 \log (c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{1143}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int\left(\frac{1}{h}-\frac{f+gx}{h\left(hx^2+gx+f\right)}\right)dx\right)+ n\int\frac{x^2\log(a+bx)}{hx^2+gx+f}dx-n\int\frac{x^2\log(c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{2009}$$

$$n\int\frac{x^2\log(a+bx)}{hx^2+gx+f}dx-n\int\frac{x^2\log(c+dx)}{hx^2+gx+f}dx- \left(\left(-\frac{(g^2-2fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h^2\sqrt{g^2-4fh}}-\frac{g\log\left(f+gx+hx^2\right)}{2h^2}+\frac{x}{h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log\right.\right.$$

$$\Big\downarrow \text{2865}$$

$$n\int\left(\frac{\log(a+bx)}{h}-\frac{(f+gx)\log(a+bx)}{h\left(hx^2+gx+f\right)}\right)dx-n\int\left(\frac{\log(c+dx)}{h}-\frac{(f+gx)\log(c+dx)}{h\left(hx^2+gx+f\right)}\right)dx- \left(\left(-\frac{(g^2-2fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h^2\sqrt{g^2-4fh}}-\frac{g\log\left(f+gx+hx^2\right)}{2h^2}+\frac{x}{h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\left(\left(-\frac{(g^2-2fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h^2\sqrt{g^2-4fh}}-\frac{g\log\left(f+gx+hx^2\right)}{2h^2}+\frac{x}{h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\right.\right.$$

$$n\left(-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2}-\frac{\left(\frac{g^2-2fh}{\sqrt{g^2-4fh}}+g\right)\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}-\frac{(g\right.$$

$$n\left(-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2}-\frac{\left(\frac{g^2-2fh}{\sqrt{g^2-4fh}}+g\right)\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}-\frac{(g\right.$$

input
```
Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

3.83. $\int \frac{x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

output
```
-((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*(x/h
- ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]])/(h^2*Sqrt[g^2 - 4
*f*h]) - (g*Log[f + g*x + h*x^2])/(2*h^2))) + n*(-(x/h) + ((a + b*x)*Log[a
 + b*x])/(b*h) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-
((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))]
)/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-((b*(
g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*
h^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(
2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))
])/(2*h^2)) - n*(-(x/h) + ((c + d*x)*Log[c + d*x])/(d*h) - ((g - (g^2 - 2*
f*h)/Sqrt[g^2 - 4*f*h])*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h
*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) - ((g + (g^2 - 2*f*h)/
Sqrt[g^2 - 4*f*h])*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/
(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) - ((g - (g^2 - 2*f*h)/Sqrt[
g^2 - 4*f*h])*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]
))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(c +
 d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^2))
```

3.83.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.83.4 Maple [F]

$$\int \frac{x^2 \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{h\,x^2+gx+f}dx$$

input
```
int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

output
```
int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

3.83.5 Fricas [F]

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{x^2 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

input
```
integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="frica
s")
```

output
```
integral(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

output
```
Timed out
```

3.83.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for
 more deta
```

3.83.8 Giac [F]

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x^2 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f} \, dx$$

input
```
integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac"
)
```

output
```
integrate(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x^2 \ln \left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f} \, dx$$

input
```
int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

output
```
int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.84 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

3.84.1 Optimal result

Integrand size = 32, antiderivative size = 685

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= -\frac{g\,\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$

output
```
-1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/h
+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^
2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*
f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))
/h+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*
f*h+g^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/
2))))*(1-g/(-4*f*h+g^2)^(1/2))/h+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^
2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2
*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(
1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g
+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x
+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-g*arctanh
((2*h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln
(d*x+c))/h/(-4*f*h+g^2)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.79

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$$

$$= \frac{\left(-g+\sqrt{g^2-4fh}\right) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) \log \left(g-\sqrt{g^2-4fh}+2hx\right)+\left(g+\sqrt{g^2-4fh}\right) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) \log (g -}{}$$

input
```
Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

output
```
((-g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2
- 4*f*h] + 2*h*x] + (g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]
*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + (g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h
*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/
(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x
] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sq
rt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h
 + d*(-g + Sqrt[g^2 - 4*f*h]))]) - (g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
 + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*c*h
 - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyL
og[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*
h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sq
rt[g^2 - 4*f*h]))]))/(2*h*Sqrt[g^2 - 4*f*h])
```

3.84.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2993, 1142, 1083, 219, 1103, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$$

$$\Big\downarrow \text{ 2993}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \frac{x}{hx^2+gx+f}dx\right)+ n \int \frac{x \log (a+bx)}{hx^2+gx+f}dx-n \int \frac{x \log (c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{ 1142}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \left(\frac{\int \frac{g+2hx}{hx^2+gx+f}dx}{2h}-\frac{g \int \frac{1}{hx^2+gx+f}dx}{2h}\right)\right)+ n \int \frac{x \log (a+bx)}{hx^2+gx+f}dx-n \int \frac{x \log (c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \left(\frac{g \int \frac{1}{g^2-(g+2hx)^2-4fh}d(g+2hx)}{h}+\frac{\int \frac{g+2hx}{hx^2+gx+f}dx}{2h}\right)\right. n \int \frac{x \log (a+bx)}{hx^2+gx+f}dx-n \int \frac{x \log (c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{ 219}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \left(\frac{\int \frac{g+2hx}{hx^2+gx+f}dx}{2h}+\frac{g \text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h\sqrt{g^2-4fh}}\right)\right)+ n \int \frac{x \log (a+bx)}{hx^2+gx+f}dx-n \int \frac{x \log (c+dx)}{hx^2+gx+f}dx$$

$$\Big\downarrow \text{ 1103}$$

$$n\int \frac{x\log(a+bx)}{hx^2+gx+f}dx-n\int \frac{x\log(c+dx)}{hx^2+gx+f}dx-$$
$$\left(\left(\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h\sqrt{g^2-4fh}}+\frac{\log\left(f+gx+hx^2\right)}{2h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)$$

$$\Big\downarrow \text{2865}$$

$$n\int \left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+2hx-\sqrt{g^2-4fh}}+\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\log(a+bx)}{g+2hx+\sqrt{g^2-4fh}}\right)dx-$$
$$n\int \left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g+2hx-\sqrt{g^2-4fh}}+\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\log(c+dx)}{g+2hx+\sqrt{g^2-4fh}}\right)dx-$$
$$\left(\left(\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h\sqrt{g^2-4fh}}+\frac{\log\left(f+gx+hx^2\right)}{2h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\left(\left(\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{h\sqrt{g^2-4fh}}+\frac{\log\left(f+gx+hx^2\right)}{2h}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\right)+$$
$$n\left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{\left(1-\right.}{}\right.$$
$$n\left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{\left(1-\right.}{}\right.$$

input
```
Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

3.84. $\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

output
```
-((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*((g*A
rcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]])/(h*Sqrt[g^2 - 4*f*h]) + Log[f + g*x
 + h*x^2]/(2*h))) + n*(((1 - g/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-((b*(g
 - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h
) + ((1 + g/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h
] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h) + ((1 - g/Sqrt[g^
2 - 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))
])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h)) - n*(((1 - g/Sqrt[g^2 - 4*f*h])*Log[c
+ d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2
- 4*f*h])))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*Log[c + d*x]*Log[-((d*(g
+ Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h)
 + ((1 - g/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - S
qrt[g^2 - 4*f*h]))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(c
 + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h))
```

3.84.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84. $\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.84.4 Maple [F]

$$\int \frac{x \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}dx$$

input
```
int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

output
```
int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

3.84.5 Fricas [F]

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{x \log \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}{hx^{2}+gx+f}\,dx$$

input
```
integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas"
)
```

output
```
integral(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \text{Timed out}$$

input
```
integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

output
```
Timed out
```

3.84.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for
 more deta
```

3.84.8 Giac [F]

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f} \, dx$$

input
```
integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

output
```
integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x\, \ln\left(e\, {\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f} \,d x$$

input
```
int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

output
```
int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.85 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

3.85.1 Optimal result

Integrand size = 31, antiderivative size = 401

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = -\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} - \frac{n\,\mathrm{PolyLog}\left(2,\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{n\,\mathrm{PolyLog}\left(2,\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}}$$

output
```
-ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b
*d*f-b*c*g-a*d*g+2*a*c*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2
)+ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*
b*d*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/
2)-n*polylog(2,2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+
2*a*c*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+n*polylog(2,2*(
c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)
*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.28

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= \frac{-n \log \left(\frac{2h(a+bx)}{-bg+2ah+b\sqrt{g^2-4fh}}\right) \log \left(g-\sqrt{g^2-4fh}+2hx\right)+\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log \left(g-\sqrt{g^2-4fh}+2hx\right)+n}{}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

output
```
(-(n*Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])]*Log[g - S
qrt[g^2 - 4*f*h] + 2*h*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g
^2 - 4*f*h] + 2*h*x] + n*Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2
- 4*f*h])]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + n*Log[(2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - Log[
e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - n*Log[(2*h
*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h]
 + 2*h*x] + n*PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(d*g) + 2*
c*h + d*Sqrt[g^2 - 4*f*h])] - n*PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*
h*x))/(2*a*h + b*(-g + Sqrt[g^2 - 4*f*h]))] + n*PolyLog[2, (b*(g + Sqrt[g^
2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - n*PolyLog[2,
(d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))])
/Sqrt[g^2 - 4*f*h]
```

3.85.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2976, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$\Big\downarrow \ 2976$$

$$(bc-ad)\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ha^2-bga+b^2f-\frac{(2bdf-bcg-adg+2ach)(a+bx)}{c+dx}+\frac{(hc^2-dgc+d^2f)(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$\downarrow$ 2804

$$(bc-ad)\int\left(\frac{2(hc^2-dgc+d^2f)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)\sqrt{g^2-4fh}\left(-\sqrt{g^2-4fh}(bc-ad)+2bdf-bcg-adg+2ach-\frac{2(hc^2-dgc+d^2f)(a+bx)}{c+dx}\right)}+\frac{}{(bc-}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(-\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}(bc-ad)}+\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{}{(c+dx)(}\right.}{\sqrt{g^2-4}}\right.$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

output
```
(b*c - a*d)*(-((Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2*(d^2*f - c*d*g +
 c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*Sqrt[
g^2 - 4*f*h])*(c + d*x))])/((b*c - a*d)*Sqrt[g^2 - 4*f*h])) + (Log[e*((a +
 b*x)/(c + d*x))^n]*Log[1 - (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*
f - b*c*g - a*d*g + 2*a*c*h + (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))])/
((b*c - a*d)*Sqrt[g^2 - 4*f*h]) - (n*PolyLog[2, (2*(d^2*f - c*d*g + c^2*h)
*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*Sqrt[g^2 - 4
*f*h])*(c + d*x))])/((b*c - a*d)*Sqrt[g^2 - 4*f*h]) + (n*PolyLog[2, (2*(d^
2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h + (b*c
 - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))])/((b*c - a*d)*Sqrt[g^2 - 4*f*h]))
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.85.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

3.85.5 Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx+hx^{2}}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{hx^{2}+gx+f}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
```

output
```
integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx+hx^{2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

output
```
Timed out
```

3.85.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for
 more deta
```

3.85.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2), x)
```

3.86 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx$

3.86.1 Optimal result

Integrand size = 34, antiderivative size = 800

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx$$
$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}$$
$$- \frac{g\,\mathrm{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}}$$
$$- \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f}$$
$$- \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f}$$
$$- \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{n\,\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)}{f}$$
$$+ \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} - \frac{n\,\mathrm{PolyLog}\left(2,1+\frac{dx}{c}\right)}{f}$$

output
```
n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*
((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c)
)^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f+n*polylog(2,1+b*x/a)/f-n*polylog(2,1+d
*x/c)/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4
*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/f+1/2*n*ln(d*x+c)*ln(-d*(g+2*h
*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)
^(1/2))/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1
-g/(-4*f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g
^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4
*f*h+g^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2)
)/f+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h
+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a
*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2
*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f-g*
arctanh((2*h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
n)-n*ln(d*x+c))/f/(-4*f*h+g^2)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.78

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx$$

$$= \frac{2\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right)-\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log}{}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

output
```
(2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - (1 + g/Sqrt[g^2 - 4*f*h])*Log[e
*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (1 - g/Sqrt
[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] +
2*h*x] - 2*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -(
(b*x)/a)] - PolyLog[2, -((d*x)/c)]) + ((g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*
h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))
/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*
x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*S
qrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*
h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((-g + Sqrt[g^2 - 4
*f*h])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[
(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4
*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h +
b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x
))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f)
```

3.86.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 719, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2993, 1144, 25, 1142, 1083, 219, 1103, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\int\frac{1}{x\left(hx^2+gx+f\right)}dx\right)+ n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx$$

$$\Big\downarrow \text{1144}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{\int-\frac{g+hx}{hx^2+gx+f}dx}{f}+\frac{\log(x)}{f}\right)\right)+ n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{\log(x)}{f}-\frac{\int\frac{g+hx}{hx^2+gx+f}dx}{f}\right)\right)+ n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx$$

$\downarrow$ 1142

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{\log(x)}{f}-\frac{\frac{1}{2}g\int\frac{1}{hx^2+gx+f}dx+\frac{1}{2}\int\frac{g+2hx}{hx^2+gx+f}dx}{f}\right)\right)+ n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx$$

$\downarrow$ 1083

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{\log(x)}{f}-\frac{\frac{1}{2}\int\frac{g+2hx}{hx^2+gx+f}dx-g\int\frac{1}{g^2-(g+2hx)^2-4fh}d(g
$$
$$n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx$$

$\downarrow$ 219

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{\log(x)}{f}-\frac{\frac{1}{2}\int\frac{g+2hx}{hx^2+gx+f}dx-\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{\sqrt{g^2-4fh}}}{f}\right)\right.$$
$$n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx$$

$\downarrow$ 1103

$$n\int\frac{\log(a+bx)}{x\left(hx^2+gx+f\right)}dx-n\int\frac{\log(c+dx)}{x\left(hx^2+gx+f\right)}dx-$$
$$\left(\left(\frac{\log(x)}{f}-\frac{\frac{1}{2}\log\left(f+gx+hx^2\right)-\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{\sqrt{g^2-4fh}}}{f}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+d\right.\right.$$

$\downarrow$ 2865

3.86. $\int\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx$

$$n\int\left(\frac{\log(a+bx)}{fx}+\frac{(-g-hx)\log(a+bx)}{f\left(hx^2+gx+f\right)}\right)dx-$$
$$n\int\left(\frac{\log(c+dx)}{fx}+\frac{(-g-hx)\log(c+dx)}{f\left(hx^2+gx+f\right)}\right)dx-$$
$$\left(\left(\left(\frac{\log(x)}{f}-\frac{\frac{1}{2}\log\left(f+gx+hx^2\right)-\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{\sqrt{g^2-4fh}}}{f}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+d\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\left(\left(\left(\frac{\log(x)}{f}-\frac{\frac{1}{2}\log\left(f+gx+hx^2\right)-\frac{g\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{\sqrt{g^2-4fh}}}{f}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+\right.\right.\right.$$
$$n\left(-\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}-\right.$$
$$n\left(-\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}-\right.$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

3.86. $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx$

output
```
-((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*(Log[
x]/f - (-((g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]])/Sqrt[g^2 - 4*f*h]) +
Log[f + g*x + h*x^2]/2)/f)) + n*((Log[-((b*x)/a)]*Log[a + b*x])/f - ((1 +
g/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h]
)*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + S
qrt[g^2 - 4*f*h])))])/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(
a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqrt[g^2 -
 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/
(2*f) + PolyLog[2, 1 + (b*x)/a]/f) - n*((Log[-((d*x)/c)]*Log[c + d*x])/f -
 ((1 + g/Sqrt[g^2 - 4*f*h])*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f) - ((1 - g/Sqrt[g^2 -
 4*f*h])*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d
*(g + Sqrt[g^2 - 4*f*h])))])/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*PolyLog[2,
 (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqr
t[g^2 - 4*f*h])*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*
h]))])/(2*f) + PolyLog[2, 1 + (d*x)/c]/f)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.86.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x\left(h\,x^{2}+gx+f\right)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

3.86.5 Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="fricas"
)
```

output
```
integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^3 + g*x^2 + f*x), x)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(h*x**2+g*x+f),x)
```

output
```
Timed out
```

3.86.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for
 more deta
```

3.86.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="giac")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx=\int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x\,\left(h\,x^2+g\,x+f\right)}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)), x)
```

3.87 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

3.87.1 Optimal result

Integrand size = 34, antiderivative size = 995

$$\begin{aligned}
&\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2 (f+gx+hx^2)}\,dx \\
&= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} \\
&- \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+dx)}{fx} \\
&+ \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} \\
&+ \frac{(g^2-2fh)\,\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2\sqrt{g^2-4fh}} \\
&+ \frac{g\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2} \\
&+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f^2} \\
&+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{gn\,\text{PolyLog}\left(2,1+\frac{bx}{a}\right)}{f^2} - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} + \frac{gn\,\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{f^2}
\end{aligned}$$

output
```
b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x-g*n*ln(-b*x/
a)*ln(b*x+a)/f^2+d*n*ln(d*x+c)/c/f+n*ln(d*x+c)/f/x+g*n*ln(-d*x/c)*ln(d*x+c
)/f^2+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/x+g*ln(x)*(n*l
n(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2-1/2*g*(n*ln(b*x+a)-ln(
e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f^2-g*n*polylog(2,1+b*
x/a)/f^2+g*n*polylog(2,1+d*x/c)/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h
+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)
^(1/2))/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g
+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylo
g(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g
^2)^(1/2))/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))
))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(
-4*f*h+g^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f
*h+g^2)^(1/2))/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c
*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*
n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/
(-4*f*h+g^2)^(1/2))/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^
2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+(-2*f*h+g^2)*arctanh((
2*h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d
*x+c))/f^2/(-4*f*h+g^2)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.72

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2 (f+gx+hx^2)} \, dx$$

$$= \frac{-\frac{2f\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} - 2g\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \frac{2fn((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} + \left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)\log}{}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

output
```
((-2*f*Log[e*((a + b*x)/(c + d*x))^n])/x - 2*g*Log[x]*Log[e*((a + b*x)/(c
+ d*x))^n] + (2*f*n*((b*c - a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d
*x]))/(a*c) + (g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c +
d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + (g + (-g^2 + 2*f*h)/Sqrt[g^2
 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + 2*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b
*x)/a)] - PolyLog[2, -((d*x)/c)]) - ((g^2 - 2*f*h + g*Sqrt[g^2 - 4*f*h])*n
*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*
(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f
*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2
*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h
*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((g^2 - 2
*f*h - g*Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g
^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])
*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h]
 + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt
[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 -
 4*f*h])/(2*f^2)
```

3.87.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 883, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2993, 1145, 25, 1200, 2009, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2 \left(f+gx+hx^2\right)} \, dx$$

$$\Big\downarrow \text{2993}$$

$$-\left(\left(-\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+n \log (a+bx)-n \log (c+dx)\right) \int \frac{1}{x^2 \left(hx^2+gx+f\right)}dx\right)+ n \int \frac{\log (a+bx)}{x^2 \left(hx^2+gx+f\right)}dx-n \int \frac{\log (c+dx)}{x^2 \left(hx^2+gx+f\right)}dx$$

$$\Big\downarrow \text{1145}$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(\frac{\int -\frac{g+hx}{x(hx^2+gx+f)}dx}{f}-\frac{1}{fx}\right)\right)+ n\int\frac{\log(a+bx)}{x^2(hx^2+gx+f)}dx-n\int\frac{\log(c+dx)}{x^2(hx^2+gx+f)}dx$$

$$\Big\downarrow \; 25$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(-\frac{\int \frac{g+hx}{x(hx^2+gx+f)}dx}{f}-\frac{1}{fx}\right)\right)+ n\int\frac{\log(a+bx)}{x^2(hx^2+gx+f)}dx-n\int\frac{\log(c+dx)}{x^2(hx^2+gx+f)}dx$$

$$\Big\downarrow \; 1200$$

$$-\left(\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)\left(-\frac{\int \left(\frac{g}{fx}+\frac{-g^2-hxg+fh}{f(hx^2+gx+f)}\right)dx}{f}-\frac{1}{fx}\right)\right)+ n\int\frac{\log(a+bx)}{x^2(hx^2+gx+f)}dx-n\int\frac{\log(c+dx)}{x^2(hx^2+gx+f)}dx$$

$$\Big\downarrow \; 2009$$

$$n\int\frac{\log(a+bx)}{x^2(hx^2+gx+f)}dx-n\int\frac{\log(c+dx)}{x^2(hx^2+gx+f)}dx- \left(\left(-\frac{\frac{(g^2-2fh)\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{f\sqrt{g^2-4fh}}-\frac{g\log(f+gx+hx^2)}{2f}+\frac{g\log(x)}{f}}{f}-\frac{1}{fx}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\,\mathrm{l}\right.\right.$$

$$\Big\downarrow \; 2865$$

$$n\int\left(-\frac{g\log(a+bx)}{f^2x}+\frac{(g^2+hxg-fh)\log(a+bx)}{f^2(hx^2+gx+f)}+\frac{\log(a+bx)}{fx^2}\right)dx- n\int\left(-\frac{g\log(c+dx)}{f^2x}+\frac{(g^2+hxg-fh)\log(c+dx)}{f^2(hx^2+gx+f)}+\frac{\log(c+dx)}{fx^2}\right)dx- \left(\left(-\frac{\frac{(g^2-2fh)\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{f\sqrt{g^2-4fh}}-\frac{g\log(f+gx+hx^2)}{2f}+\frac{g\log(x)}{f}}{f}-\frac{1}{fx}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\,\mathrm{l}\right.\right.$$

$$\Big\downarrow \; 2009$$

3.87. $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

$$-\left(\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\left(-\frac{\frac{(g^2-2fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)}{f\sqrt{g^2-4fh}}+\frac{g\log(x)}{f}-\frac{g\log(hx^2+gx}{2f}}{f}\right.\right.$$

$$n\left(\frac{b\log(x)}{af}-\frac{g\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2}-\frac{b\log(a+bx)}{af}-\frac{\log(a+bx)}{fx}+\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)\log(a+bx)\log\left(-\frac{b(g+2hx}{2ah-b(g}\right.}{2f^2}\right.$$

$$n\left(\frac{d\log(x)}{cf}-\frac{g\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2}-\frac{d\log(c+dx)}{cf}-\frac{\log(c+dx)}{fx}+\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)\log(c+dx)\log\left(-\frac{d(g+2h}{2ch-d(}\right.}{2f^2}\right.$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

output
```
-((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*(-(1/
(f*x)) - (((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]])/(f*Sqrt[g
^2 - 4*f*h]) + (g*Log[x])/f - (g*Log[f + g*x + h*x^2])/(2*f))/f)) + n*((b*
Log[x])/(a*f) - (b*Log[a + b*x])/(a*f) - Log[a + b*x]/(f*x) - (g*Log[-((b*
x)/a)]*Log[a + b*x])/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[a +
b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 -
4*f*h])))])/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[a + b*x]*
Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h
])))])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(a
 + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2
- 4*f*h]))])/(2*f^2) - (g*PolyLog[2, 1 + (b*x)/a])/f^2) - n*((d*Log[x])/(c
*f) - (d*Log[c + d*x])/(c*f) - Log[c + d*x]/(f*x) - (g*Log[-((d*x)/c)]*Log
[c + d*x])/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[c + d*x]*Log[-
((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))]
)/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[c + d*x]*Log[-((d*(
g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*
f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*PolyLog[2, (2*h*(c + d*x))/(
2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h...
```

3.87. $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

3.87. $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

3.87.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x^{2}\left(h\,x^{2}+gx+f\right)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

3.87.5 Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{x^{2}\left(f+gx+hx^{2}\right)}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(hx^{2}+gx+f)x^{2}}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="frica
s")
```

output
```
integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^4 + g*x^3 + f*x^2), x)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{x^{2}\left(f+gx+hx^{2}\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(h*x**2+g*x+f),x)
```

output
```
Timed out
```

3.87.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for
 more deta
```

3.87.8 Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x^2}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="giac"
)
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x^2), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx = \int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x^2\,\left(h\,x^2+g\,x+f\right)}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)), x)
```

3.88 $\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} - \frac{\operatorname{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}$$

output
```
-ln(a/(b*x+a))*ln(c*x/(b*x+a))/b-polylog(2,1-a/(b*x+a))/b
```

3.88.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \frac{\log\left(-\frac{bx}{a}\right)\log\left(\frac{a}{a+bx}\right)}{b} + \frac{\log^2\left(\frac{a}{a+bx}\right)}{2b} - \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} - \frac{\operatorname{PolyLog}\left(2,\frac{a+bx}{a}\right)}{b}$$

input
```
Integrate[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

output
```
(Log[-((b*x)/a)]*Log[a/(a + b*x)])/b + Log[a/(a + b*x)]^2/(2*b) - (Log[a/(
a + b*x)]*Log[(c*x)/(a + b*x)])/b - PolyLog[2, (a + b*x)/a]/b
```

3.88.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2944, 2858, 25, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{cx}{a+bx}\right)}{a+bx} \, dx$$

$$\Big\downarrow \text{2944}$$

$$\frac{a \int \frac{\log \left(\frac{a}{a+bx}\right)}{x(a+bx)}dx}{b}-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2858}$$

$$\frac{a \int \frac{\log \left(\frac{a}{a+bx}\right)}{x(a+bx)}d(a+bx)}{b^2}-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a \int -\frac{\log \left(\frac{a}{a+bx}\right)}{x(a+bx)}d(a+bx)}{b^2}-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a \int -\frac{\log \left(\frac{a}{a+bx}\right)}{bx(a+bx)}d(a+bx)}{b}-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2778}$$

$$\frac{a \int -\frac{(a+bx) \log \left(\frac{a}{a+bx}\right)}{bx}d\frac{1}{a+bx}}{b}-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2005}$$

$$\frac{a \int \frac{\log \left(\frac{a}{a+bx}\right)}{\frac{a}{a+bx}-1}d\frac{1}{a+bx}}{b}-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\log \left(\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}$$

input
```
Int[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

output
```
-((Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)])/b) - PolyLog[2, 1 - a/(a + b*x)]
/b
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.88. $\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$

rule 2944
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

3.88.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

method	result	size
parts	$\frac{\ln\left(\frac{cx}{bx+a}\right)\ln(bx+a)}{b}-\frac{-\frac{c\ln(bx+a)^2}{2}+c\left(\operatorname{dilog}\left(-\frac{xb}{a}\right)+\ln(bx+a)\ln\left(-\frac{xb}{a}\right)\right)}{bc}$	69
derivativedivides	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97
default	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97
risch	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97

input
```
int(ln(c*x/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(c*x/(b*x+a))*ln(b*x+a)/b-1/b/c*(-1/2*c*ln(b*x+a)^2+c*(dilog(-x/a*b)+ln(
b*x+a)*ln(-x/a*b)))
```

3.88.5 Fricas [F]

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx=\int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}\,dx$$

input
```
integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

output
```
integral(log(c*x/(b*x + a))/(b*x + a), x)
```

3.88.6 Sympy [F]

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$$

input
```
integrate(ln(c*x/(b*x+a))/(b*x+a),x)
```

output
```
Integral(log(c*x/(a + b*x))/(a + b*x), x)
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \frac{\log(bx+a)\log\left(\frac{cx}{bx+a}\right)}{b} - \frac{\frac{c\log(bx+a)^2}{b} - \frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)c}{b}}{2c} + \frac{(c\log(bx+a) - c\log(x))\log(bx+a)}{bc}$$

input
```
integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

output
```
log(b*x + a)*log(c*x/(b*x + a))/b - 1/2*(c*log(b*x + a)^2/b - 2*(log(b*x/a
 + 1)*log(x) + dilog(-b*x/a))*c/b)/c + (c*log(b*x + a) - c*log(x))*log(b*x
 + a)/(b*c)
```

3.88.8 Giac [F]

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}\,dx$$

input
```
integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(c*x/(b*x + a))/(b*x + a), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)}{a+b\,x}\,dx$$

input
```
int(log((c*x)/(a + b*x))/(a + b*x),x)
```

output
```
int(log((c*x)/(a + b*x))/(a + b*x), x)
```

3.89 $\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

3.89.1 Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx=\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

output
```
1/3*ln(c*x/(b*x+a))^3/a
```

3.89.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx=\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

input
```
Integrate[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

output
```
Log[(c*x)/(a + b*x)]^3/(3*a)
```

3.89.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{\int \frac{(a+bx)\log^2\left(\frac{cx}{a+bx}\right)}{x}d\frac{x}{a+bx}}{a}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int \frac{x^2}{(a+bx)^2}d\log\left(\frac{cx}{a+bx}\right)}{a}$$

$$\Big\downarrow \; 15$$

$$\frac{x^3}{3a(a+bx)^3}$$

input
```
Int[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

output
```
x^3/(3*a*(a + b*x)^3)
```

3.89.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

3.89.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(\frac{cx}{bx+a}\right)^3}{3a}$	19
default	$\frac{\ln\left(\frac{cx}{bx+a}\right)^3}{3a}$	19
norman	$\frac{\ln\left(\frac{cx}{bx+a}\right)^3}{3a}$	19
risch	$\frac{\ln\left(\frac{cx}{bx+a}\right)^3}{3a}$	19
parallelrisch	$\frac{\ln\left(\frac{cx}{bx+a}\right)^3}{3a}$	19

input
```
int(ln(c*x/(b*x+a))^2/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*x/(b*x+a))^3/a
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log\left(\frac{cx}{bx+a}\right)^3}{3\,a}$$

input
```
integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")
```

output
```
1/3*log(c*x/(b*x + a))^3/a
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log\left(\frac{cx}{a+bx}\right)^3}{3a}$$

input
```
integrate(ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

output
```
log(c*x/(a + b*x))**3/(3*a)
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(18) = 36.

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\begin{aligned}
&\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx \\
&= -\left(\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}\right)\log\left(\frac{cx}{bx+a}\right)^2 \\
&\quad - \frac{\left(c\log(bx+a)^2 - 2\,c\log(bx+a)\log(x) + c\log(x)^2\right)\log\left(\frac{cx}{bx+a}\right)}{ac} \\
&\quad - \frac{c^2\log(bx+a)^3 - 3\,c^2\log(bx+a)^2\log(x) + 3\,c^2\log(bx+a)\log(x)^2 - c^2\log(x)^3}{3\,ac^2}
\end{aligned}$$

input
```
integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima")
```

output
```
-(log(b*x + a)/a - log(x)/a)*log(c*x/(b*x + a))^2 - (c*log(b*x + a)^2 - 2*
c*log(b*x + a)*log(x) + c*log(x)^2)*log(c*x/(b*x + a))/(a*c) - 1/3*(c^2*lo
g(b*x + a)^3 - 3*c^2*log(b*x + a)^2*log(x) + 3*c^2*log(b*x + a)*log(x)^2 -
 c^2*log(x)^3)/(a*c^2)
```

3.89.8 Giac [F]

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2}{(bx+a)x}\,dx$$

input
```
integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(c*x/(b*x + a))^2/((b*x + a)*x), x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^3}{3\,a}$$

input
```
int(log((c*x)/(a + b*x))^2/(x*(a + b*x)),x)
```

output
```
log((c*x)/(a + b*x))^3/(3*a)
```

3.90 $\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

3.90.1 Optimal result

Integrand size = 34, antiderivative size = 82

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{a} + \frac{2\log\left(\frac{cx}{a+bx}\right)\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right)}{a} - \frac{2\,\text{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

output
```
-ln(c*x/(b*x+a))^2*polylog(2,1-a/(b*x+a))/a+2*ln(c*x/(b*x+a))*polylog(3,1-
a/(b*x+a))/a-2*polylog(4,1-a/(b*x+a))/a
```

3.90.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{PolyLog}\left(2,\frac{bx}{a+bx}\right)}{a} + \frac{2\log\left(\frac{cx}{a+bx}\right)\text{PolyLog}\left(3,\frac{bx}{a+bx}\right)}{a} - \frac{2\,\text{PolyLog}\left(4,\frac{bx}{a+bx}\right)}{a}$$

input
```
Integrate[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

output
```
-((Log[(c*x)/(a + b*x)]^2*PolyLog[2, (b*x)/(a + b*x)])/a) + (2*Log[(c*x)/(
a + b*x)]*PolyLog[3, (b*x)/(a + b*x)])/a - (2*PolyLog[4, (b*x)/(a + b*x)])
/a
```

3.90.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2988, 2990, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{a}{a+bx}\right) \log ^2\left(\frac{cx}{a+bx}\right)}{x (a+bx)} \, dx$$

$$\Big\downarrow \text{2988}$$

$$2 \int \frac{\log \left(\frac{cx}{a+bx}\right) \text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{x (a+bx)}dx-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right) \log ^2\left(\frac{cx}{a+bx}\right)}{a}$$

$$\Big\downarrow \text{2990}$$

$$2 \left(\frac{\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{a}-\int \frac{\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right)}{x (a+bx)}dx\right)-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right) \log ^2\left(\frac{cx}{a+bx}\right)}{a}$$

$$\Big\downarrow \text{7164}$$

$$2 \left(\frac{\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right) \log \left(\frac{cx}{a+bx}\right)}{a}-\frac{\text{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}\right)-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right) \log ^2\left(\frac{cx}{a+bx}\right)}{a}$$

input
```
Int[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

output
```
-((Log[(c*x)/(a + b*x)]^2*PolyLog[2, 1 - a/(a + b*x)])/a) + 2*((Log[(c*x)/
(a + b*x)]*PolyLog[3, 1 - a/(a + b*x)])/a - PolyLog[4, 1 - a/(a + b*x)]/a)
```

3.90.3.1 Defintions of rubi rules used

rule 2988
```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Simp[h*p
*r*s   Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/
((a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e
, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 2990
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Simp[h*p*
r*s   Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.90.4 Maple [F]

$$\int \frac{\ln\left(\frac{a}{bx+a}\right)\ln\left(\frac{cx}{bx+a}\right)^{2}}{x\,(bx+a)}dx$$

input
```
int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

output
```
int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

3.90.5 Fricas [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

input
```
integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas
")
```

output
```
integral(log(c*x/(b*x + a))^2*log(a/(b*x + a))/(b*x^2 + a*x), x)
```

3.90.6 Sympy [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)^2}{x\,(a+bx)}\,dx$$

input
```
integrate(ln(a/(b*x+a))*ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

output
```
Integral(log(a/(a + b*x))*log(c*x/(a + b*x))**2/(x*(a + b*x)), x)
```

3.90.7 Maxima [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

input
```
integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima
")
```

output
```
1/4*(log(b*x + a)^4 - 4*log(b*x + a)^3*log(x))/a + integrate((a*log(a)*log
(c)^2 + 2*a*log(a)*log(c)*log(x) + a*log(a)*log(x)^2 + (a*(log(a) + 2*log(
c)) + (3*b*x + 2*a)*log(x))*log(b*x + a)^2 - (2*a*(log(a) + log(c))*log(x)
 + a*log(x)^2 + (2*log(a)*log(c) + log(c)^2)*a)*log(b*x + a))/(a*b*x^2 + a
^2*x), x)
```

3.90.8 Giac [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

input
```
integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(c*x/(b*x + a))^2*log(a/(b*x + a))/((b*x + a)*x), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^2\ln\left(\frac{a}{a+b\,x}\right)}{x\,\left(a+b\,x\right)}\,dx$$

input
```
int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)),x)
```

output
```
int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)), x)
```

3.91 $\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx$

3.91.1 Optimal result

Integrand size = 55, antiderivative size = 150

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2\operatorname{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

output
```
-ln(e*(b*x+a)/(d*x+c))^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+2*l
n(e*(b*x+a)/(d*x+c))*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g-2*polyl
og(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

3.91.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \frac{-\log^2\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)+2\log\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-2\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{(bc-ad)g}$$

input
```
Integrate[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/
((c + d*x)*(a*g + b*g*x)),x]
```

output
```
(-(Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
 + 2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]
- 2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g)
```

3.91.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.055, Rules used = {2988, 2990, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}dx$$

$$\Big\downarrow \text{ 2988}$$

$$\frac{2\int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right)\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{g}-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}$$

$$\Big\downarrow \text{ 2990}$$

$$\frac{2\left(\frac{\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bc-ad}-\int \frac{\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx\right)}{g}-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{2\left(\frac{\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bc-ad}-\frac{\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{bc-ad}\right)}{g}-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}$$

input
```
Int[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/((c +
d*x)*(a*g + b*g*x)),x]
```

output
```
-((Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))
])/((b*c - a*d)*g)) + (2*((Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, 1 - (b*
c - a*d)/(b*(c + d*x))])/(b*c - a*d) - PolyLog[4, 1 - (b*c - a*d)/(b*(c +
d*x))]/(b*c - a*d)))/g
```

3.91.3.1 Defintions of rubi rules used

rule 2988
```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Simp[h*p
*r*s   Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/
((a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e
, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 2990
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Simp[h*p*
r*s   Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.91.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\frac{\ln\left(-\frac{\frac{e(bx+a)d}{dx+c}-be}{be}\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)^{3}}{3}-\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)^{3}\ln\left(1-\frac{d(bx+a)}{b(dx+c)}\right)}{3}-\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}\mathrm{Li}_2\left(\frac{d(bx+a)}{b(dx+c)}\right)+2\ln\left(\frac{e(bx+a)}{dx+c}\right)\mathrm{Li}_3\left(\frac{d(bx+a)}{b(dx+c)}\right)-}{g(ad-cb)}$

input
```
int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x
,method=_RETURNVERBOSE)
```

output
```
-1/g/(a*d-b*c)*(1/3*ln(-(e*(b*x+a)/(d*x+c)*d-b*e)/b/e)*ln(e*(b*x+a)/(d*x+c
))^3-1/3*ln(e*(b*x+a)/(d*x+c))^3*ln(1-d*(b*x+a)/b/(d*x+c))-ln(e*(b*x+a)/(d
*x+c))^2*polylog(2,d*(b*x+a)/b/(d*x+c))+2*ln(e*(b*x+a)/(d*x+c))*polylog(3,
d*(b*x+a)/b/(d*x+c))-2*polylog(4,d*(b*x+a)/b/(d*x+c)))
```

3.91.5 Fricas [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

input
```
integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*
x+a*g),x, algorithm="fricas")
```

output
```
integral(log((b*c - a*d)/(b*d*x + b*c))*log((b*e*x + a*e)/(d*x + c))^2/(b*
d*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

3.91.6 Sympy [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{d\int \frac{\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^3}{c+dx}\,dx}{3g\,(ad-bc)} - \frac{\log\left(\frac{-ad+bc}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adg-3bcg}$$

input
```
integrate(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))**2/(d*x+c)/(b*g*x
+a*g),x)
```

output
```
-d*Integral(log(a*e/(c + d*x) + b*e*x/(c + d*x))**3/(c + d*x), x)/(3*g*(a*
d - b*c)) - log((-a*d + b*c)/(b*(c + d*x)))*log(e*(a + b*x)/(c + d*x))**3/
(3*a*d*g - 3*b*c*g)
```

3.91.7 Maxima [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

input
```
integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*
x+a*g),x, algorithm="maxima")
```

output
```
-1/4*(4*log(b*x + a)*log(d*x + c)^3 - log(d*x + c)^4)/(b*c*g - a*d*g) - in
tegrate((((d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*
b)*log(b*x + a)^2 + ((d*log(b*c - a*d) - d*log(b) + 2*d*log(e))*a - (c*(lo
g(b*c - a*d) + 2*log(e)) - c*log(b))*b - (3*b*d*x + 2*b*c + a*d)*log(b*x +
 a))*log(d*x + c)^2 + (d*log(b*c - a*d)*log(e)^2 - d*log(b)*log(e)^2)*a -
(c*log(b*c - a*d)*log(e)^2 - c*log(b)*log(e)^2)*b + 2*((d*log(b*c - a*d)*l
og(e) - d*log(b)*log(e))*a - (c*log(b*c - a*d)*log(e) - c*log(b)*log(e))*b
)*log(b*x + a) + ((b*c - a*d)*log(b*x + a)^2 - (2*d*log(b*c - a*d)*log(e)
- 2*d*log(b)*log(e) + d*log(e)^2)*a - (2*c*log(b)*log(e) - (2*log(b*c - a*
d)*log(e) + log(e)^2)*c)*b - 2*((d*log(b*c - a*d) - d*log(b) + d*log(e))*a
 - (c*(log(b*c - a*d) + log(e)) - c*log(b))*b)*log(b*x + a))*log(d*x + c))
/(a*b*c^2*g - a^2*c*d*g + (b^2*c*d*g - a*b*d^2*g)*x^2 + (b^2*c^2*g - a^2*d
^2*g)*x), x)
```

3.91.8 Giac [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

input
```
integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*
x+a*g),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)*e/(d*x + c))^2*log((b*c - a*d)/((d*x + c)*b))/((b*
g*x + a*g)*(d*x + c)), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\ln\left(-\frac{a\,d-b\,c}{b\,(c+d\,x)}\right)}{(a\,g+b\,g\,x)\,\left(c+d\,x\right)}\,dx$$

input
```
int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g
 + b*g*x)*(c + d*x)),x)
```

output
```
int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g
 + b*g*x)*(c + d*x)), x)
```

3.92 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$

3.92.1 Optimal result

Integrand size = 58, antiderivative size = 160

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2n^2\operatorname{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

output
```
-ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
+2*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)
/g-2*n^2*polylog(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

3.92.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 785 vs. 2(160) = 320.

Time = 0.31 (sec) , antiderivative size = 785, normalized size of antiderivative = 4.91

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$$
$$= \frac{\log\left(\frac{a+bx}{c+dx}\right)\left(3\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 3n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{a+bx}{c+dx}\right) + n^2\log^2\left(\frac{a+bx}{c+dx}\right)\right)\log\left(\frac{bc-ad}{bc+bdx}\right) + \frac{3}{2}\left(\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{}$$

input
```
Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))]
)/((c + d*x)*(a*g + b*g*x)),x]
```

output
```
(Log[(a + b*x)/(c + d*x)]*(3*Log[e*((a + b*x)/(c + d*x))^n]^2 - 3*n*Log[e*
((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)] + n^2*Log[(a + b*x)/(c +
 d*x)]^2)*Log[(b*c - a*d)/(b*c + b*d*x)] + (3*(Log[e*((a + b*x)/(c + d*x))
^n] - n*Log[(a + b*x)/(c + d*x)])^2*(-Log[c/d + x]^2 - 2*Log[a/b + x]*Log[
c + d*x] + 2*Log[c/d + x]*Log[c + d*x] + 2*Log[(a + b*x)/(c + d*x)]*Log[c
+ d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[2, (d*(
a + b*x))/(-(b*c) + a*d)]))/2 + n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
(a + b*x)/(c + d*x)])*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b + x] +
Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d + x] + Log
[(a + b*x)/(c + d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))
/(b*c - a*d)] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] +
3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2
 - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x)
)/(b*c - a*d)]) - n^2*(Log[(a + b*x)/(c + d*x)]^3*Log[(b*c - a*d)/(b*c + b
*d*x)] + 3*Log[(a + b*x)/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))] - 6*Log[(a + b*x)/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] +
 6*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))]))/(3*(b*c - a*d)*g)
```

3.92.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2988, 2990, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)(ag+bgx)}dx$$

$$\Big\downarrow \text{2988}$$

$$\frac{2n\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{g}-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}$$

$$\Big\downarrow \text{2990}$$

$$\frac{2n\left(\frac{\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}-n\int\frac{\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx\right)}{g}-\frac{\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{2n\left(\frac{\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}-\frac{n\operatorname{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{bc-ad}\right)}{g}-\frac{\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}$$

input
```
Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/((c
+ d*x)*(a*g + b*g*x)),x]
```

output
```
-((Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x
))])/((b*c - a*d)*g)) + (2*n*((Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, 1
 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (n*PolyLog[4, 1 - (b*c - a*d)
/(b*(c + d*x))])/(b*c - a*d)))/g
```

3.92.3.1 Defintions of rubi rules used

rule 2988
```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Simp[h*p
*r*s   Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/
((a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e
, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 2990
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Simp[h*p*
r*s   Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.92.4 Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\ln\left(\frac{-ad+cb}{b(dx+c)}\right)}{(dx+c)\,(bgx+ag)}dx$$

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*
g),x)
```

output
```
int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*
g),x)
```

3.92.5 Fricas [F]

$$\int \frac{\log^{2}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(
b*g*x+a*g),x, algorithm="fricas")
```

output
```
integral(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/(b*d*x + b*c))/(
b*d*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

3.92.6 Sympy [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \frac{\int \frac{\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2\log\left(-\frac{ad}{bc+bdx}+\frac{bc}{bc+bdx}\right)}{ac+adx+bcx+bdx^2}\,dx}{g}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(
b*g*x+a*g),x)
```

output
```
Integral(log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2*log(-a*d/(b*c + b*d*x)
 + b*c/(b*c + b*d*x))/(a*c + a*d*x + b*c*x + b*d*x**2), x)/g
```

3.92.7 Maxima [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(
b*g*x+a*g),x, algorithm="maxima")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/
((b*g*x + a*g)*(d*x + c)), x)
```

3.92.8 Giac [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(
b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/
((b*g*x + a*g)*(d*x + c)), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\ln\left(-\frac{a\,d-b\,c}{b\,(c+d\,x)}\right)\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{(a\,g+b\,g\,x)\,(c+d\,x)}\,dx$$

input
```
int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a
*g + b*g*x)*(c + d*x)),x)
```

output
```
int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a
*g + b*g*x)*(c + d*x)), x)
```

3.93 $\int \log \left(\frac{c(b+ax)}{x}\right) dx$

3.93.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \log \left(\frac{c(b+ax)}{x}\right) dx = x \log \left(ac+\frac{bc}{x}\right)+\frac{b \log(b+ax)}{a}$$

output
```
x*ln(a*c+b*c/x)+b*ln(a*x+b)/a
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \log \left(\frac{c(b+ax)}{x}\right) dx = \frac{b \log(x)}{a}+\frac{(b+ax) \log \left(\frac{c(b+ax)}{x}\right)}{a}$$

input
```
Integrate[Log[(c*(b + a*x))/x],x]
```

output
```
(b*Log[x])/a + ((b + a*x)*Log[(c*(b + a*x))/x])/a
```

3.93.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2903, 2898, 27, 795, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\frac{c(ax+b)}{x}\right)dx$$

$$\Big\downarrow \text{2903}$$

$$\int \log\left(ac+\frac{bc}{x}\right)dx$$

$$\Big\downarrow \text{2898}$$

$$bc\int \frac{1}{c\left(a+\frac{b}{x}\right)x}dx+x\log\left(ac+\frac{bc}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{1}{\left(a+\frac{b}{x}\right)x}dx+x\log\left(ac+\frac{bc}{x}\right)$$

$$\Big\downarrow \text{795}$$

$$b\int \frac{1}{b+ax}dx+x\log\left(ac+\frac{bc}{x}\right)$$

$$\Big\downarrow \text{16}$$

$$x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(ax+b)}{a}$$

input
```
Int[Log[(c*(b + a*x))/x],x]
```

output
```
x*Log[a*c + (b*c)/x] + (b*Log[b + a*x])/a
```

3.93.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2903
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

3.93.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
risch	$x \ln\left(\frac{c(ax+b)}{x}\right)+\frac{b\ln(ax+b)}{a}$	26
parts	$x \ln\left(\frac{c(ax+b)}{x}\right)+\frac{b\ln(ax+b)}{a}$	26
parallelrisch	$-\frac{-\ln\left(\frac{c(ax+b)}{x}\right)xab-\ln(x)b^2-b^2\ln\left(\frac{c(ax+b)}{x}\right)}{ab}$	49
derivativedivides	$-cb\left(\frac{\ln\left(-\frac{bc}{x}\right)}{ac}-\frac{\ln\left(ca+\frac{bc}{x}\right)\left(ca+\frac{bc}{x}\right)x}{a\,c^2b}\right)$	54
default	$-cb\left(\frac{\ln\left(-\frac{bc}{x}\right)}{ac}-\frac{\ln\left(ca+\frac{bc}{x}\right)\left(ca+\frac{bc}{x}\right)x}{a\,c^2b}\right)$	54

input
```
int(ln(c*(a*x+b)/x),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a*x+b)/x)+b*ln(a*x+b)/a
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \log \left(\frac{c(b+ax)}{x}\right) \, dx = \frac{ax \log \left(\frac{acx+bc}{x}\right) + b \log (ax+b)}{a}$$

input
```
integrate(log(c*(a*x+b)/x),x, algorithm="fricas")
```

output
```
(a*x*log((a*c*x + b*c)/x) + b*log(a*x + b))/a
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \log \left(\frac{c(b+ax)}{x}\right) \, dx = x \log \left(\frac{c(ax+b)}{x}\right) + \frac{b \log (ax+b)}{a}$$

input
```
integrate(ln(c*(a*x+b)/x),x)
```

output
```
x*log(c*(a*x + b)/x) + b*log(a*x + b)/a
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{c(b+ax)}{x}\right) \, dx = x \log \left(\frac{(ax+b)c}{x}\right) + \frac{b \log (ax+b)}{a}$$

input
```
integrate(log(c*(a*x+b)/x),x, algorithm="maxima")
```

output
```
x*log((a*x + b)*c/x) + b*log(a*x + b)/a
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.12

$$\int \log\left(\frac{c(b+ax)}{x}\right)\,dx = \frac{b^2c^2\left(\frac{\log\left(\frac{|acx+bc|}{|x|}\right)}{ac}-\frac{\log\left(\left|-ac+\frac{acx+bc}{x}\right|\right)}{ac}\right)-\frac{b^2c^2\log\left(-\left(b-\frac{a}{\frac{a}{b}-\frac{acx+bc}{bcx}}\right)c\left(\frac{a}{b}-\frac{acx+bc}{bcx}\right)\right)}{ac-\frac{acx+bc}{x}}}{bc}$$

input
```
integrate(log(c*(a*x+b)/x),x, algorithm="giac")
```

output
```
(b^2*c^2*(log(abs(a*c*x + b*c)/abs(x))/(a*c) - log(abs(-a*c + (a*c*x + b*c
)/x))/(a*c)) - b^2*c^2*log(-(b - a/(a/b - (a*c*x + b*c)/(b*c*x)))*c*(a/b -
 (a*c*x + b*c)/(b*c*x)))/(a*c - (a*c*x + b*c)/x))/(b*c)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{c(b+ax)}{x}\right)\,dx = x\,\ln\left(\frac{c\,(b+a\,x)}{x}\right)+\frac{b\,\ln\left(b+a\,x\right)}{a}$$

input
```
int(log((c*(b + a*x))/x),x)
```

output
```
x*log((c*(b + a*x))/x) + (b*log(b + a*x))/a
```

3.94 $\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx$

3.94.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a}-\frac{2b\,\text{PolyLog}\left(2,1+\frac{b}{ax}\right)}{a}$$

output
```
(a*x+b)*ln(a*c+b*c/x)^2/a-2*b*ln(c*(a+b/x))*ln(-b/a/x)/a-2*b*polylog(2,1+b
/a/x)/a
```

3.94.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{\log\left(\frac{c(b+ax)}{x}\right)\left(-2b\log\left(-\frac{b}{ax}\right)+(b+ax)\log\left(\frac{c(b+ax)}{x}\right)\right)-2b\,\text{PolyLog}\left(2,1+\frac{b}{ax}\right)}{a}$$

input
```
Integrate[Log[(c*(b + a*x))/x]^2,x]
```

output
```
(Log[(c*(b + a*x))/x]*(-2*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))/
x]) - 2*b*PolyLog[2, 1 + b/(a*x)])/a
```

3.94.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2903, 2899, 2904, 2841, 27, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(\frac{c(ax+b)}{x}\right) dx$$

$$\Big\downarrow \text{2903}$$

$$\int \log^2\left(ac+\frac{bc}{x}\right) dx$$

$$\Big\downarrow \text{2899}$$

$$\frac{2b\int \frac{\log\left(ac+\frac{bc}{x}\right)}{x}dx}{a}+\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}$$

$$\Big\downarrow \text{2904}$$

$$\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\int x\log\left(ac+\frac{bc}{x}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2841}$$

$$\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\left(\log\left(-\frac{b}{ax}\right)\log\left(ac+\frac{bc}{x}\right)-bc\int \frac{\log\left(-\frac{b}{ax}\right)}{c\left(a+\frac{b}{x}\right)}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\left(\log\left(-\frac{b}{ax}\right)\log\left(ac+\frac{bc}{x}\right)-b\int \frac{\log\left(-\frac{b}{ax}\right)}{a+\frac{b}{x}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\left(\log\left(-\frac{b}{ax}\right)\log\left(ac+\frac{bc}{x}\right)+\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\right)}{a}$$

input
```
Int[Log[(c*(b + a*x))/x]^2,x]
```

output
```
((b + a*x)*Log[a*c + (b*c)/x]^2)/a - (2*b*(Log[a*c + (b*c)/x]*Log[-(b/(a*x
))] + PolyLog[2, 1 + b/(a*x)]))/a
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2899
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Simp[b*e*p*(q/d)   I
nt[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}
, x] && IGtQ[q, 0]
```

rule 2903
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

3.94.4 Maple [F]

$$\int \ln\left(\frac{c(ax+b)}{x}\right)^2 dx$$

input
```
int(ln(c*(a*x+b)/x)^2,x)
```

output
```
int(ln(c*(a*x+b)/x)^2,x)
```

3.94.5 Fricas [F]

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log\left(\frac{(ax+b)c}{x}\right)^2\,dx$$

input
```
integrate(log(c*(a*x+b)/x)^2,x, algorithm="fricas")
```

output
```
integral(log((a*c*x + b*c)/x)^2, x)
```

3.94.6 Sympy [F]

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx = 2b\int \frac{\log\left(ac+\frac{bc}{x}\right)}{ax+b}\,dx + x\log\left(\frac{c(ax+b)}{x}\right)^2$$

input
```
integrate(ln(c*(a*x+b)/x)**2,x)
```

output
```
2*b*Integral(log(a*c + b*c/x)/(a*x + b), x) + x*log(c*(a*x + b)/x)**2
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx &= x\log\left(\frac{(ax+b)c}{x}\right)^2 + \frac{2\,b\log(ax+b)\log\left(\frac{(ax+b)c}{x}\right)}{a} \\ &\quad + \frac{\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2\,(c\log(ax+b)-c\log(x))b\log(ax+b)}{a}}{c}\end{aligned}$$

input
```
integrate(log(c*(a*x+b)/x)^2,x, algorithm="maxima")
```

output
```
x*log((a*x + b)*c/x)^2 + 2*b*log(a*x + b)*log((a*x + b)*c/x)/a + ((c*log(a
*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(
a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

3.94.8 Giac [F]

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log\left(\frac{(ax+b)c}{x}\right)^2\,dx$$

input
```
integrate(log(c*(a*x+b)/x)^2,x, algorithm="giac")
```

output
```
integrate(log((a*x + b)*c/x)^2, x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx = \int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^2\,dx$$

input
```
int(log((c*(b + a*x))/x)^2,x)
```

output
```
int(log((c*(b + a*x))/x)^2, x)
```

3.95 $\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx$

3.95.1 Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a}-\frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{b}{ax}\right)}{a}+\frac{6b\operatorname{PolyLog}\left(3,1+\frac{b}{ax}\right)}{a}$$

output
```
(a*x+b)*ln(a*c+b*c/x)^3/a-3*b*ln(c*(a+b/x))^2*ln(-b/a/x)/a-6*b*ln(c*(a+b/x
))*polylog(2,1+b/a/x)/a+6*b*polylog(3,1+b/a/x)/a
```

3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx$$
$$=\frac{\log^2\left(\frac{c(b+ax)}{x}\right)\left(-3b\log\left(-\frac{b}{ax}\right)+(b+ax)\log\left(\frac{c(b+ax)}{x}\right)\right)-6b\log\left(\frac{c(b+ax)}{x}\right)\operatorname{PolyLog}\left(2,1+\frac{b}{ax}\right)+6b\,\mathrm{PolyL}}{a}$$

input
```
Integrate[Log[(c*(b + a*x))/x]^3,x]
```

output
```
(Log[(c*(b + a*x))/x]^2*(-3*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x)
)/x]) - 6*b*Log[(c*(b + a*x))/x]*PolyLog[2, 1 + b/(a*x)] + 6*b*PolyLog[3,
1 + b/(a*x)])/a
```

3.95.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2903, 2899, 2904, 2843, 27, 2881, 27, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3\left(\frac{c(ax+b)}{x}\right) dx$$

$$\Big\downarrow \text{2903}$$

$$\int \log^3\left(ac+\frac{bc}{x}\right) dx$$

$$\Big\downarrow \text{2899}$$

$$\frac{3b\int \frac{\log^2\left(ac+\frac{bc}{x}\right)}{x}dx}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}$$

$$\Big\downarrow \text{2904}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\int x\log^2\left(ac+\frac{bc}{x}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2843}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\left(\log\left(-\frac{b}{ax}\right)\log^2\left(ac+\frac{bc}{x}\right)-2bc\int \frac{\log\left(ac+\frac{bc}{x}\right)\log\left(-\frac{b}{ax}\right)}{c\left(a+\frac{b}{x}\right)}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\left(\log\left(-\frac{b}{ax}\right)\log^2\left(ac+\frac{bc}{x}\right)-2b\int \frac{\log\left(ac+\frac{bc}{x}\right)\log\left(-\frac{b}{ax}\right)}{a+\frac{b}{x}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{2881}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\left(\log\left(-\frac{b}{ax}\right)\log^2\left(ac+\frac{bc}{x}\right)-\frac{2\int cx\log\left(ac+\frac{bc}{x}\right)\log\left(-\frac{b}{ax}\right)d\left(ac+\frac{bc}{x}\right)}{c}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b(\log\left(-\frac{b}{ax}\right)\log^2\left(ac+\frac{bc}{x}\right)-2\int x\log\left(ac+\frac{bc}{x}\right)\log\left(-\frac{b}{ax}\right)d(ac+\frac{bc}{x}))}{a}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\left(\log\left(-\frac{b}{ax}\right)\log^2\left(ac+\frac{bc}{x}\right)-2\left(\int x\,\mathrm{PolyLog}\left(2,\frac{ac+\frac{bc}{x}}{ac}\right)d\left(ac+\frac{bc}{x}\right)-\mathrm{PolyLog}\left(2,\frac{ac+\frac{bc}{x}}{ac}\right)\log\left(ac+\frac{bc}{x}\right)\right)\right)}{a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\left(\log\left(-\frac{b}{ax}\right)\log^2\left(ac+\frac{bc}{x}\right)-2\left(\mathrm{PolyLog}\left(3,\frac{ac+\frac{bc}{x}}{ac}\right)-\mathrm{PolyLog}\left(2,\frac{ac+\frac{bc}{x}}{ac}\right)\log\left(ac+\frac{bc}{x}\right)\right)\right)}{a}$$

input
```
Int[Log[(c*(b + a*x))/x]^3,x]
```

output
```
((b + a*x)*Log[a*c + (b*c)/x]^3)/a - (3*b*(Log[a*c + (b*c)/x]^2*Log[-(b/(a
*x))] - 2*(-(Log[a*c + (b*c)/x]*PolyLog[2, (a*c + (b*c)/x)/(a*c)]) + PolyL
og[3, (a*c + (b*c)/x)/(a*c)])))/a
```

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2899
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Simp[b*e*p*(q/d)   I
nt[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}
, x] && IGtQ[q, 0]
```

rule 2903
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.95.4 Maple [F]

$$\int \ln\left(\frac{c(ax+b)}{x}\right)^{3} dx$$

input
```
int(ln(c*(a*x+b)/x)^3,x)
```

output
```
int(ln(c*(a*x+b)/x)^3,x)
```

3.95.5 Fricas [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx = \int \log\left(\frac{(ax+b)c}{x}\right)^3 dx$$

input
```
integrate(log(c*(a*x+b)/x)^3,x, algorithm="fricas")
```

output
```
integral(log((a*c*x + b*c)/x)^3, x)
```

3.95.6 Sympy [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx = 3b\int \frac{\log\left(ac+\frac{bc}{x}\right)^2}{ax+b}\, dx + x\log\left(\frac{c(ax+b)}{x}\right)^3$$

input
```
integrate(ln(c*(a*x+b)/x)**3,x)
```

output
```
3*b*Integral(log(a*c + b*c/x)**2/(a*x + b), x) + x*log(c*(a*x + b)/x)**3
```

3.95.7 Maxima [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx = \int \log\left(\frac{(ax+b)c}{x}\right)^3 dx$$

input
```
integrate(log(c*(a*x+b)/x)^3,x, algorithm="maxima")
```

output
```
((a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - a*x*log(x))*log(a*x + b)^2)/a
+ integrate((a*x*log(c)^3 + b*log(c)^3 - (a*x + b)*log(x)^3 + 3*(a*x*log(c
) + b*log(c))*log(x)^2 + 3*((log(c)^2 - 2*log(c))*a*x + b*log(c)^2 + (a*x
+ b)*log(x)^2 - 2*(a*x*(log(c) - 1) + b*log(c))*log(x))*log(a*x + b) - 3*(
a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

3.95.8 Giac [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx = \int \log\left(\frac{(ax+b)c}{x}\right)^3 dx$$

input
```
integrate(log(c*(a*x+b)/x)^3,x, algorithm="giac")
```

output
```
integrate(log((a*x + b)*c/x)^3, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx = \int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^3 dx$$

input
```
int(log((c*(b + a*x))/x)^3,x)
```

output
```
int(log((c*(b + a*x))/x)^3, x)
```

3.96 $\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx$

3.96.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = \frac{2b\log(b+ax)}{a} + x\log\left(\frac{c(b+ax)^2}{x^2}\right)$$

output
```
2*b*ln(a*x+b)/a+x*ln(c*(a*x+b)^2/x^2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = \frac{2b\log(b+ax)}{a} + x\log\left(\frac{c(b+ax)^2}{x^2}\right)$$

input
```
Integrate[Log[(c*(b + a*x)^2)/x^2],x]
```

output
```
(2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

3.96.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2936, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\frac{c(ax+b)^2}{x^2}\right)dx$$

$$\Big\downarrow \ 2936$$

$$2b\int \frac{1}{b+ax}dx + x\log\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \ 16$$

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right) + \frac{2b\log(ax+b)}{a}$$

input
```
Int[Log[(c*(b + a*x)^2)/x^2],x]
```

output
```
(2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

3.96.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

3.96.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{2b\ln(ax+b)}{a}+x\ln\left(\frac{c(ax+b)^2}{x^2}\right)$	29
parts	$\frac{2b\ln(ax+b)}{a}+x\ln\left(\frac{c(ax+b)^2}{x^2}\right)$	29
derivativedivides	$x\ln\left(c\left(a+\frac{b}{x}\right)^2\right)-2b\left(\frac{\ln\left(\frac{1}{x}\right)}{a}-\frac{\ln\left(a+\frac{b}{x}\right)}{a}\right)$	41
default	$x\ln\left(c\left(a+\frac{b}{x}\right)^2\right)-2b\left(\frac{\ln\left(\frac{1}{x}\right)}{a}-\frac{\ln\left(a+\frac{b}{x}\right)}{a}\right)$	41
parallelrisch	$-\frac{-2\ln\left(\frac{c(ax+b)^2}{x^2}\right)xa-4\ln(x)b-2b\ln\left(\frac{c(ax+b)^2}{x^2}\right)}{2a}$	45

input
```
int(ln(c*(a*x+b)^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
2*b*ln(a*x+b)/a+x*ln(c*(a*x+b)^2/x^2)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx=\frac{ax\log\left(\frac{a^2cx^2+2\,abcx+b^2c}{x^2}\right)+2\,b\log\left(ax+b\right)}{a}$$

input
```
integrate(log(c*(a*x+b)^2/x^2),x, algorithm="fricas")
```

output
```
(a*x*log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2) + 2*b*log(a*x + b))/a
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log\left(\frac{c(ax+b)^2}{x^2}\right) + \frac{2b\log(ax+b)}{a}$$

input
```
integrate(ln(c*(a*x+b)**2/x**2),x)
```

output
```
x*log(c*(a*x + b)**2/x**2) + 2*b*log(a*x + b)/a
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log\left(\frac{(ax+b)^2c}{x^2}\right) + \frac{2\,b\log(ax+b)}{a}$$

input
```
integrate(log(c*(a*x+b)^2/x^2),x, algorithm="maxima")
```

output
```
x*log((a*x + b)^2*c/x^2) + 2*b*log(a*x + b)/a
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log\left(\frac{(ax+b)^2c}{x^2}\right) + \frac{2\,b\log(|ax+b|)}{a}$$

input
```
integrate(log(c*(a*x+b)^2/x^2),x, algorithm="giac")
```

output
```
x*log((a*x + b)^2*c/x^2) + 2*b*log(abs(a*x + b))/a
```

3.96.9 Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\,\ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right) + \frac{2\,b\,\ln\left(b+a\,x\right)}{a}$$

input
```
int(log((c*(b + a*x)^2)/x^2),x)
```

output
```
x*log((c*(b + a*x)^2)/x^2) + (2*b*log(b + a*x))/a
```

3.97 $\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx$

3.97.1 Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\,\text{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}$$

output
```
-4*b*ln(b/(a*x+b))*ln(c*(a*x+b)^2/x^2)/a+x*ln(c*(a*x+b)^2/x^2)^2+8*b*polyl
og(2,1-b/(a*x+b))/a
```

3.97.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = -\frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{4b\log^2\left(\frac{b}{b+ax}\right)}{a} - \frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\,\text{PolyLog}\left(2,\frac{b+ax}{b}\right)}{a}$$

input
```
Integrate[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

output
```
(-8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)/a - (
4*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/x
^2]^2 + (8*b*PolyLog[2, (b + a*x)/b])/a
```

3.97.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2936, 2942, 2858, 25, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(\frac{c(ax+b)^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2936}$$

$$4b\int \frac{\log\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}dx + x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2942}$$

$$4b\left(-\frac{2b\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}dx}{a} - \frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right) + x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2858}$$

$$4b\left(-\frac{2b\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}d(b+ax)}{a^2} - \frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right) + x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$4b\left(\frac{2b\int -\frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}d(b+ax)}{a^2} - \frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right) + x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$4b\left(\frac{2b\int -\frac{\log\left(\frac{b}{b+ax}\right)}{ax(b+ax)}d(b+ax)}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2778}$$

$$4b\left(-\frac{2b\int -\frac{(b+ax)\log\left(\frac{b}{b+ax}\right)}{ax}d\frac{1}{b+ax}}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2005}$$

$$4b\left(-\frac{2b\int \frac{\log\left(\frac{b}{b+ax}\right)}{\frac{b}{b+ax}-1}d\frac{1}{b+ax}}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$4b\left(\frac{2\,\text{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)$$

input
```
Int[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

output
```
x*Log[(c*(b + a*x)^2)/x^2]^2 + 4*b*(-((Log[b/(b + a*x)]*Log[(c*(b + a*x)^2
)/x^2])/a) + (2*PolyLog[2, 1 - b/(b + a*x)])/a)
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.97.4 Maple [F]

$$\int \ln\left(\frac{c(ax+b)^2}{x^2}\right)^2 dx$$

input
```
int(ln(c*(a*x+b)^2/x^2)^2,x)
```

output
```
int(ln(c*(a*x+b)^2/x^2)^2,x)
```

3.97.5 Fricas [F]

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx=\int \log\left(\frac{(ax+b)^2c}{x^2}\right)^2\,dx$$

input
```
integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="fricas")
```

output
```
integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^2, x)
```

3.97.6 Sympy [F]

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx=4b\int \frac{\log\left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)}{ax+b}\,dx+x\log\left(\frac{c(ax+b)^2}{x^2}\right)^2$$

input
```
integrate(ln(c*(a*x+b)**2/x**2)**2,x)
```

output
```
4*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)/(a*x + b), x) + x*log(c
*(a*x + b)**2/x**2)**2
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\begin{aligned}&\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx\\&=x\log\left(\frac{(ax+b)^2c}{x^2}\right)^2+\frac{4\,b\log\left(ax+b\right)\log\left(\frac{(ax+b)^2c}{x^2}\right)}{a}\\&\quad+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\left(c\log(ax+b)-c\log(x)\right)b\log(ax+b)}{a}\right)}{c}\end{aligned}$$

input
```
integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="maxima")
```

output
```
x*log((a*x + b)^2*c/x^2)^2 + 4*b*log(a*x + b)*log((a*x + b)^2*c/x^2)/a + 4
*((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b -
 2*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

3.97.8 Giac [F]

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^2 dx$$

input
```
integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="giac")
```

output
```
integrate(log((a*x + b)^2*c/x^2)^2, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = \int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^2 dx$$

input
```
int(log((c*(b + a*x)^2)/x^2)^2,x)
```

output
```
int(log((c*(b + a*x)^2)/x^2)^2, x)
```

3.98 $\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx$

3.98.1 Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{6b\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a} + \frac{48b\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

output
```
x*ln(c*(a*x+b)^2/x^2)^3-6*b*ln(c*(a*x+b)^2/x^2)^2*ln(1-a*x/(a*x+b))/a+24*b
*ln(c*(a*x+b)^2/x^2)*polylog(2,a*x/(a*x+b))/a+48*b*polylog(3,a*x/(a*x+b))/
a
```

3.98.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a} + \frac{48b\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

input
```
Integrate[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

output
```
(-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^
2)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)])/a
+ (48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

3.98.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2936, 2950, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3\left(\frac{c(ax+b)^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2936}$$

$$6b\int \frac{\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}dx+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2950}$$

$$6b\int \frac{x\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{(b+ax)\left(a-\frac{b+ax}{x}\right)}d\frac{b+ax}{x}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2779}$$

$$6b\left(\frac{4\int \frac{x\log\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{b+ax}d\frac{b+ax}{x}}{a}-\frac{\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{2821}$$

$$6b\left(\frac{4\left(\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)-2\int \frac{x\,\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{b+ax}d\frac{b+ax}{x}\right)}{a}-\frac{\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+$$
$$x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)$$

$$\Big\downarrow \text{7143}$$

$$6b\left(\frac{4\left(\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)+2\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)\right)}{a}-\frac{\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}\right)+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)$$

input
```
Int[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

output
```
x*Log[(c*(b + a*x)^2)/x^2]^3 + 6*b*(-((Log[(c*(b + a*x)^2)/x^2]^2*Log[1 -
(a*x)/(b + a*x)])/a) + (4*(Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b +
a*x)] + 2*PolyLog[3, (a*x)/(b + a*x)]))/a)
```

3.98.3.1 Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.98.4 Maple [F]

$$\int \ln\left(\frac{c(ax+b)^2}{x^2}\right)^3 dx$$

input
```
int(ln(c*(a*x+b)^2/x^2)^3,x)
```

output
```
int(ln(c*(a*x+b)^2/x^2)^3,x)
```

3.98.5 Fricas [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right) dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3 dx$$

input
```
integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="fricas")
```

output
```
integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^3, x)
```

3.98.6 Sympy [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = 6b\int \frac{\log\left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)^2}{ax+b}\,dx + x\log\left(\frac{c(ax+b)^2}{x^2}\right)^3$$

input
```
integrate(ln(c*(a*x+b)**2/x**2)**3,x)
```

output
```
6*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)**2/(a*x + b), x) + x*lo
g(c*(a*x + b)**2/x**2)**3
```

3.98.7 Maxima [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3\,dx$$

input
```
integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="maxima")
```

output
```
4*(2*(a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - 2*a*x*log(x))*log(a*x + b)
^2)/a + integrate((a*x*log(c)^3 + b*log(c)^3 - 8*(a*x + b)*log(x)^3 + 12*(
a*x*log(c) + b*log(c))*log(x)^2 + 6*((log(c)^2 - 4*log(c))*a*x + b*log(c)^
2 + 4*(a*x + b)*log(x)^2 - 4*(a*x*(log(c) - 2) + b*log(c))*log(x))*log(a*x
 + b) - 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

3.98.8 Giac [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3\,dx$$

input
```
integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(log((a*x + b)^2*c/x^2)^3, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^3\,dx$$

input
```
int(log((c*(b + a*x)^2)/x^2)^3,x)
```

output
```
int(log((c*(b + a*x)^2)/x^2)^3, x)
```

3.99 $\int \log \left(\frac{cx^2}{(b+ax)^2}\right) \, dx$

3.99.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) \, dx = x \log \left(\frac{cx^2}{(b+ax)^2}\right) - \frac{2b\log(b+ax)}{a}$$

output
```
x*ln(c*x^2/(a*x+b)^2)-2*b*ln(a*x+b)/a
```

3.99.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) \, dx = x \log \left(\frac{cx^2}{(b+ax)^2}\right) - \frac{2b\log(b+ax)}{a}$$

input
```
Integrate[Log[(c*x^2)/(b + a*x)^2],x]
```

output
```
x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

3.99.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2936, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\frac{cx^2}{(ax+b)^2}\right) dx$$

$$\Big\downarrow \; 2936$$

$$x \log \left(\frac{cx^2}{(ax+b)^2}\right) - 2b \int \frac{1}{b+ax} dx$$

$$\Big\downarrow \; 16$$

$$x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log(ax+b)}{a}$$

input
```
Int[Log[(c*x^2)/(b + a*x)^2],x]
```

output
```
x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

3.99.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
risch	$x\ln\left(\frac{c\,x^{2}}{(ax+b)^{2}}\right)-\frac{2b\ln(ax+b)}{a}$	29
parts	$x\ln\left(\frac{c\,x^{2}}{(ax+b)^{2}}\right)-\frac{2b\ln(ax+b)}{a}$	29
parallelrisch	$-\frac{-2\ln\left(\frac{c\,x^{2}}{(ax+b)^{2}}\right)xab+4\ln(x)b^{2}-2b^{2}\ln\left(\frac{c\,x^{2}}{(ax+b)^{2}}\right)}{2ab}$	53
derivativedivides	$-\frac{-(ax+b)\ln\left(\frac{c\left(\frac{b}{ax+b}-1\right)^{2}}{a^{2}}\right)+2b\left(-\ln\left(\frac{1}{ax+b}\right)+\ln\left(\frac{b}{ax+b}-1\right)\right)}{a}$	59
default	$-\frac{-(ax+b)\ln\left(\frac{c\left(\frac{b}{ax+b}-1\right)^{2}}{a^{2}}\right)+2b\left(-\ln\left(\frac{1}{ax+b}\right)+\ln\left(\frac{b}{ax+b}-1\right)\right)}{a}$	59

input
```
int(ln(c*x^2/(a*x+b)^2),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*x^2/(a*x+b)^2)-2*b*ln(a*x+b)/a
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \log\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\frac{ax\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)-2\,b\log(ax+b)}{a}$$

input
```
integrate(log(c*x^2/(a*x+b)^2),x, algorithm="fricas")
```

output
```
(a*x*log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2)) - 2*b*log(a*x + b))/a
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log (ax+b)}{a}$$

input
```
integrate(ln(c*x**2/(a*x+b)**2),x)
```

output
```
x*log(c*x**2/(a*x + b)**2) - 2*b*log(a*x + b)/a
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2\,b \log (ax+b)}{a}$$

input
```
integrate(log(c*x^2/(a*x+b)^2),x, algorithm="maxima")
```

output
```
x*log(c*x^2/(a*x + b)^2) - 2*b*log(a*x + b)/a
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2\,b \log (|ax+b|)}{a}$$

input
```
integrate(log(c*x^2/(a*x+b)^2),x, algorithm="giac")
```

output
```
x*log(c*x^2/(a*x + b)^2) - 2*b*log(abs(a*x + b))/a
```

3.99.9 Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{cx^2}{(b+ax)^2} \right) \, dx = x \ln \left(\frac{c\,x^2}{(b+a\,x)^2} \right) - \frac{2\,b\,\ln (b+a\,x)}{a}$$

input
```
int(log((c*x^2)/(b + a*x)^2),x)
```

output
```
x*log((c*x^2)/(b + a*x)^2) - (2*b*log(b + a*x))/a
```

3.100 $\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx$

3.100.1 Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{8b\,\text{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}$$

output
```
x*ln(c*x^2/(a*x+b)^2)^2+4*b*ln(c*x^2/(a*x+b)^2)*ln(b/(a*x+b))/a+8*b*polylo
g(2,1-b/(a*x+b))/a
```

3.100.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \frac{ax\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)-4b\log\left(\frac{b}{b+ax}\right)\left(2\log\left(-\frac{ax}{b}\right)+\log\left(\frac{b}{b+ax}\right)\right)+8b\,\text{PolyLog}\,(2}{a}$$

input
```
Integrate[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

output
```
(a*x*Log[(c*x^2)/(b + a*x)^2]^2 + 4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b +
a*x)] - 4*b*Log[b/(b + a*x)]*(2*Log[-((a*x)/b)] + Log[b/(b + a*x)]) + 8*b*
PolyLog[2, 1 + (a*x)/b])/a
```

3.100.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2936, 2942, 2858, 25, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(\frac{cx^2}{(ax+b)^2}\right)dx$$

$$\Big\downarrow \; 2936$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}dx$$

$$\Big\downarrow \; 2942$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(\frac{2b\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}dx}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \; 2858$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(\frac{2b\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}d(b+ax)}{a^2}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \; 25$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(-\frac{2b\int -\frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}d(b+ax)}{a^2}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \; 27$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(-\frac{2b\int -\frac{\log\left(\frac{b}{b+ax}\right)}{ax(b+ax)}d(b+ax)}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \text{2778}$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(\frac{2b\int -\frac{(b+ax)\log\left(\frac{b}{b+ax}\right)}{ax}d\frac{1}{b+ax}}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \text{2005}$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(\frac{2b\int \frac{\log\left(\frac{b}{b+ax}\right)}{\frac{b}{b+ax}-1}d\frac{1}{b+ax}}{a}-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \text{2752}$$

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)-4b\left(-\frac{\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{2\,\text{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}\right)$$

input
```
Int[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

output
```
x*Log[(c*x^2)/(b + a*x)^2]^2 - 4*b*(-((Log[(c*x^2)/(b + a*x)^2]*Log[b/(b +
 a*x)])/a) - (2*PolyLog[2, 1 - b/(b + a*x)])/a)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.100.4 Maple [F]

$$\int \ln \left(\frac{c\,x^{2}}{\left(a x +b \right)^{2}}\right)^{2}d x$$

input
```
int(ln(c*x^2/(a*x+b)^2)^2,x)
```

output
```
int(ln(c*x^2/(a*x+b)^2)^2,x)
```

3.100.5 Fricas [F]

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^2\,dx$$

input
```
integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="fricas")
```

output
```
integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^2, x)
```

3.100.6 Sympy [F]

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=-4b\int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)}{ax+b}\,dx+x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2$$

input
```
integrate(ln(c*x**2/(a*x+b)**2)**2,x)
```

output
```
-4*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))/(a*x + b), x) + x*l
og(c*x**2/(a*x + b)**2)**2
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx$$
$$=x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2-\frac{4\,b\log(ax+b)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\,(c\log(ax+b)-c\log(x))b\log(ax+b)}{a}\right)}{c}$$

input
```
integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="maxima")
```

output
```
x*log(c*x^2/(a*x + b)^2)^2 - 4*b*log(a*x + b)*log(c*x^2/(a*x + b)^2)/a + 4
*((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b -
 2*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

3.100.8 Giac [F]

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^2\,dx$$

input
```
integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="giac")
```

output
```
integrate(log(c*x^2/(a*x + b)^2)^2, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)^2\,dx$$

input
```
int(log((c*x^2)/(b + a*x)^2)^2,x)
```

output
```
int(log((c*x^2)/(b + a*x)^2)^2, x)
```

3.101 $\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx$

3.101.1 Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\operatorname{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a}-\frac{48b\operatorname{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

output
```
x*ln(c*x^2/(a*x+b)^2)^3+6*b*ln(c*x^2/(a*x+b)^2)^2*ln(b/(a*x+b))/a+24*b*ln(
c*x^2/(a*x+b)^2)*polylog(2,a*x/(a*x+b))/a-48*b*polylog(3,a*x/(a*x+b))/a
```

3.101.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\operatorname{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a}-\frac{48b\operatorname{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

input
```
Integrate[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

output
```
x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b +
a*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a -
 (48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

3.101.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2936, 2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3\left(\frac{cx^2}{(ax+b)^2}\right)dx$$

$$\Big\downarrow \text{2936}$$

$$x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)-6b\int \frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}dx$$

$$\Big\downarrow \text{2952}$$

$$x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)-6b\int \frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{1-\frac{ax}{b+ax}}d\frac{x}{b+ax}$$

$$\Big\downarrow \text{2754}$$

$$x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)-6b\left(\frac{4\int \frac{(b+ax)\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{x}d\frac{x}{b+ax}}{a}-\frac{\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \text{2821}$$

$$x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)-6b\left(\frac{4\left(2\int \frac{(b+ax)\,\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{x}d\frac{x}{b+ax}-\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)\right)}{a}-\frac{\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

$$\Big\downarrow \text{7143}$$

$$x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)- 6b\left(\frac{4\left(2\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)-\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)\right)}{a}-\frac{\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\right)$$

input
```
Int[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

output
```
x*Log[(c*x^2)/(b + a*x)^2]^3 - 6*b*(-((Log[(c*x^2)/(b + a*x)^2]^2*Log[1 -
(a*x)/(b + a*x)])/a) + (4*(-(Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b
+ a*x)]) + 2*PolyLog[3, (a*x)/(b + a*x)]))/a)
```

3.101.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.101.4 Maple [F]

$$\int \ln\left(\frac{c\,x^2}{\left(ax+b\right)^2}\right)^3 dx$$

input
```
int(ln(c*x^2/(a*x+b)^2)^3,x)
```

output
```
int(ln(c*x^2/(a*x+b)^2)^3,x)
```

3.101.5 Fricas [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

input
```
integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="fricas")
```

output
```
integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^3, x)
```

3.101.6 Sympy [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = -6b\int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)^2}{ax+b}\,dx + x\log\left(\frac{cx^2}{(ax+b)^2}\right)^3$$

input
```
integrate(ln(c*x**2/(a*x+b)**2)**3,x)
```

output
```
-6*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))**2/(a*x + b), x) +
x*log(c*x**2/(a*x + b)**2)**3
```

3.101.7 Maxima [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

input
```
integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="maxima")
```

output
```
-4*(2*(a*x + b)*log(a*x + b)^3 - 3*(a*x*log(c) + 2*a*x*log(x))*log(a*x + b
)^2)/a - integrate(-(a*x*log(c)^3 + b*log(c)^3 + 8*(a*x + b)*log(x)^3 + 12
*(a*x*log(c) + b*log(c))*log(x)^2 - 6*((log(c)^2 + 4*log(c))*a*x + b*log(c
)^2 + 4*(a*x + b)*log(x)^2 + 4*(a*x*(log(c) + 2) + b*log(c))*log(x))*log(a
*x + b) + 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

3.101.8 Giac [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

input
```
integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="giac")
```

output
```
integrate(log(c*x^2/(a*x + b)^2)^3, x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)^3\,dx$$

input
```
int(log((c*x^2)/(b + a*x)^2)^3,x)
```

output
```
int(log((c*x^2)/(b + a*x)^2)^3, x)
```

3.102 $\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

3.102.1 Optimal result

Integrand size = 38, antiderivative size = 35

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=-\frac{\text{PolyLog}\left(3,1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

output
```
-polylog(3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

3.102.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{-bc+ad}$$

input
```
Integrate[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),
x]
```

output
```
PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]/(-(b*c) + a*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{ 7164}$$

$$-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

input
```
Int[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x]
```

output
```
-(PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))
```

3.102.3.1 Defintions of rubi rules used

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.102.4 Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\text{Li}_3\left(1-\frac{ad-cb}{d(bx+a)}\right)}{ad-cb}$	36
default	$\frac{\text{Li}_3\left(1-\frac{ad-cb}{d(bx+a)}\right)}{ad-cb}$	36

input
```
int(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERB
OSE)
```

output
```
1/(a*d-b*c)*polylog(3,1-(a*d-b*c)/d/(b*x+a))
```

3.102.5 Fricas [F]

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="
fricas")
```

output
```
integral(dilog((b*c - a*d)/(b*d*x + a*d) + 1)/(b*d*x^2 + a*c + (b*c + a*d)
*x), x)
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.102.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="
maxima")
```

output
```
integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

3.102.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="
giac")
```

output
```
integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{polylog}\left(2,1-\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)),x)
```

output
```
int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)), x)
```

3.103 $\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

3.103.1 Optimal result

Integrand size = 50, antiderivative size = 85

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}-\frac{\operatorname{PolyLog}\left(3,1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

output
```
ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)-polylog
(3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

3.103.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)-\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

input
```
Integrate[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])
/((a + b*x)*(c + d*x)),x]
```

output
```
(Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - Po
lyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b*c - a*d)
```

3.103.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2988, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2988}$$

$$\int \frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{(a+bx)(c+dx)}dx+\frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

input
```
Int[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/((a +
 b*x)*(c + d*x)),x]
```

output
```
(Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(
b*c - a*d) - PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d)
```

3.103.3.1 Defintions of rubi rules used

rule 2988
```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Simp[h*p
*r*s   Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/
((a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e
, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.103.4 Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

method	result	size
default	$\frac{\frac{\ln\left(-\frac{\frac{e(dx+c)b}{bx+a}-de}{de}\right)\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{2}-\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2\ln\left(1-\frac{b(dx+c)}{d(bx+a)}\right)}{2}-\ln\left(\frac{e(dx+c)}{bx+a}\right)\mathrm{Li}_2\left(\frac{b(dx+c)}{d(bx+a)}\right)+\mathrm{Li}_3\left(\frac{b(dx+c)}{d(bx+a)}\right)}{ad-cb}$	156

input
```
int(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x,method
=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)*(1/2*ln(-(e*(d*x+c)/(b*x+a)*b-d*e)/d/e)*ln(e*(d*x+c)/(b*x+a))^
2-1/2*ln(e*(d*x+c)/(b*x+a))^2*ln(1-b*(d*x+c)/d/(b*x+a))-ln(e*(d*x+c)/(b*x+
a))*polylog(2,b*(d*x+c)/d/(b*x+a))+polylog(3,b*(d*x+c)/d/(b*x+a)))
```

3.103.5 Fricas [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),
x, algorithm="fricas")
```

output
```
integral(log(-(b*c - a*d)/(b*d*x + a*d))*log((d*e*x + c*e)/(b*x + a))/(b*d
*x^2 + a*c + (b*c + a*d)*x), x)
```

3.103.6 Sympy [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{b\int \frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx}{2(ad-bc)} + \frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{2ad-2bc}$$

input
```
integrate(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
b*Integral(log(c*e/(a + b*x) + d*e*x/(a + b*x))**2/(a + b*x), x)/(2*(a*d -
 b*c)) + log((a*d - b*c)/(d*(a + b*x)))*log(e*(c + d*x)/(a + b*x))**2/(2*a
*d - 2*b*c)
```

3.103.7 Maxima [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),
x, algorithm="maxima")
```

output
```
integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
 + a)*(d*x + c)), x)
```

3.103.8 Giac [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),
x, algorithm="giac")
```

output
```
integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
 + a)*(d*x + c)), x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*
x)*(c + d*x)),x)
```

output
```
int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*
x)*(c + d*x)), x)
```

3.104 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx$

3.104.1 Optimal result

Integrand size = 42, antiderivative size = 140

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = -\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/b-2*l
n((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+
2*polylog(3,b*(d*x+c)/d/(b*x+a))/b
```

3.104.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \frac{-\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) - 2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right) + 2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

input
```
Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x)
,x]
```

output
```
(-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c
*f)*(a + b*x))]^2) - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*
(a + b*x))])/b
```

3.104.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{a+bx}dx$$

$$\Big\downarrow \text{2952}$$

$$\int \frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2754}$$

$$\frac{2\int \frac{(a+bx)\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

$$\Big\downarrow \text{2821}$$

$$\frac{2\left(\int \frac{(a+bx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}-\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2\left(\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

input
```
Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),x]
```

output
```
-((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - (b*(c +
d*x))/(d*(a + b*x))])/b) + (2*(-(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*
(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]) + PolyLog[3, (b*(c +
d*x))/(d*(a + b*x))]))/b
```

3.104.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(140) = 280$.

Time = 3.77 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.99

method	result
derivativedivides	$\frac{(cf-de)\left(\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)}{-adf+bde}\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}\right.\right.}{-bcf}$
default	$\frac{(cf-de)\left(\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)}{-adf+bde}\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}\right.\right.}{-bcf}$
risch	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)}{-adf+bde}\right)cf}{-bcf+bde}-\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af}{(cf-}\right.$

input
```
int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x,method=_RETURNVE
RBOSE)
```

output
```
(c*f-d*e)/(-b*c*f+b*d*e)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c
*f-d*e)*(a*f-b*e)/b)^2*ln(1-(-b*c*f+b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d
-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))+2*ln(-(a*f-b*e)*(a*d-b
*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,(-b*c*f+b*d*e)/
(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-
b*e)/b))-2*polylog(3,(-b*c*f+b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b
/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)))
```

3.104.5 Fricas [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorith
m="fricas")
```

output
```
integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
 b*c*f)*x))^2/(b*x + a), x)
```

3.104.6 Sympy [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx$$
$$= \int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{a+bx}\,dx$$

input
```
integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a),x)
```

output
```
Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
 + a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x)
 + b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(a + b*x), x)
```

3.104.7 Maxima [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorith
m="maxima")
```

output
```
log(d*x + c)^3/a - integrate(-((log(-b*e + a*f)^2 - 2*log(-b*e + a*f)*log(
-d*e + c*f) + log(-d*e + c*f)^2)*b*d*x + (log(-b*e + a*f)^2 - 2*log(-b*e +
 a*f)*log(-d*e + c*f) + log(-d*e + c*f)^2)*b*c + (b*d*x + b*c)*log(b*x + a
)^2 - 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f)) + b*c*(log(-b*e + a*f)
- log(-d*e + c*f)))*log(b*x + a) + 2*(b*d*x*(log(-b*e + a*f) - log(-d*e +
c*f)) + b*c*(log(-b*e + a*f) - log(-d*e + c*f)) - (2*b*d*x + b*c + a*d)*lo
g(b*x + a))*log(d*x + c))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

3.104.8 Giac [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorith
m="giac")
```

output
```
integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/(b*x + a),
x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{a+b\,x}\,dx$$

input
```
int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x),x)
```

output
```
int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x), x)
```

3.105 $\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

3.105.1 Optimal result

Integrand size = 62, antiderivative size = 109

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{PolyLog}\left(2,1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} - \frac{\operatorname{PolyLog}\left(3,1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

output
```
ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(
-a*d+b*c)-polylog(3,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

3.105.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)-\operatorname{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

input
```
Integrate[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d
*e - c*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

output
```
(Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c
*f)*(a + b*x))] - PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))])/(b*c - a*d)
```

3.105.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2988, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{e(c+dx)}{a+bx}\right) \log \left(\frac{(e+fx)(ad-bc)}{(a+bx)(de-cf)}\right)}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \text{ 2988}$$

$$\int \frac{\text{PolyLog}\left(2,\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{(a+bx)(c+dx)}dx+\frac{\log \left(\frac{e(c+dx)}{a+bx}\right) \text{PolyLog}\left(2,\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\log \left(\frac{e(c+dx)}{a+bx}\right) \text{PolyLog}\left(2,\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad}$$

input
```
Int[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c
*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

output
```
(Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/((d*e
 - c*f)*(a + b*x))])/(b*c - a*d) - PolyLog[3, 1 + ((b*c - a*d)*(e + f*x))/
((d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

3.105.3.1 Defintions of rubi rules used

rule 2988
```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Simp[h*p
*r*s   Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/
((a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e
, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(109) = 218$.

Time = 25.00 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.88

method	result
default	$\frac{\ln\left(-\frac{\frac{e(dx+c)af}{bx+a}-\frac{e^2(dx+c)b}{bx+a}-cef+d\,e^2}{e(cf-de)}\right)\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{2}-\frac{af\left(\ln\left(\frac{e(dx+c)}{bx+a}\right)^2\ln\left(1-\frac{(af-be)e(dx+c)}{(bx+a)\left(cef-d\,e^2\right)}\right)+2\ln\left(\frac{e(dx+c)}{bx+a}\right)\mathrm{Li}_2\left(\frac{(af-be)e(dx}{(bx+a)\left(cef-\right.}\right.\right.}{2(af-be)}$

input
```
int(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)
/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)*(1/2*ln(-(e*(d*x+c)/(b*x+a)*a*f-e^2*(d*x+c)/(b*x+a)*b-c*e*f+d*
e^2)/e/(c*f-d*e))*ln(e*(d*x+c)/(b*x+a))^2-1/2*a*f/(a*f-b*e)*(ln(e*(d*x+c)/
(b*x+a))^2*ln(1-(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))+2*ln(e*(d*x+c)/
(b*x+a))*polylog(2,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))-2*polylog(3,
(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2)))+1/2*b*e/(a*f-b*e)*(ln(e*(d*x+c
)/(b*x+a))^2*ln(1-(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))+2*ln(e*(d*x+c
)/(b*x+a))*polylog(2,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))-2*polylog(
3,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2)))
```

3.105.5 Fricas [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))
/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))*log((d*e*x + c*e)/(b*x + a))/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

3.105.6 Sympy [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)^2\log\left(\frac{(e+fx)(ad-bc)}{(a+bx)(-cf+de)}\right)}{2ad-2bc} - \frac{(af-be)\int \frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{ae+afx+bex+bfx^2}\,dx}{2\,(ad-bc)}$$

input
```
integrate(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(
b*x+a)/(d*x+c),x)
```

output
```
log(e*(c + d*x)/(a + b*x))**2*log((e + f*x)*(a*d - b*c)/((a + b*x)*(-c*f +
 d*e)))/(2*a*d - 2*b*c) - (a*f - b*e)*Integral(log(c*e/(a + b*x) + d*e*x/(
a + b*x))**2/(a*e + a*f*x + b*e*x + b*f*x**2), x)/(2*(a*d - b*c))
```

3.105.7 Maxima [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))
/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(log(b*x + a)^2 - 2*(log(b*x + a) - log(e))*log(d*x + c) + log(d*x +
c)^2 - 2*log(b*x + a)*log(e))*log(f*x + e)/(b*c - a*d) + integrate(1/2*(2*
(e*log(-b*c + a*d)*log(e) - e*log(d*e - c*f)*log(e))*b*c + (b*d*f*x^2 + 2*
b*c*e - (2*d*e - c*f)*a + (3*b*c*f - a*d*f)*x)*log(b*x + a)^2 - 2*(d*e*log
(-b*c + a*d)*log(e) - d*e*log(d*e - c*f)*log(e))*a + 2*((f*log(-b*c + a*d)
*log(e) - f*log(d*e - c*f)*log(e))*b*c - (d*f*log(-b*c + a*d)*log(e) - d*f
*log(d*e - c*f)*log(e))*a)*x - 2*(b*d*f*x^2*log(e) - (e*(log(d*e - c*f) -
log(e)) - e*log(-b*c + a*d))*b*c + (d*e*(log(d*e - c*f) - log(e)) - d*e*lo
g(-b*c + a*d) + c*f*log(e))*a + ((f*log(-b*c + a*d) - f*log(d*e - c*f) + 2
*f*log(e))*b*c - (d*f*log(-b*c + a*d) - d*f*log(d*e - c*f))*a)*x)*log(b*x
+ a) + 2*(b*d*f*x^2*log(e) + (e*log(-b*c + a*d) - e*log(d*e - c*f))*b*c -
(d*e*log(-b*c + a*d) - d*e*log(d*e - c*f) - c*f*log(e))*a + ((f*log(-b*c +
 a*d) - f*log(d*e - c*f) + f*log(e))*b*c - (d*f*log(-b*c + a*d) - (f*log(d
*e - c*f) + f*log(e))*d)*a)*x - (b*d*f*x^2 + 2*b*c*f*x + b*c*e - (d*e - c*
f)*a)*log(b*x + a))*log(d*x + c))/(a*b*c^2*e - a^2*c*d*e + (b^2*c*d*f - a*
b*d^2*f)*x^3 - (a*b*d^2*e + a^2*d^2*f - (c*d*e + c^2*f)*b^2)*x^2 + (b^2*c^
2*e + a*b*c^2*f - (d^2*e + c*d*f)*a^2)*x), x)
```

3.105.8 Giac [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))
/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)*(f*x + e)/((d*e - c*
f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(-\frac{(e+f\,x)\,(a\,d-b\,c)}{(c\,f-d\,e)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e
)*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

output
```
int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e
)*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

3.106 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx$

3.106.1 Optimal result

Integrand size = 49, antiderivative size = 204

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2\,\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}$$

output
```
-ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f
+d*e)/(b*x+a))/(-a*f+b*e)-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*poly
log(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)+2*polylog(3,(-a*f+
b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)
```

3.106.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1636 vs. $2(204) = 408$.

Time = 0.63 (sec) , antiderivative size = 1636, normalized size of antiderivative = 8.02

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \text{Too large to display}$$

input

```
Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x
)*(e + f*x)),x]
```

output

```
(-2*Log[a/b + x]^3 + 3*Log[a/b + x]^2*Log[a + b*x] - 6*Log[a/b + x]*Log[c/
d + x]*Log[a + b*x] + 3*Log[c/d + x]^2*Log[a + b*x] + 6*Log[a/b + x]*Log[c
/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 3*Log[c/d + x]^2*Log[(d*(a + b
*x))/(-(b*c) + a*d)] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3
*Log[a/b + x]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 6*L
og[a/b + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))] - 6*Log[c/d + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)
*(a + b*x))] + 6*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[((b*e
- a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 3*Log[(-(b*c) + a*d)/(d*(a +
b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + 3*Log[a +
b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 3*Log[a/b +
x]^2*Log[e + f*x] + 6*Log[a/b + x]*Log[c/d + x]*Log[e + f*x] - 3*Log[c/d +
 x]^2*Log[e + f*x] - 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]*Log[e + f*x] + 6*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Log[e + f*x] - 3*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]^2*Log[e + f*x] + 3*Log[a/b + x]^2*Log[(b*(e + f*x))/(b
*e - a*f)] - 6*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e +
f*x))/(b*e - a*f)] + 3*Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x
))/(b*e - a*f)] + 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*
(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] - 6*Log[(f*(c + d*x))/(-(d*e...
```

3.106.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2966, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{(a+bx)(e+fx)}dx$$

$$\Big\downarrow \text{2966}$$

$$\int \frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{-\frac{(c+dx)(be-af)}{a+bx}-cf+de}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2754}$$

$$\frac{2\int \frac{(a+bx)\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{be-af}-\frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

$$\Big\downarrow \text{2821}$$

$$\frac{2\left(\int \frac{(a+bx)\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}-\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\right)}{be-af}-\frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2\left(\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)-\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\right)}{be-af}-\frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

input
```
Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)*(e +
 f*x)),x]
```

output
```
-((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e -
a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*e - a*f)) + (2*(-(Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]) + PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]))/(b*e - a*f)
```

3.106.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2966
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c
*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c
+ d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && EqQ[n + mn
, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0] &&
 EqQ[d*h - c*i, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.106.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2 \ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)\mathrm{Li}_2\left(-\frac{(af-be)}{b(cf-de}\right.}{af-be}$
default	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2 \ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)\mathrm{Li}_2\left(-\frac{(af-be)}{b(cf-de}\right.}{af-be}$
risch	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2 \ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)}{af-be}+\frac{2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)\mathrm{Li}_2\left(-\frac{(af-}{b(cf}\right.}{af-be}$

input
```
int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x,method=_
RETURNVERBOSE)
```

output
```
1/(a*f-b*e)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-
b*e)/b)^2*ln((a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)
/b+1)+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/
b)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e
)/b)-2*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f
-b*e)/b))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.29

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = -\frac{\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right)^2 \log\left(-\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+(bde-bcf)x}\right)+2\,\mathrm{Li}_2\left(\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+(bde-bcf)x}+1\right)\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right.}{be-af}$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x,
algorithm="fricas")
```

output
```
-(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e - b*c*f)
*x))^2*log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e - b*
c*f)*x)) + 2*dilog(((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d
*e - b*c*f)*x) + 1)*log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f
 + (b*d*e - b*c*f)*x)) - 2*polylog(3, (b*c*e - a*c*f + (b*d*e - a*d*f)*x)/
(a*d*e - a*c*f + (b*d*e - b*c*f)*x)))/(b*e - a*f)
```

3.106.6 Sympy [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx$$
$$=\int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex}-\frac{adfx}{-acf+ade-bcfx+bdex}+\frac{bce}{-acf+ade-bcfx+bdex}+\frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{(a+bx)\,(e+fx)}\,dx$$

input
```
integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a)/(f*x+e),x)
```

output
```
Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
 + a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x)
 + b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/((a + b*x)*(e + f*x)),
 x)
```

3.106.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x,
algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: Memory limit reached. Please j
ump to an outer pointer, quit program and enlarge thememory limits before
executing the program again.
```

3.106.8 Giac [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{(bx+a)(fx+e)}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x,
algorithm="giac")
```

output
```
integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/((b*x + a)*
(f*x + e)), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{(e+f\,x)\,(a+b\,x)}\,dx$$

input
```
int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
 b*x)),x)
```

output
```
int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
 b*x)), x)
```

3.107 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$

3.107.1 Optimal result

Integrand size = 42, antiderivative size = 322

$$\begin{aligned}\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = & -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} \\ & +\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} \\ & -\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{f} \\ & +\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} \\ & +\frac{2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{f}-\frac{2\,\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f}\end{aligned}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/f+ln(
(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+d*e
)/(b*x+a))/f-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+
c)/d/(b*x+a))/f+2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*
f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f+2*polylog(3,b*(d*x+c)/d/(b*x+a))/f-2*
polylog(3,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f
```

3.107.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1080 vs. $2(322) = 644$.

Time = 0.40 (sec) , antiderivative size = 1080, normalized size of antiderivative = 3.35

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$$
$$= \frac{-\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)+\log^2\left(\frac{a}{b}+x\right)\log(e+fx)-2\log\left(\frac{a}{b}+x\right)\log\left(\frac{c}{d}+x\right)\log(e+fx)+1}{}$$

input
```
Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x)
,x]
```

output
```
(-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c
*f)*(a + b*x))]^2) + Log[a/b + x]^2*Log[e + f*x] - 2*Log[a/b + x]*Log[c/d
+ x]*Log[e + f*x] + Log[c/d + x]^2*Log[e + f*x] + 2*Log[a/b + x]*Log[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] - 2*Log[c/d + x]*
Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] + Log[((
b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[e + f*x] - Log[a/b +
x]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(
d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*x))/(-(d*e) +
c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[a/b + x]*Log[((b*e - a*f)*(
c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[
(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c + d*x))/((d*
e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log
[c/d + x]*Log[(d*(e + f*x))/(d*e - c*f)] - Log[c/d + x]^2*Log[(d*(e + f*x)
)/(d*e - c*f)] - 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(d*(
e + f*x))/(d*e - c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*(e + f
*x))/(d*e - c*f)] + 2*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(d*e)
 + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e +
f*x))/(d*e - c*f)] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x)...
```

3.107.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2954, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{e+fx}dx$$

$$\Big\downarrow \text{2954}$$

$$-\left((bc-ad)\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)\left(de-cf-\frac{(be-af)(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}\right)$$

$$\Big\downarrow \text{2804}$$

$$-\left((bc-ad)\int \left(\frac{b\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(bc-ad)f\left(\frac{b(c+dx)}{a+bx}-d\right)}+\frac{(be-af)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(bc-ad)f\left(de-cf-\frac{(be-af)(c+dx)}{a+bx}\right)}\right)d\frac{c+dx}{a+bx}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\left((bc-ad)\left(\frac{2\,\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f(bc-ad)}+\frac{2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f(bc-ad)}-\frac{2\,\text{PolyLog}\left(2,\frac{(be-af)(}{(de-cf)(}\right.}{f(b}\right.\right.$$

input
```
Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),x]
```

output
```
-((b*c - a*d)*((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log
[1 - (b*(c + d*x))/(d*(a + b*x))])/((b*c - a*d)*f) - (Log[((b*e - a*f)*(c
+ d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a*f)*(c + d*x))/((d*e -
 c*f)*(a + b*x))])/((b*c - a*d)*f) + (2*Log[((b*e - a*f)*(c + d*x))/((d*e
- c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/((b*c - a*d)*f
) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b
*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((b*c - a*d)*f) - (2*PolyLo
g[3, (b*(c + d*x))/(d*(a + b*x))])/((b*c - a*d)*f) + (2*PolyLog[3, ((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((b*c - a*d)*f)))
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(321) = 642$.

Time = 3.18 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.52

method	result
derivativedivides	$(af-be)(ad-cb)\left(\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-}{b(cf-de)(bx-}\right.}{(af-be)f}\right.$
default	$(af-be)(ad-cb)\left(\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-}{b(cf-de)(bx-}\right.}{(af-be)f}\right.$
risch	Expression too large to display

input
```
int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x,method=_RETURNVE
RBOSE)
```

output
```
(a*f-b*e)*(a*d-b*c)*(1/(a*f-b*e)/f/(a*d-b*c)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c
*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln((a*f-b*e)*(a*d-b*c)/b/(c*f-d
*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b+1)+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*
e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-
d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)-2*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(
c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))-b*(c*f-d*e)/(b*c*f-b*d*e)/(a*f-
b*e)/f/(a*d-b*c)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*
(a*f-b*e)/b)^2*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(
c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*
f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b
*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))-
2*polylog(3,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e
)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))))
```

3.107.5 Fricas [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{fx+e}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorith
m="fricas")
```

output
```
integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
 b*c*f)*x))^2/(f*x + e), x)
```

3.107.6 Sympy [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$$
$$= \int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{e+fx}\,dx$$

input
```
integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(f*x+e),x)
```

output
```
Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
 + a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x)
 + b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(e + f*x), x)
```

3.107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: Memory limit reached. Please j
ump to an outer pointer, quit program and enlarge thememory limits before
executing the program again.
```

3.107.8 Giac [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{fx+e}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorith
m="giac")
```

output
```
integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/(f*x + e),
x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{e+f\,x}\,dx$$

input
```
int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x),x)
```

output
```
int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x), x)
```

3.108 $\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$

3.108.1 Optimal result

Integrand size = 65, antiderivative size = 433

$$\begin{aligned}\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx =& -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} \\ & -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} \\ & +\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} \\ & -\frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad} \\ & +\frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad} \\ & +\frac{\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}-\frac{\operatorname{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}\end{aligned}$$

output
```
-1/2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(
-a*d+b*c)-1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(b*(f*x+e)/(-a
*f+b*e))/(-a*d+b*c)+1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(
-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)-ln((-a*f+b*e)*(d*x+c)/(-c
*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)+ln((-a*f+b*e)*(
d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a)
)/(-a*d+b*c)+polylog(3,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)-polylog(3,(-a*f+b*e
)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

3.108.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1855 vs. $2(433) = 866$.

Time = 0.44 (sec) , antiderivative size = 1855, normalized size of antiderivative = 4.28

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e
+ f*x))/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

output
```
(2*Log[c/d + x]*Log[e/f + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a/b
 + x]*Log[e/f + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*(Log[a + b*x] - Log[
c + d*x])*(Log[a/b + x] - Log[c/d + x] + Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))])*(Log[e/f + x] - Log[(b*(e + f*x))/(b*e - a*f)]) + (Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[(f*(a + b*x))/(-(b*e) + a*f)])*Log[(
b*(e + f*x))/(b*e - a*f)]*(-2*Log[c/d + x] + Log[(b*(e + f*x))/(b*e - a*f)
]) + Log[a/b + x]^2*(-Log[e/f + x] + Log[(b*(e + f*x))/(b*e - a*f)]) + (Lo
g[(b*(c + d*x))/(b*c - a*d)] - Log[(f*(c + d*x))/(-(d*e) + c*f)])*Log[(d*(
e + f*x))/(d*e - c*f)]*(-2*Log[a/b + x] + Log[(d*(e + f*x))/(d*e - c*f)])
+ Log[c/d + x]^2*(-Log[e/f + x] + Log[(d*(e + f*x))/(d*e - c*f)]) + 2*(-Lo
g[(b*(c + d*x))/(b*c - a*d)] + Log[(f*(c + d*x))/(-(d*e) + c*f)])*Log[(d*(
e + f*x))/(d*e - c*f)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*
x))] + (Log[(-(b*e) + a*f)/(f*(a + b*x))] + Log[(b*(c + d*x))/(b*c - a*d)]
 - Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*Log[((-(b*c) + a*
d)*(e + f*x))/((d*e - c*f)*(a + b*x))]^2 + 2*(-Log[(d*(a + b*x))/(-(b*c) +
 a*d)] + Log[(f*(a + b*x))/(-(b*e) + a*f)])*Log[(b*(e + f*x))/(b*e - a*f)]
*Log[((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))] + (Log[(d*(a + b*x))
/(-(b*c) + a*d)] + Log[(-(d*e) + c*f)/(f*(c + d*x))] - Log[((d*e - c*f)*(a
 + b*x))/((b*e - a*f)*(c + d*x))])*Log[((b*c - a*d)*(e + f*x))/((b*e - a*f
)*(c + d*x))]^2 + 2*(Log[e/f + x] - Log[((-(b*c) + a*d)*(e + f*x))/((d*...
```

3.108.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2989, 2954, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{b(e+fx)}{be-af}\right) \log \left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \text{2989}$$

$$\frac{f \int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}dx}{2(bc-ad)}-\frac{\log \left(\frac{b(e+fx)}{be-af}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}$$

$$\Big\downarrow \text{2954}$$

$$-\frac{1}{2}f\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)\left(de-cf-\frac{(be-af)(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}-\frac{\log \left(\frac{b(e+fx)}{be-af}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}$$

$$\Big\downarrow \text{2804}$$

$$-\frac{1}{2} f \int \left(\frac{b \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(bc-ad) f \left(\frac{b(c+dx)}{a+bx}-d\right)}+\frac{(be-af) \log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(bc-ad) f \left(de-cf-\frac{(be-af)(c+dx)}{a+bx}\right)}\right) d\frac{c+dx}{a+bx}-\frac{\log\left(\frac{b(e+fx)}{be-af}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2} f \left(\frac{2 \operatorname{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f(bc-ad)}+\frac{2 \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f(bc-ad)}-\frac{2 \operatorname{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) \text{lo}}{f(bc-ad)}\right.$$

$$\frac{\log\left(\frac{b(e+fx)}{be-af}\right) \log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}$$

input
```
Int[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x)
)/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

output
```
-1/2*(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f
*x))/(b*e - a*f)])/(b*c - a*d) - (f*((Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/((b*c - a*d)*f) -
 (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((b*c - a*d)*f) + (2*Log[((b*e -
a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a +
b*x))])/((b*c - a*d)*f) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a +
 b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((b*c
 - a*d)*f) - (2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/((b*c - a*d)*f) +
 (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((b*c - a
*d)*f)))/2
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.108. $\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

rule 2989
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Simp[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d)))   Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &
& EqQ[p + q, 0] && NeQ[s, -1]
```

3.108.4 Maple [F]

$$\int \frac{\ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)\ln\left(\frac{b(fx+e)}{-af+be}\right)}{(bx+a)\,(dx+c)}dx$$

input
```
int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*
x+a)/(d*x+c),x)
```

output
```
int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*
x+a)/(d*x+c),x)
```

3.108.5 Fricas [F]

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(fx+e)b}{be-af}\right)\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b
*e))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
 b*c*f)*x))*log((b*f*x + b*e)/(b*e - a*f))/(b*d*x^2 + a*c + (b*c + a*d)*x)
, x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e
))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b
*e))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: Memory limit reached. Please j
ump to an outer pointer, quit program and enlarge thememory limits before
executing the program again.
```

3.108.8 Giac [F]

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(fx+e)b}{be-af}\right)\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b
*e))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log((f*x + e)*b/(b*e - a*f))*log((b*e - a*f)*(d*x + c)/((d*e - c
*f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(-\frac{b\,(e+f\,x)}{a\,f-b\,e}\right)\,\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d
*e)*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

output
```
int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d
*e)*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```